FAIRCHILD'S SOLUTION BOOK

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J. T. Fairchild.

A

COMPLETE AND PRACTICAL

SOLUTION BOOK

FOR THE

Common School Teacher

BY

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DEDICATED

TO MY INSTRUCTOR,

MRS. EVA MAGLOTT, A. M.,

PROFESSOR OF MATHEMATICS

IN THE

OHIO NORMAL UNIVERSITY.

PREFACE.

An attempt is made in this small volume to bring the science of Arithmetic and Geometry directly to the comprehension of the learner, and to accomplish this end it is necessary to give a complete solution to the problems. This volume is the outgrowth of a long course of study and long experience in teaching. The experience of the author teaches him that most books of this class now in use treat the subject in too brief or too difficult a manner. Most of our solution books treat on problems that are out of the reach of common school teachers. It is the aim of the author to give them the solution of problems that they can comprehend.

More time is spent on the study of Arithmetic in our schools than on any other one branch of study; and still the results are no better. It has been our aim to give solutions to every peculiar problem that usually gives our teachers trouble, and if we have omitted any they will be given space in later editions of this work.

The method employed for the solution of problems in Stocks and Bonds and Commission, I received from Prof. Ed. M. Mills while he was my instructor. In my opinion, it is the best method. Many hints on the subject of mensuration were received from him. I also acknowledge my indebtedness to Prof. G. B. M. Zerr and Prof. J. C. Gregg for aiding me in some difficult work in trigonometrical *regula falsi*, or Double Position.

I have taken problems from the *School Visitor*, from the *Teachers' Review*, published by Stonebrook & Maurer, from the *American Mathematical Monthly*, published by Professors B. F. Finkel and J. M. Colaw, by the permission of these eminent mathematicians. I submit this book to my fellow teachers. Any correction will be thankfully received, knowing that it is almost impossible to put out a book whose first edition is free from error.

Crawfis College, Ohio, August, 1898.

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FAIRCHILD'S

SOLUTION BOOK.

CHAPTER I.

PROPORTION DISCUSSED.

1. There are few subjects in Arithmetic more easily mastered and more valuable in developing the reasoning powers than **Proportion**, and at the same time few students are in love with this subject. Students should put more time on this fascinating part of Arithmetic.

2. Ratio is the quotient arising from dividing one quantity by another of the same kind.

Thus, the ratio of 8 apples to 2 apples is 8 apples $\div$ 2 apples $=$ 4. We may inquire how many times 2 apples are contained times in 8 apples, but not how many times 2 peaches are contained in 8 apples, because no number of times one will produce the other, *i. e.*, no one of these quantities can be multiplied and the other obtained. $\therefore$ There is no ratio of 8 apples to 2 peaches.

3. Proportion is the equality of two ratios.

The ratio of 8 apples to 2 apples is 4, or $\frac{8 \text{ apples}}{2 \text{ apples}} = 4$.

The ratio of 12 peaches to 3 peaches is 4, or $\frac{12 \text{ peaches}}{3 \text{ peaches}} = 4$.

Then, by definition, $\frac{8 \text{ apples}}{2 \text{ apples}} = \frac{12 \text{ peaches}}{3 \text{ peaches}}$.

Or, 8 apples : 2 apples :: 12 peaches : 3 peaches. The first and fourth terms are the *extremes*. The second and third are the *means*. Now, in any problem in Simple Proportion, if three of these four quantities be given, the fourth can be found. It is evident that if we have the two means and one extreme, we can find the other extreme by dividing the product of the means by the given extreme. In the same manner we can find the other mean if we have the two extremes and one of the means.

4. In solving problems by Simple Proportion, we must always have the following relations:

Greater : Less :: Greater : Less
or, Less : Greater :: Less : Greater

Take for the third term the number which is of the same kind as the answer sought. That is, if peaches are required, put for the third term the given number of peaches. After this is done, we must ascertain whether more or less of the required quantity is necessary to fulfill the conditions of the problem; if less, we have, greater : less :: greater : less; but if greater, less : greater :: less : greater.

6. We shall illustrate the method.

If 28 men mow a field of grass in 12 days, how many men will be required to mow it in 8 days?

If 28 men mow a field in 12 days, will it take a greater or less number of men to mow it in 8 days? Answer, greater; therefore, we must have:

Less : Greater :: Less : Greater
8 days : 12 days :: 28 : x = 42 men.

7. The author believes this to be the simplest method of illustrating **Compound Proportion.**

If 28 men dig a ditch 120 rods long, 15 feet wide and 12 feet deep, how many men will dig a ditch 360 rods long, 9 feet wide and 10 feet deep?

If 28 men dig a ditch 120 rods long, will it take a greater or less number of men to dig a ditch 360 rods long? Answer, greater. Then we have:

Less	: Greater	:: Less	: Greater
120 rods	: 360 rods	:: 24 men	: x

If 28 men dig a ditch 15 feet wide, will it take a greater or less number of men to dig a ditch 9 feet wide? Answer, less.

Greater	: Less	:: Greater	: Less
15 feet	: 9 feet	:: 28 men	: x

If 28 men dig a ditch 12 feet deep, will it take a greater or less number of men to dig a ditch 10 feet deep? Answer, less.

Greater	: Less	:: Greater	: Less
12 feet	: 10 feet	:: 28 men	: x

Now write out the three simple proportions in the form of a compound proportion. Thus:

$$\left.\begin{matrix} 120 \text{ rods} : 360 \text{ rods} \\ 15 \text{ feet} : 9 \text{ feet} \\ 12 \text{ feet} : 10 \text{ feet} \end{matrix}\right\} :: 28 \text{ men} : x = 42 \text{ men.}$$

A few lessons given on the above method will fix Proportion firmly in the student's mind.

CHAPTER II.

ANALYSIS.

8. **Arithmetical Analysis** is the process of developing problems by comparison of them through their relations to the unit.

NOTE.—Nothing is of more importance than Analysis. The reason why Arithmetic is so difficult to many students is that they do not pay proper attention to arithmetical analysis.

9. $\frac{2}{3} \div \frac{3}{4}$. Demonstrate.

One fourth is contained in one unit 4 times, but 3 fourths are 3 times as much. Then it is plain that it is contained in a unit $\frac{1}{3}$ of 4 times, or $\frac{4}{3}$ of 1 time, and in $\frac{2}{3}$ of a unit it is contained $\frac{4}{3}$ of $\frac{2}{3}$, or $\frac{8}{9}$ of 1 time.

Rule.—*Invert the divisor and multiply the fractions together.*

PROBLEM 1.

What number is that, to which if you add $\frac{3}{7}$ of itself the sum will be 20?

Solution.

(1) $\frac{7}{7}$ = the required number.
(2) $\frac{7}{7} + \frac{3}{7} = \frac{10}{7}$.
(3) $\frac{10}{7} = 20$.
(4) $\frac{1}{7} = \frac{1}{10}$ of $20 = 2$.
(5) $\frac{7}{7} = 7$ times $2 = 14$.

$\therefore$ 14 = required number.

PROBLEM 2.

If $\frac{2}{3}$ of a yard cost \$$\frac{2}{7}$, what will $\frac{3}{3}$ of a yard cost?

Solution.

(1) If $\frac{2}{3}$ of a yard cost \$$\frac{2}{7}$, $\frac{1}{3}$ of a yard will cost $\frac{1}{2}$ of $\frac{2}{7}$, or \$$\frac{1}{7}$.
(2) $\frac{3}{3}$, or a yard, will cost 3 times $\frac{1}{7}$ = \$$\frac{3}{7}$.

(3) Hence, $\frac{2}{5}$ of a yard will cost $\frac{2}{5}$ of $\$\frac{3}{7} = \$\frac{6}{35}$.

$\therefore \frac{2}{5}$ of a yard cost $\$\frac{6}{35}$.

PROBLEM 3.

A and B together have \$930; $\frac{3}{4}$ of A's money equals $\frac{4}{5}$ of B's: how much has each?

Solution.

(1) $\frac{3}{4}$ A's $= \frac{4}{5}$ B's.
(2) $\frac{1}{4}$ A's $= \frac{1}{3}$ of $\frac{4}{5}$ B's $= \frac{4}{15}$ B's.
(3) $\frac{4}{4}$ A's $=$ 4 times $\frac{4}{15}$ B's $= \frac{16}{15}$ B's, value of A's in terms of B's.
(4) $\frac{15}{15} =$ B's money.
(5) $\frac{16}{15} =$ A's money.
(6) $\frac{15}{15} + \frac{16}{15} = \frac{31}{15} =$ what both have.
(7) \$930 = what both have.
(8) $\frac{31}{15} = \$930$.
(9) $\frac{1}{15} = \frac{1}{31}$ of \$930 = \$30.
(10) $\frac{15}{15} =$ 15 times \$30 = \$450.
(11) $\frac{16}{15} =$ 16 times \$30 = \$480.

$\therefore$ \$450 = A's money; \$480 = B's money.

PROBLEM 4.

A man owning $\frac{4}{15}$ of a farm sold $\frac{1}{2}$ of his share for \$1840: what was the farm worth at that rate?

Solution.

(1) $\frac{1}{2}$ of $\frac{4}{15} = \frac{2}{15} =$ part sold.
(2) \$1840 = value of part sold.
(3) $\frac{2}{15} = \$1840$.
(4) $\frac{1}{15} = \frac{1}{2}$ of \$1840 = \$920.
(5) $\frac{15}{15} =$ 15 times \$920 = \$13800.

$\therefore$ The farm is worth \$13800.

PROBLEM 5.

\$20 is $\frac{2}{5}$ of the cost of a barrel of whisky: what did it cost?

Solution.

(1) $\frac{5}{5} =$ the cost of the whisky per barrel.
(2) $\frac{2}{5}$ of the cost = \$20.
(3) $\frac{1}{5}$ of the cost $= \frac{1}{2}$ of \$20 = \$10.
(4) $\frac{5}{5}$ of the cost = 5 times \$10 = \$50.

$\therefore$ \$50 = cost of whisky.

PROBLEM 6.

I spent $\frac{2}{3}$ of my money and then received \$36; after losing $\frac{3}{4}$ of what I then had, I had left \$2 less than I had at first: what sum had I at first?

Solution.

(1) $\frac{12}{12}$ = my money.
(2) $\frac{2}{3}$ of $\frac{12}{12}$ my money = $\frac{2}{3}$ of my money.
(3) $\frac{12}{12}$ my money — $\frac{2}{3}$ of my money = $\frac{1}{3}$ of my money.
(4) $\frac{1}{3}$ + \$36 = what I had by second condition.
(5) $\frac{3}{4}$ of ($\frac{1}{3}$ of my money + \$36) = $\frac{3}{12}$ of my money + \$27 = what I lost.
(6) ($\frac{1}{3}$ of my money + \$36) — ($\frac{3}{12}$ of my money + \$27) = $\frac{1}{12}$ of my money + \$9. This is what I had left, but this is \$2 less than what I had at first.
(7) ∴ $\frac{12}{12}$ my money = ($\frac{1}{12}$ of my money + \$9) + \$2.
(8) Hence, $\frac{11}{12}$ = \$11.
(9) $\frac{1}{12} = \frac{1}{11}$ of \$11 = \$1.
(10) $\frac{12}{12}$ my money = 12 times \$1 = \$12.
∴ \$12 = the money I had at first.

PROBLEM 7.

Nine men can do a piece of work in $8\frac{1}{3}$ days: how many days may 3 men remain away and yet finish the work in the same time by bringing 5 more with them?

Solution.

(1) $8\frac{1}{3}$ days = time in which 9 men can do the work.
(2) $8\frac{1}{3}$ days × 9 = 75, the number of days work.
(3) $8\frac{1}{3}$ days × 6 = 50, the number of days work that the 6 men do who work all the time.
(4) 75 days — 50 days = 25 days.
(5) Hence, the 8 men do 25 days work in 25 days ÷ 8 = $3\frac{1}{8}$ days.
(6) $8\frac{1}{3}$ days — $3\frac{1}{8}$ days = $5\frac{5}{24}$ days.
∴ They can stay away $5\frac{5}{24}$ days.

PROBLEM 8.

If $\frac{2}{3}$ of 6 be 3, what will $\frac{1}{2}$ of 20 be?

Solution.

(1) $\frac{2}{3}$ of 6 = 4.
(2) $\frac{1}{2}$ of 20 = 10. By the condition of the problem,
(3) 4 = 3.
(4) 1 = $\frac{3}{4}$.

(5) 10 = 10 times $\frac{3}{4}$, or $7\frac{1}{2}$.

∴

PROBLEM 9.

John, William and Frank can do a piece of work in 30 days. If John does $\frac{3}{4}$ as much as William and William $\frac{2}{3}$ as much as Frank, how long will it take each to do it?

Solution.

(1) $\frac{3}{3}$ = amount of work that Frank does.
(2) $\frac{2}{3}$ of $\frac{3}{3}$ Frank's work = $\frac{2}{3}$ of William's work.
(3) $\frac{3}{4}$ of $\frac{2}{3}$ William's work = $\frac{1}{2}$ John's work.
(4) Hence, all do $\frac{3}{3} + \frac{2}{3} + \frac{1}{2} = \frac{13}{6}$, or $2\frac{1}{2}$ times Frank's work.
(5) In 1 day Frank does $\frac{6}{13}$ of $\frac{1}{30} = \frac{1}{65}$ of the work.
(6) $\frac{65}{65}$ is what Frank can do in $\frac{65}{65} \div \frac{1}{65}$, or 65 days.
(7) $\frac{2}{3}$ William's work = 65 days.
(8) $\frac{1}{3} = \frac{1}{2}$ of 65 = $32\frac{1}{2}$ days.
(9) $\frac{3}{3}$ = 3 times $32\frac{1}{2}$ = $97\frac{1}{2}$ days.
(10) $\frac{1}{2}$ John's work = 2 times 65 days = 130 days.

∴ John can do the work in 130 days; William in $97\frac{1}{2}$ days; Frank in 65 days.

NOTE.—We can readily see that Frank does 2 times as much as John and $\frac{1}{2}$ times as much as William.

PROBLEM 10.

A pole, the length of which is 39 feet, is in the air and water; $\frac{9}{8}$ of the length in the air + 6 feet equals $1\frac{1}{2}$ times the length in the water: find the length of each part.

Solution.

(1) $\frac{4}{4}$ = length of the part in the air.
(2) $\frac{9}{8}$ of $\frac{4}{4}$ the length of the pole in the air = $\frac{9}{8}$ of the length of the pole in the air.
(3) $\frac{9}{8}$ + 6 feet = $1\frac{1}{2}$, or $\frac{3}{4}$ the length in the air + 4 feet = the length in the water.
(4) ∴ $\frac{4}{4} + \frac{3}{4} + 4$ = length of the pole.
(5) $\frac{4}{4} + \frac{3}{4} + 4$ = 39 feet.
(6) $\frac{7}{4}$ = 35 feet.
(7) $\frac{1}{4} = \frac{1}{7}$ of 35 feet = 5 feet.
(8) $\frac{4}{4}$ = 4 times 5 feet = 20 feet.
(9) 39 feet — 20 feet = 19 feet.

∴ 20 feet is the length in the air, and 19 feet the length in the water.

PROBLEM 11.

A and B can do a piece of work in 20 days, A and C in 15 days, B and C in 12 days. In what time can all do it? Each?

Solution.

(1) 20 days = time it takes A and B to do the work.
(2) $\frac{1}{20}$ = part they do in 1 day.
(3) 15 days = time it takes A and C to do the work.
(4) $\frac{1}{15}$ = part they do in 1 day.
(5) 12 days = time it takes B and C to do the work.
(6) $\frac{1}{12}$ = part they do in 1 day.
(7) Then, $\frac{1}{20}+\frac{1}{15}+\frac{1}{12}=\frac{1}{5}$ = part A and B, A and C, and B and C do in 1 day = twice the work A, B and C do in 1 day.
(8) $\frac{1}{10}=\frac{1}{2}$ of $\frac{1}{5}$ = part A, B and C do in 1 day.
(9) $\frac{10}{10}$ = part A, B and C do in $\frac{10}{10}\div\frac{1}{10}$, or 10 days.
(10) $\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$ = part A, B and C do in 1 day — part A and B do in 1 day = part C does in 1 day.
(11) $\frac{20}{20}$ = part C does in $\frac{20}{20}\div\frac{1}{20}$, or 20 days.
(12) $\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$ = part A, B and C do in 1 day — part A and C do in 1 day = part B does in 1 day.
(13) $\frac{30}{30}$ = part A does in $\frac{30}{30}\div\frac{1}{30}$ = 30 days.
(14) $\frac{1}{10}-\frac{1}{12}=\frac{1}{60}$ = part A, B and C do in 1 day — part B and C do in 1 day = part A does in 1 day.
(15) $\frac{60}{60}$ = part A does in $\frac{60}{60}\div\frac{1}{60}$, or 60 days.

∴ A, B and C do the work in 10 days, A in 60 days, B in 30 days, and C in 20 days.

PROBLEM 12.

A man and a boy can do a certain work in 10 hours; if the boy rests $2\frac{5}{8}$ hours, it takes them $11\frac{1}{6}$ hours: in what time can each do the work?

Solution.

(1) $11\frac{1}{6}$ hr. — $2\frac{5}{8}$ hr. = $8\frac{13}{24}$ hr. = time they both work together by second condition of the problem.
(2) 10 hr. = time in which they both do the work.
(3) $\frac{1}{10}$ = part they do in 1 hour.
(4) $8\frac{13}{24}\div 10=\frac{205}{240}$ = part they do in $8\frac{13}{24}$ hours.
(5) $\frac{240}{240}-\frac{205}{240}=\frac{35}{240}$ = part the man does while the boy rests in $2\frac{5}{8}$ hours.
(6) $\frac{35}{240}\div 2\frac{5}{8}=\frac{1}{18}$, part the man does in 1 hour.
(7) $\frac{18}{18}$ = part the man can do in $\frac{18}{18}\div 18$, or 18 hours.
(8) $\frac{1}{10}-\frac{1}{18}=\frac{8}{180}$, part the boy does in 1 hour.

(9) $\frac{180}{180}$ = part the boy does in $\frac{180}{180} \div \frac{8}{180}$, or $22\frac{1}{2}$ hours.

∴ It will take the man 18 hours and the boy $22\frac{1}{2}$ hours.

PROBLEM 13.

A can do as much work in 2 days as B can in $2\frac{1}{2}$; and B can do as much in 2 days as C can do in $2\frac{1}{2}$. They work together and earn $91.50: what amount do they each receive?

Solution.

(1) Let 20 units or parts represent A's work in 2 days.
(2) In 1 day A can do $\frac{1}{2}$ of the same work, of which B can do $\frac{2}{5}$ in a day.
(3) ∴ $\frac{1}{2}$ of 20 units = 10 units, part that A does in 1 day.
(4) $\frac{2}{5}$ of 20 units = 8 units, part that B does in 1 day.
(5) In 1 day B can do $\frac{1}{2}$ of the same work, of which C can do $\frac{2}{5}$ in a day.
(6) 16 units = part that B can do in 2 days.
(7) $\frac{2}{5}$ of 16 = $6\frac{2}{5}$, part that C can do in a day.
(8) ∴ The parts are 25, 20 and 16; and A receives $\frac{25}{61}$ of $91.50, or $37.50; B, $\frac{20}{61}$ of $91.50, or $39; and C, $\frac{16}{61}$ of $91.50, or $24.

∴ A receives $37.50; B, $30; C, $24.

PROBLEM 14.

If $\frac{2}{3}$ of A's turkeys, plus $\frac{3}{4}$ of B's, equals 900, how many turkeys has each, provided $\frac{3}{4}$ of B's number is twice $\frac{2}{3}$ of A's number?

Solution.

(1) $\frac{3}{3}$ = A's number of turkeys.
(2) $\frac{2}{3}$ of $\frac{3}{3} = \frac{2}{3} \times 2 = \frac{4}{3}$, twice A's number of turkeys = $\frac{3}{4}$ of B's number of turkeys.
(3) $\frac{2}{3}$ of A's + $\frac{4}{3}$ or $\frac{3}{4}$ of B's = $\frac{6}{3}$ = 900 turkeys.
(4) $\frac{1}{3} = \frac{1}{6}$ of 900 = 150 turkeys.
(5) $\frac{3}{3}$ = 3 times 150 turkeys = 450, A's turkeys.
(6) $\frac{2}{3}$ of 450, A's number = 300 turkeys.
(7) $\frac{3}{4}$ of B's turkeys = 300 × 2 = 600 turkeys.
(8) $\frac{1}{4} = \frac{1}{3}$ of 600 = 200 turkeys.
(9) $\frac{4}{4}$ = 4 times 200 = 800 turkeys.

∴ A has 450 turkeys; B, 800.

PROBLEM 15.

A man after doing $\frac{3}{5}$ of a piece of work in 30 days engages an an assistant and both together complete it in 6 days: in what time could the assistant do it alone?

Solution.

(1) $\frac{5}{5}$ = the work.

(2) $\frac{3}{5}$ = the part the man does in 30 days.

(3) $\frac{3}{5}$ = 30 days.

(4) $\frac{1}{5} = \frac{1}{3}$ of 30 days = 10 days.

(5) $\frac{5}{5}$ = 5 times 10, or 50 days, the time in which the man can do the whole work.

(6) $\frac{5}{5} - \frac{3}{5} = \frac{2}{5}$, the part the man and assistant do in 6 days.

(7) $\frac{2}{5}$ = 6 days.

(8) $\frac{1}{5} = \frac{1}{2}$ of 6, or 3 days.

(9) $\frac{5}{5}$ = 5 times 3, or 15 days, the time in which the man and assistant can do the whole work.

(10) $\frac{1}{50}$ = the part the man does in 1 day.

(11) $\frac{1}{15}$ = the part the man and assistant do in 1 day.

(12) $\frac{150}{150}$ = what the assistant does in $\frac{150}{150} \div \frac{7}{150} = 21\frac{3}{7}$ days.

∴ The assistant can do the work alone in $21\frac{3}{7}$ days.

PROBLEM 16.

Six men can do a piece of work in $4\frac{1}{3}$ days; after working two days, how many men must join them to complete it in $3\frac{2}{5}$ days from the time of beginning the work?

Solution.

(1) $4\frac{1}{3}$ days = time in which 6 men can do the work.

(2) $4\frac{1}{3}$ days × 6 = 26 days, the time 1 man can do it.

(3) 2 days × 6 = 12 days, time they all work from the start.

(4) ∴ 26 days — 12 days, or 14 days work remain to be done in $3\frac{2}{5}$ days — 2 days = $1\frac{2}{5}$ days.

(5) 14 days ÷ $1\frac{2}{5}$ = 10, the number of men required to finish the work.

(6) Hence, 10 men — 6 men = 4 men.

∴ 4 men must join them.

PROBLEM 17.

If 3 times Susan's age is $\frac{3}{8}$ of Daisy's age, in how many years will Daisy be just twice as old as Susan? (*Raub's Arith., p. 323.*)

Solution.

(1) Let D = Daisy's present age, and S = Susan's.

(2) By the condition of the problem, D = 8S.

(3) ∴ 2S + the required number of years = D, or 8S.

(4) Hence, the required number of years must be 6S, or 6 times Susan's present age.

(5) It is no matter what Susan's present age may be, when it is increased by 6 times itself it will be $\frac{1}{2}$ of Daisy's.

(6) Suppose Susan's age is 2, Daisy's 16.

(7) Susan's age added to 6 times itself = 14 years; this is $\frac{1}{2}$ of Daisy's, the latter being 16 + 12, or 28 years.

NOTE.—Many teachers stumble over this problem.

PROBLEM 18.

The age of Jane is twice the age of Mary, and $\frac{3}{5}$ of Mary's age plus 44 years equal $2\frac{1}{2}$ times the age of Jane: what is the age of each?

Solution.

(1) $\frac{5}{5}$ = Mary's age.

(2) $\frac{10}{5}$ = Jane's age.

(3) $\frac{3}{5}$ of $\frac{5}{5}$ Mary's age = $\frac{3}{5}$ of Mary's age.

(4) $\frac{3}{5}$ of Mary's age + 44 years = $2\frac{1}{2}$ times the age of Jane. $(2\frac{1}{2} \times \frac{10}{5} = \frac{25}{5})$

(5) $\therefore \frac{3}{5} + 44 = \frac{25}{5}$.

(6) Hence, $\frac{22}{5}$ = 44 years.

(7) $\frac{1}{5} = \frac{1}{22}$ of 44 = 2 years.

(8) $\therefore \frac{5}{5}$ Mary's age = 5 times 2 years, or 10 years.

(9) 10 years × 2 = 20 years, the age of Jane.

$\therefore$ Mary is 10 years old and Jane 20.

PROBLEM 19.

C's age at A's birth was $5\frac{1}{2}$ times B's, and now is the sum of A's and B's, but if A were now 3 years younger and B 4 years older, A's age would be $\frac{3}{4}$ of B's: find their ages.

Solution.

(1) Since C's age at A's birth + A's present age = A's present age + B's;

(2) $\therefore$ C's age at A's birth = B's present age. By the second condition, A's age + 3 = $\frac{3}{4}$ (B's + 4), from which A's age = $\frac{3}{4}$ B's age + 6 years.

(3) Now, the difference between A's and B's present age = B's at the birth of A.

(4) $\therefore \frac{1}{4}$ B's present age − 6 years = B's age at A's birth, and $5\frac{1}{2}$ ($\frac{1}{4}$ B's age − 6 years) = $\frac{11}{8}$ B's present age, from which $\frac{11}{8}$ B's age − 33 years = B's age, or 88 years.

(5) $\therefore$ C's age was 88 years at A's birth, B's 88 ÷ $5\frac{1}{2}$ = 16 years; and A's present age is 88 — 16 = 72 years, B's 88, and C's 88 + 72 = 160 years.

$\therefore$ A is 72 years old, B is 88, and C 160.

PROBLEM 20.

A man engaging in trade lost $\frac{2}{5}$ of his money invested, after which he gained $840, and then had $3680: how much did he lose?

Solution.

(1) $\frac{5}{5}$ = what he had at first.
(2) $\frac{2}{5}$ = part lost.
(3) $\frac{5}{5} - \frac{2}{5} = \frac{3}{5}$, part left.
(4) $840 = amount gained afterward.
(5) $\frac{3}{5}$ + $840 = amount he had after gaining $840.
(6) $3680 = amount he had after gaining $840.
(7) $\frac{3}{5}$ + $840 = $3680.
(8) $\frac{3}{5}$ = $2840.
(9) $\frac{1}{5} = \frac{1}{3}$ of $2840 = 946\frac{2}{3}$.
(10) $\frac{5}{5}$ = 5 times 946\frac{2}{3}$ = 4733\frac{1}{3}$, his money.
(11) $\frac{2}{5}$ of 4733\frac{1}{3}$ = 1893\frac{1}{3}$.
∴ He lost 1893\frac{1}{3}$.

PROBLEM 21.

John spent $\frac{2}{3}$ of $\frac{4}{5}$ more than half his money, and had $210 left: how much had he at first?

Solution.

(1) $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$.
(2) $\frac{2}{2}$ = his money.
(3) $\frac{1}{2}$ = half of his money.
(4) $\frac{8}{15}$ of $\frac{1}{2} = \frac{4}{15}$.
(5) $\frac{1}{2} + \frac{4}{15} = \frac{23}{30}$, part spent.
(6) $\frac{30}{30} - \frac{23}{30} = \frac{7}{30}$, part left.
(7) $\frac{7}{30}$ = $210.
(8) $\frac{1}{30} = \frac{1}{7}$ of $210 = $30.
(9) $\frac{30}{30}$ = 30 times $30 = $900.
∴ $900 = what he had at first.

NOTE.—$\frac{8}{15}$ more than $\frac{1}{2}$ is $\frac{8}{15}$ of $\frac{1}{2}$ plus $\frac{1}{2}$.

PROBLEM 22.

If $\frac{3}{4}$ of the time past noon, plus $3\frac{1}{2}$ hours, equals $\frac{5}{8}$ of the time to midnight, plus $2\frac{1}{8}$ hours, what is the time?

Solution.

(1) $\frac{4}{4}$ = time past noon.
(2) 12 hours = time from noon to midnight.
(3) $\frac{3}{4}$ of $\frac{4}{4}$ time past noon = $\frac{3}{4}$ of time past noon.

(4) 12 hours — $\frac{4}{4}$ time past noon = time to midnight.
(5) $\frac{3}{4}$ of time past noon + $3\frac{1}{2}$ hours = $\frac{2}{3}$ of (12 hours — $\frac{4}{4}$ time past noon) + $2\frac{1}{3}$ hours.
(6) ∴ $\frac{3}{4}$ of time past noon + $3\frac{1}{2}$ hours = 8 hours — $\frac{8}{12}$ of time past noon + $2\frac{1}{3}$ hours.
(7) $\frac{17}{12}$ of time past noon = $\frac{41}{6}$ hours.
(8) $\frac{1}{12}$ = $\frac{1}{17}$ of $\frac{41}{6}$, or $\frac{41}{102}$ hours.
(9) $\frac{12}{12}$ = 12 times $\frac{41}{102}$ = $4\frac{14}{17}$ hours.

∴ $4\frac{14}{17}$ hours, or 4 hr. 49 min. $24\frac{12}{17}$ sec. is the time past noon.

PROBLEM 23.

A man spent $\frac{1}{5}$ more than $\frac{3}{4}$ of his money and had \$500 left: how much did he have?

Solution.

(1) $\frac{4}{4}$ = his money.
(2) $\frac{1}{5}$ of $\frac{3}{4}$ = $\frac{3}{20}$.
(3) $\frac{3}{4}$ + $\frac{3}{20}$ = $\frac{18}{20}$, or $\frac{9}{10}$, the amount spent.
(4) $\frac{10}{10}$ — $\frac{9}{10}$ = $\frac{1}{10}$, the amount left.
(5) $\frac{1}{10}$ = \$500.
(6) $\frac{10}{10}$ = 10 times \$500 = \$5000.

∴ He had \$5000.

PROBLEM 24.

Ten years ago the age of John was $\frac{3}{4}$ of the age of Henry, and ten years hence the age of John will be $\frac{5}{6}$ of the age of Henry: find the age of each.

Solution.

(1) $\frac{4}{4}$ = Henry's age 10 years ago.
(2) $\frac{3}{4}$ = John's age 10 years ago.
(3) $\frac{4}{4}$ + 20 years = Henry's age 10 years hence.
(4) $\frac{3}{4}$ + 20 years = John's age 10 years hence.
(5) $\frac{5}{6}$ of ($\frac{4}{4}$ + 20 years) = $\frac{5}{6}$ Henry's + $16\frac{2}{3}$ years.
(6) $\frac{5}{6}$ Henry's + $16\frac{2}{3}$ years = $\frac{3}{4}$ Henry's + 20 years.
(7) $\frac{1}{12}$ Henry's = $3\frac{1}{3}$ years.
(8) $\frac{12}{12}$ = 12 times $3\frac{1}{3}$ years = 40, Henry's age 10 years ago.
(9) $\frac{3}{4}$ of 40 years = 30 years, John's age 10 years ago.
(10) 40 years + 10 years = 50 years, Henry's present age.
(11) 30 years + 10 years = 40 years, John's present age.

∴ Henry's age is 50 years, John's age 40 years.

PROBLEM 25.

A hare starts 30 yards before a hound, but is not seen by him till she has been running 40 seconds. The hare runs at the rate of 8 miles an hour, and the hound pursues her at the rate of 10 miles per hour: how long will the chase continue, and how far must the hound run?

Solution.

(1) 10 miles = distance the hound runs in 1 hour.

(2) 8 miles = distance the hare runs in 1 hour.

(3) 10 miles — 8 miles = 2 miles, distance the hound gains on the hare in 1 hour.

(4) 1760 yards = the number of yards in 1 mile.

(5) (1760 yards $\times$ 2) $\div$ (60 $\times$ 60) = $\frac{352}{360}$ yards, distance the hound gains on the hare in one second.

(6) 30 yards + (40 $\times$ 8 $\times$ 1760) $\div$ (60 $\times$ 60) = 186$\frac{4}{9}$ yards, the distance to be gained by the hound.

(7) 186$\frac{4}{9}$ yards $\div$ $\frac{352}{360}$ = 190$\frac{15}{22}$ seconds, or 3 min. 10$\frac{15}{22}$ sec.

$\therefore$ 186$\frac{4}{9}$ yards is the distance the hound runs, and 3 min. 10$\frac{15}{22}$ sec. is the time of the chase.

PROBLEM 26.

A fox is 120 leaps before a hound and takes 5 leaps to the hound's 2, but 4 of the hound's leaps equal 12 of the fox's: how many leaps must the hound take to catch the fox?

Solution.

(1) 2 of the hound's leaps = 6 of the fox's, but while the hound makes 6 of the fox's the fox takes only 5, hence, the hound gains 1 of the fox's leaps in making 2, and in 120 the hound must take 120 $\times$ 2 = 240 leaps.

$\therefore$ The hound takes 240 leaps to catch the fox.

PROBLEM 27.

A fish's head is 4 inches long; its tail is as long as its head and $\frac{1}{3}$ of its body; the body is as long as its head and tail: what is its length?

Solution.

(1) $\frac{3}{3}$ = length of body.

(2) 4 inches = length of head.

(3) 4 inches + $\frac{1}{3}$ of body = tail.

(4) 8 inches + $\frac{1}{3}$ of body = body.

(5) $\frac{2}{3}$ of body = 8 inches.

(6) $\frac{1}{3}$ of body = $\frac{1}{2}$ of 8 in. = 4 in.

(7) $\frac{3}{3}$ length of body = 3 times 4 in. = 12 in.

(8) 4 in. + $\frac{1}{3}$ of body (or tail) = 8 in.

(9) ∴ 4 in. + 8 in. + 12 in. = 24 in., length of fish.

∴ The fish is 24 inches long.

PROBLEM 28.

There is coal now on the dock and coal is running on also from a chute at a uniform rate. Six men can clear the doek in one hour, but 11 men can clear it in 20 minutes: how long will it take 4 men?

Solution.

(1) Suppose 1 man's work to be 6 tons.
(2) Then 6 tons × 6 = 36 tons, whole quantity on dock 1 hr.
(3) 11 × 6 × $\frac{1}{3}$ hr. (or 20 minutes) = 22 tons on dock in 20 minutes.
(4) 36 tons — 22 tons = 14 tons, quantity running on in 1 — $\frac{1}{3}$ = $\frac{2}{3}$ of an hour.
(5) 14 tons ÷ $\frac{2}{3}$ = 21 tons run on in 1 hour.
(6) 36 tons — 21 tons = 15 tons, original quantity on dock.
(7) 21 tons ÷ 6 = $3\frac{1}{2}$, or $3\frac{1}{2}$ times a man's work to clear what is running on.
(8) 4 — $3\frac{1}{2}$ = $\frac{1}{2}$ of 1 man's work left to clear original quantity, or 15 tons.
(9) Half a man's work being 3 tons an hour, it will take 15 ÷ 3, or 5 hours.

∴ It will take 4 men 5 hours to clear the dock.

PROBLEM 29.

A man at his marriage agreed that if at his death he should leave only a daughter, his wife should have $\frac{3}{4}$ of his estate; and if he should leave a son, she should have $\frac{1}{4}$. He left a son and a daughter. What fractional part of the estate should each receive, and how much was each one's portion, the estate being worth $4000?

Solution.

(1) The wife in the first condition was to have $\frac{3}{4}$, or 3 times as much as the daughter.
(2) In the second condition the son was to have $\frac{3}{4}$, or 3 times as much as the wife.
(3) Daughter has 1 part, wife 3 parts, son 9 parts, or $\frac{1}{13} + \frac{3}{13} + \frac{9}{13} = \frac{13}{13}$ = whole estate.
(4) $\frac{1}{13}$ of $4000 = $$307\frac{9}{13}$, daughter's share.
(5) $\frac{3}{13}$ of $4000 = $$923\frac{1}{13}$, wife's share.
(6) $\frac{9}{13}$ of $4000 = $$2769\frac{3}{13}$, son's share.

∴ Daughter's share is $$307\frac{9}{13}$, wife's $$923\frac{1}{13}$, son's $$2769\frac{3}{13}$.

PROBLEM 30.

A was engaged for a year at $80 and a suit of clothes; he served 7 months and received for his wages the clothes and $35: what was the value of the clothes?

Solution.

(1) $\frac{12}{12}$ = the value of the clothes.
(2) $\frac{12}{12}$ + $80 = wages for 12 months.
(3) $\frac{1}{12}$ of ($\frac{12}{12}$ + $80) = $\frac{1}{12}$ + 6\frac{2}{3}$, wages for 1 month.
(4) ($\frac{1}{12}$ + 6\frac{2}{3}$) × 7 = $\frac{7}{12}$ + 46\frac{2}{3}$, wages for 7 months.
(5) $\frac{12}{12}$ + $35 = wages for 7 months.
(6) $\frac{12}{12}$ + $35 = $\frac{7}{12}$ + 46\frac{2}{3}$.
(7) $\frac{5}{12}$ = 11\frac{2}{3}$.
(8) $\frac{1}{12}$ = $\frac{1}{5}$ of 11\frac{2}{3}$ = 2\frac{1}{3}$.
(9) $\frac{12}{12}$ = 12 times 2\frac{1}{3}$ = $28.

∴ $28 was the value of the clothes.

PROBLEM 31.

A teacher has books in three rooms; $\frac{2}{3}$ of the number in the first room equals $\frac{3}{4}$ of the number in the second room, and $\frac{2}{3}$ of the number in the second room equals $\frac{3}{4}$ of the number in the third room. If the entire number is 651, how many are in each room?

Solution.

(1) $\frac{2}{3}$ of the number in the first room = $\frac{3}{4}$ the number in the second room.
(2) $\frac{1}{3}$ of the number in the first room = $\frac{1}{2}$ of $\frac{3}{4}$ = $\frac{3}{8}$ of number in second room.
(3) $\frac{3}{3}$, or number in first room = 3 times $\frac{3}{8}$ = $\frac{9}{8}$ of number in second room.
(4) Then $\frac{8}{8}$ = number in second room.
(5) $\frac{3}{4}$ of number in 3d room = $\frac{2}{3}$ of number in 2d room.
(6) $\frac{1}{4}$ of number in 3d room = $\frac{1}{3}$ of $\frac{2}{3}$ = $\frac{2}{9}$ of no. in 2d room.
(7) $\frac{4}{4}$, or number in 3d room = 4 times $\frac{2}{9}$ = $\frac{8}{9}$ of number in 2d room.
(8) ∴ $\frac{9}{8} + \frac{8}{8} + \frac{8}{9} = \frac{217}{72}$ = 651, number in the 3 rooms.
(9) $\frac{1}{72} = \frac{1}{217}$ of 651 = 3.
(10) $\frac{81}{72}$ = 81 times 3, or 243, number in 1st room.
(11) $\frac{72}{72}$ = 72 times 3, or 216, number in 2d room.
(12) $\frac{64}{72}$ = 64 times 3, or 192, number in 3d room.

∴ There are 243 books in 1st room, 216 in 2d, 192 in 3d.

PROBLEM 32.

Four masons, A, B, C and D, engage to build a wall; A, B and C can build it in 18 days; B, C and D in 20 days; A, C and D in 24 days; and A, B and D in 27 days: in what time can they build it jointly and separately? (*Schuyler's H. A., p. 421, prob. 18.*)

Solution.

(1) 18 days = time it takes A, B and C to build the wall.
(2) $\therefore \frac{1}{18}$ = part they do in 1 day.
(3) 20 days = time it takes B, C and D to build the wall.
(4) $\frac{1}{20}$ = part they do in 1 day.
(5) 24 days = time it takes A, C and D to build the wall.
(6) $\frac{1}{24}$ = part they do in 1 day.
(7) 27 days = time it takes A, B and D to build the wall.
(8) $\frac{1}{27}$ = part they do in 1 day.
(9) $\therefore \frac{1}{18} + \frac{1}{20} + \frac{1}{24} + \frac{1}{27} = \frac{199}{1080}$, part A, B and C; B, C and D; A, C and D; and A, B and D do in 1 day = 3 times the work A, B, C and D do in 1 day.
(10) $\therefore \frac{1}{3}$ of $\frac{199}{1080} = \frac{199}{3240}$, part A, B, C and D do in 1 day.
(11) $\frac{199}{3240} - \frac{1}{18} = \frac{19}{3240}$, part A, B, C and D do in 1 day — part A, B and C do in 1 day = part D does in 1 day.
(12) $\frac{3240}{3240}$ = part D does in $\frac{3240}{3240} \div \frac{19}{3240} = 170\frac{10}{19}$ days.
(13) $\frac{199}{3240} - \frac{1}{20} = \frac{37}{3240}$, part A, B, C and D do in 1 day — part B, C and D do in 1 day = part A does in 1 day.
(14) $\frac{3240}{3240}$ = part A does in $\frac{3240}{3240} \div \frac{37}{3940} = 87\frac{21}{37}$ days.
(15) $\frac{199}{3340} - \frac{1}{24} = \frac{35}{3240}$, part A, B, C and D do in 1 day — part A, C and D do in 1 day = part B does in 1 day.
(16) $\frac{3240}{3240}$ = part B does in $\frac{3240}{3240} \div \frac{35}{3240} = 50\frac{5}{8}$ days.
(17) $\frac{199}{3240} - \frac{1}{27} = \frac{79}{3240}$, part A, B, C and D do in 1 day — part A, B and D do in 1 day = part C does in 1 day.
(18) $\frac{3240}{3240}$ = part C does in $\frac{3240}{3240} \div \frac{199}{3240} = 41\frac{1}{79}$ days.
(19) $\frac{3240}{3240}$ = part A, B, C and D do in $\frac{3240}{3240} \div \frac{199}{3240} = 16\frac{56}{199}$ days.

$\therefore$ They can jointly do the work in $16\frac{56}{199}$ days. A can do the work in $87\frac{21}{37}$ days, B in $50\frac{5}{8}$ days, C in $41\frac{1}{79}$ days, and D in $170\frac{10}{19}$ days.

PROBLEM 33.

A, B and C dine on 8 loaves of bread; A furnishes 5 loaves; B, 3 loaves; C pays the others 8 dimes for his share: how must A and B divide the money?

Solution.

(1) 8 = the number of loaves they eat.
(2) $8 \div 3 = 2\frac{2}{3}$, the number each eats.

(3) 5 loaves — $2\frac{2}{3}$ = $2\frac{1}{3}$ loaves, what A furnished for C.
(4) 3 loaves — $2\frac{2}{3}$ loaves = $\frac{1}{3}$ loaf, what B furnished for C.
(5) $\frac{7}{3}$: $\frac{1}{3}$, or A's portion is to B's as 7 : 1.

∴ A's = $\frac{7}{8}$ of 8 d. = 7 d., A's portion of the 8 d.; and B's is $\frac{1}{8}$ of 8 d. = 1 d., B's portion.

PROBLEM 34.

A is twelve years old, and if to his age be added $\frac{5}{8}$ of the ages of A and B, the sum will be B's age: what is B's age?

Solution.

(1) Let $\frac{8}{8}$ = B's age.
(2) 12 + $\frac{5}{8}$ of (12 + $\frac{8}{8}$, B's age) = $\frac{8}{8}$, B's.
(3) 12 + $\frac{60}{8}$ + $\frac{5}{8}$ B's = $\frac{8}{8}$, B's.
(4) 96 + 60 = 3 B's, or 3 B's = 156.
(5) B's age = 52 years.

∴ 52 years = B's age.

PROBLEM 35.

What is the time of day when $\frac{2}{5}$ of the time past noon equals $\frac{14}{15}$ of the time till midnight?

Solution.

(1) $\frac{5}{5}$ = time past noon.
(2) 12 hours = time from noon to midnight.
(3) $\frac{2}{5}$ of $\frac{5}{5}$, time past noon = $\frac{14}{25}$ of (12 — $\frac{5}{5}$, time past noon.)
(4) $\frac{2}{5}$ of time past noon = $\frac{168 \text{ hr.} - 14}{25}$ of time past noon.
(5) 10 times past noon = 168 hours — 14 times past noon.
(6) 24 times past noon = 168 hours.

∴ Time past noon = 7 hrs.

PROBLEM 36.

A lady spent $40 more than $\frac{1}{2}$ the money in her purse; then $30 more than $\frac{1}{3}$ of the remainder; then $10 more than $\frac{1}{4}$ the remainder, after which she had $32: how much had she at first?

Solution.

(1) Let $\frac{2}{2}$ = her money at first.
(2) $\frac{1}{2}$ of $\frac{2}{2}$ of her money = $\frac{1}{2}$ of her money.
(3) $\frac{1}{2}$ + $40 = amount spent.
(4) $\frac{2}{2}$ her money — ($\frac{1}{2}$ her money + $40) = $\frac{1}{2}$ her money — $40, the remainder.
(5) $\frac{1}{3}$ of $\frac{1}{2}$ of her money — $40 = $\frac{1}{6}$ of her money — $\$\frac{40}{3}$ + $30 = $\frac{1}{6}$ of her money + $\$\frac{50}{3}$, amount spent.

(6) $\frac{1}{2}$ of her money $+$ \$40 — $\frac{1}{6}$ or her money $+$ \$$\frac{50}{3}$ $=$ $\frac{1}{3}$ of her money — \$$\frac{170}{3}$, remainder.

(7) $\frac{1}{4}$ of $\frac{1}{3}$ of her money — \$$\frac{170}{12}$ $+$ \$10 $=$ $\frac{1}{12}$ of her money — \$$\frac{50}{12}$, amount spent.

(8) $\frac{1}{3}$ of her money — \$$\frac{170}{12}$ — $\frac{1}{12}$ of her money — \$$\frac{50}{12}$ $=$ $\frac{1}{4}$ of her money — \$$\frac{630}{12}$, remainder.

(9) $\frac{1}{4}$ of her money — \$$\frac{630}{12}$ $=$ \$32, or $\frac{1}{4}$ of her money $=$ \$84$\frac{1}{2}$.

(10) $\frac{4}{4}$ her money $=$ 4 times \$84$\frac{1}{2}$ $=$ \$338.

∴ She had \$338 at first.

PROBLEM 37.

Charles, when asked his age, replied: "My father was born in 1843 and my mother in 1847. The sum of their ages at the time of my birth was 5 times my age in 1887." In what year will Charles be 25 years of age?

Solution.

If the father was born in 1843 and the mother in 1847, the sum of their ages in 1887 was 84 years; and since the sum of their ages at Charles's birth was 5 times his age in 1887, and the parents each increased in age as fast as he did, in 1887 the sum of their ages must have been 5 + 1 + 1, or 7 times the age of Charles; hence, he was 84 ÷ 7, or 12 years old in 1887, making the time of his birth 1875, and in 1900 he will be 25, if living.

TIME PROBLEMS.

PROBLEM 38

At what time between 5 and 6 o'clock will the hour and minute hands be together?

Solution.

(1) Let $\frac{2}{2}$ $=$ distance the hour hand moves.

(2) $\frac{24}{2}$ $=$ the distance the minute hand moves.

(3) $\frac{24}{2} - \frac{2}{2} = \frac{22}{2}$ distance gained by minute hand.

(4) 25 minutes $=$ distance gained by minute hand.

(5) $\frac{22}{2}$ $=$ 25 minutes.

(6) $\frac{1}{2} = \frac{1}{22}$ of 25 minutes $=$ $\frac{25}{22}$ minutes.

(7) $\frac{24}{2}$ $=$ 24 times $\frac{25}{22}$ $=$ $\frac{600}{22}$ min. $=$ $27\frac{3}{11}$ min.

∴ $27\frac{3}{11}$ minutes past 5 $=$ time required.

PROBLEM 39.

At what time between 7 and 8 will the hour and minute hand of a clock be opposite each other?

Solution.

(1) Let $\frac{2}{2}$ = distance moved by the hour hand.
(2) $\frac{24}{2}$ = distance moved by the minute hand.
(3) $\frac{24}{2} - \frac{2}{2} = \frac{22}{2}$ = distance gained by minute hand.
(4) $\frac{22}{2}$ = 5 minutes.
(5) $\frac{1}{2} = \frac{1}{22}$ of 5 minutes = $\frac{5}{22}$ minutes.
(6) $\frac{24}{2}$ = 24 times $\frac{5}{22} = \frac{120}{22}$ min. = $5\frac{5}{11}$ min.

∴ $5\frac{5}{11}$ minutes past 7 = time required.

PROBLEM 40.

At what time between 6 and 7 o'clock is the minute hand the same distance from 9 as the hour hand is from 4?

Solution.

(1) $\frac{2}{2}$ = distance moved by the hour hand.
(2) $\frac{24}{2}$ = distance moved by the minute hand.
(3) $\frac{26}{2}$ = distance moved by both.
(4) 35 minutes = distance moved by both.
(5) $\frac{26}{2}$ = 35 minutes.
(6) $\frac{1}{2} = \frac{1}{26}$ of 35 minutes = $\frac{35}{26}$ minutes.
(7) $\frac{24}{2}$ = 24 times $\frac{35}{26}$ min. = $\frac{840}{26}$ min. = $32\frac{4}{13}$ min.

∴ $32\frac{4}{13}$ minutes past 6 = time.

NOTE.—Locate the minute hand at 12 and the hour hand at 6. When situated this way, the hour hand is 10 minutes from 4. If the hour hand remains stationary the minute hand must move 35 minutes to be within 10 minutes of 9. The hour hand does not remain stationary, but moves farther from 4; hence the minute hand must stop short of 35 minutes just the same distance that the hour hand moves farther from 4.

PROBLEM 41.

At what time between 5 and 6 o'clock will the minute hand be at right angles with the hour hand?

Solution.

(1) $\frac{2}{2}$ = distance hour hand moves past 5.
(2) $\frac{24}{2}$ = distance minute hand moves past 12.
(3) $\frac{24}{2} - \frac{2}{2} = \frac{22}{2}$ distance gained by minute hand.
(4) $\frac{22}{2}$ = 10 min., or 40 min.
(5) $\frac{1}{2} = \frac{1}{22}$ of 10 min., or $\frac{1}{22}$ of 40 min. = $\frac{10}{22}$ min., or $\frac{40}{22}$ min.

(6) $\frac{24}{2}$ = 24 times $\frac{10}{22}$, or 24 times $\frac{40}{22}$ min. = $10\frac{10}{11}$ min., or $43\frac{7}{11}$ min.

∴ The minute hand will be at right angles with the hour hand at $10\frac{10}{11}$ minutes, or $43\frac{7}{11}$ minutes past 5 o'clock.

PROBLEM 42.

At what time between 4 and 5 will the minute hand be $\frac{1}{4}$ of the distance from 12 to the hour hand?

Solution.

(1) $\frac{4}{4}$ = distance moved by hour hand.
(2) 20 minutes + $\frac{4}{4}$ = distance from 12 to hour hand.
(3) $\frac{1}{4}$ of (20 min. + $\frac{4}{4}$) = distance minute hand moves.
(4) $\frac{48}{4}$ = distance minute hand moves.
(5) $\frac{48}{4}$ = 5 minutes + $\frac{1}{4}$.
(6) $\frac{47}{4}$ = 5 minutes.
(7) $\frac{1}{4}$ = $\frac{1}{47}$ of 5 minutes = $\frac{5}{47}$ minutes.
(8) $\frac{48}{4}$ = 48 times $\frac{5}{47}$ minutes = $5\frac{5}{47}$ minutes.

∴ $5\frac{5}{47}$ minutes past 4 = time required.

PROBLEM 43.

At what time between 4 and 5 will the minute hand be as far from 12 on the left side of the dial plate as the hour hand is from 12 on the right side?

Solution.

(1) $\frac{2}{2}$ = distance moved by the hour hand.
(2) $\frac{24}{2}$ = distance moved by the minute hand.
(3) $\frac{26}{2}$ = distance moved by both.
(4) 40 minutes = distance moved by both.
(5) $\frac{26}{2}$ = 40 minutes.
(6) $\frac{1}{2}$ = $\frac{1}{26}$ of 40 minutes = $\frac{40}{26}$ minutes.
(7) $\frac{24}{2}$ = 24 times $\frac{40}{26}$ minutes = $36\frac{12}{13}$ minutes.

∴ $36\frac{12}{13}$ minutes past 4 = the time required.

PROBLEM 44.

At what time between 5 and 6 will the minute hand be 10 minutes ahead of the hour hand?

Solution.

(1) $\frac{2}{2}$ = distance hour hand moves while minute hand is moving to be 10 minutes ahead.
(2) $\frac{24}{2}$ = distance moved by the minute hand while the hour hand moves $\frac{2}{2}$.

(3) $\frac{24}{2}$ — $\frac{2}{2}$ = distance the minute hand gains on the hour hand.
(4) 35 minutes = distance gained by minute hand.
(5) $\frac{22}{2}$ = 35 minutes.
(6) $\frac{1}{2}$ = $\frac{1}{22}$ of 35 min. = $\frac{35}{22}$ minutes.
(7) $\frac{24}{2}$ = 24 times $\frac{35}{22}$ = $38\frac{2}{11}$ minutes.
∴ $38\frac{2}{11}$ minutes past 5 = the time required.

PROBLEM 45.

Solar time at Cincinnati is 22 min. 16 sec. faster than Central Standard time: what is the longitude of Cincinnati? (*Putnam Co.*)

Solution.

22 min. 16 sec., the difference of time = 5° 34′, difference of longitude. Cincinnati is east of the Central time meridian, or 90th. ∴ 90° — 5° 34′ = 84° 26′ west longitude, or the longitude of Cincinnati.

PROBLEM 46.

At what time between 7 and 8 o'clock are the hour and minute hands of a watch together? (*Brooks' Arithmetic.*)

Solution.

(1) Let $\frac{2}{2}$ = distance moved by the hour hand.
(2) $\frac{24}{2}$ = distance moved by the minute hand.
(3) $\frac{24}{2}$ — $\frac{2}{2}$ = $\frac{22}{2}$, distance gained by the minute hand.
(4) 35 minutes = distance gained by the minute hand.
(5) $\frac{22}{2}$ = 35 minutes, and $\frac{1}{2}$ = $\frac{1}{22}$ of 35 minutes, or $\frac{35}{22}$.
(6) $\frac{24}{2}$ = 24 times $\frac{35}{22}$ or $38\frac{2}{11}$ min. past 7.

CHAPTER III.

LONGITUDE AND TIME, AND LATITUDE AND TIME.

10. The **Latitude** of a place is its distance from the equator, north or south. It is measured in degrees, minutes and seconds, and can not exceed a quadrant, or 90°.

11. The **Longitude** of a place is its distance east or west from a given meridian. It is reckoned in degrees, minutes and seconds, and can not exceed a semi-circumference, or 180°.

Rule.—*When the latitudes or longitudes are both east or west, subtract the less from the greater; when one is east and the other west, take their sum.*

NOTE.—When we add two longitudes, if their sum is 180°, it must be subtracted from 360° for the difference of longitude.

PROBLEM 47.

The longitude of New Orleans is 90° W., and of Geneva 6° 9′ 29″ E.: what is the difference of longitude? (*Brooks' Arithmetic.*)

Solution.

90° + 6° 9′ 5″ = 96° 9′ 5″, or the difference of longitude.

PROBLEM 48.

The longitude of Portland is 70° 13′ 34″ W., and of Mobile 88° 1′ 29″ W.: what is the difference of longitude? (*Brooks' Arithmetic.*)

Solution.

88° 1′ 29″ — 70° 13′ 34″ = 17° 47′ 55″, the difference of longitude.

12. The circumference of the earth contains 360°; hence, the sun appears to travel through 360° in 24 hours.

In 1 hour it travels $\frac{1}{24}$ of 360° = 15°.

In 1 minute it travels $\frac{1}{60}$ of 15° = 15′.
In 1 second it travels $\frac{1}{60}$ of 15′ = 15″.

13. To find the difference of time of two places.

Rule.—*Divide the difference of longitude by* 15 *and mark the quotient* hr. min. sec. *instead of* ° ′ ″.

14. To find the difference of longitude of two places.

Rule.—*Multiply the difference of time by* 15 *and mark the product* ° ′ ″ *instead of* hr. min. sec.

PROBLEM 49.

The longitude of New York is 74° 3′ W., and of New Orleans 90° W.: required the difference in time. (*Brooks' Arithmetic.*)

Solution.

(1) 90° — 74° 3′ = 15° 57′, the difference of longitude.
(2) 15° 57′ ÷ 15 = 1 hr. 3 min. 48 sec., the difference of time.

PROBLEM 50.

The longitude of Philadelphia is 75° 9′ 5″ W., and of Cincinnati 84° 29′ 31″ W.: what is the time at Cincinnati when it is 10 A.M. at Philadelphia? (*Brooks' Arithmetic.*)

Solution.

(1) Cincinnati is 84° 29′ 31″ W.; Philadelphia, 75° 9′ 5″ W. The latter is 9° 20′ 26″ farther east.
(2) 9° 20′ 26″ ÷ 15 = 37 min. 21$\frac{11}{15}$ sec.
(3) ∴ 10 hr. — 37 min 21$\frac{11}{15}$ sec. = 9 hr 22 min. 38$\frac{4}{15}$ sec. A. M., time at Cincinnati.

PROBLEM 51.

The longitude of Rome is 12° 27′ E., and San Francisco 122° 26′ 15″ W.: what time is it in the latter place when it is 4 P. M. in the former? (*Brooks' Arithmetic.*)

Solution.

(1) Rome 12° 27′ E., San Francisco 122° 26′ 15″ W., difference of longitude is 134° 53′ 15″.
(2) 134° 53′ 15″ ÷ 15 = 8 hr. 59 min. 33 sec.
(3) 4 P. M. = 16 hr.
(4) ∴ 16 hr. — 8 hr. 59 min. 33 sec. = 7 hr. 27 sec. A. M., the time at San Francisco.

PROBLEM 52.

The difference of time between Philadelphia and Cincinnati is about 37 min, 20 sec.: what is the difference of longitude? (*Brooks' Arithmetic.*)

Solution.

The difference of time, 37 min. 20 sec. × 15 = 9° 20′, difference of longitude.

PROBLEM 53.

When it is 11 A. M. at a place 30° E. of Greenwich, it is 3 hr. 44 min. 20 sec. a. m. at Buffalo: what is the longitude of Buffalo?

Solution.

(1) 11 hr. — 3 hr. 44 min. 20 sec. = 7 hr. 15 min. 40 sec., difference of time.

(2) 7 hr. 15 min. 40 sec. × 15 = 108° 55′, difference of long.

(3) ∴ The longitude of Buffalo is 108° 55′ — 30° = 78° 55′.

PROBLEM 54.

The longitude of Honolulu is 157° 52′ W., and that of Sydney 151° 11′ E. When it is 5 min. after 4 o'clock on Sunday morning at Honolulu, what is the time at Sydney? (*Ray's Higher.*)

Solution.

(1) The difference of longitude is 157° 52′ + 151° 11′ = 309° 3′.

(2) The difference of time is 309° 3′ ÷ 15 = 20 hr. 36 min. 52 sec. This added to 4 hr 5 min. gives 24 hr. 41 min. 52 sec. past midnight, or 41 min. 52 sec. A. M. Monday.

PROBLEM 55.

Locate and name a noted city whose sun time is 6:40 A. M. when it is noon at Greenwich.

Solution.

The difference of time is $5\frac{1}{3}$ hr., and the city must be $5\frac{1}{3}$ × 15, or 80° west of Greenwich. Pittsburg, Pa., has that longitude.

PROBLEM 56.

If we consider a degree of longitude in our latitude 53 miles, how far and in what direction had I gone on a parallel when I found my watch 2 hr. 40 min. too fast?

Solution.

(1) The difference of longitude is (2 hr. 40 min.) × 15 = 40°.

(2) Since my time was too fast, I must have gone west 40 × 53, or 2120 miles.

PROBLEM 57.

The distance from Boston to Chicago is about 840 miles, and a degree of longitude at Boston contains about 51 miles; when it is noon at Boston what is the time at Chicago?

Solution.

(1) The distance, 840 miles $\div$ 51 $= 16\frac{8}{17}°$, diff. of longitude.
(2) $16\frac{8}{17}° \div 15 = 1\frac{25}{225}$ hr. before noon.

PROBLEM 58.

Yesterday I was in longitude 86° 18′ W. and set my watch; to-day the sun is on the meridian at 36 min. past 11 o'clock: what is my longitude?

Solution.

The difference of time is 24 min., which reduced to degrees of longitude is $24 \times 15 \div 60 = 6°$; then my longitude is now 6° less, or 80° 18′ W. Beaver Falls, Pa., has that longitude, and I am in the longitude of South Bend, Ind.

NOTES TO REMEMBER.

15. If the inclination of the earth's axis were 30° instead of $23\frac{1}{2}°$, the width of the North Temperate Zone would be $90° - (30° + 30°) = 30°$, in fact, the remaining zones would be 30° wide.

16. Standard Railroad Time Explained.—Fifteen degrees of longitude equal one hour of time. Railroad men, to avoid trouble and accident, have adopted the plan of making all places on the earth have the time of certain meridians, each one being 15° apart. The following meridians are used:

0° is called Universal time, west 15° West African, 30° Central Atlantic, 45° East Brazilian, 60° La Plata, 75° Eastern, 90° Central, 105° Mountain, 120° Pacific, 135° East Alaskan, 150° Central Alaskan, 165° West Alaskan, 180° Transitional; east 165° is called New Caledonian, 150° East Australian, 135° Corean, 120° East Asian, 105° Siam, 90° East Hindostan, 75° West Hindostan, 60° Ural, 45° Caucasus, 30° Bosporus, 15° Scandinavian.

Columbus, O., Lat. 39° 57′ N., Long. 83° 3′ W.

Washington, D. C., Lat. 38° 39′ N., Long. 77° 3′ W. Exact, 77° 2′ 48″ W.

St. Helena is an island in the Atlantic 15° 55′ 26″ S. Lat., and 5° 42′ 30″ W. Long.

Chicago has a longer day than Constantinople, because it is farther north by 50′.

17. A **Geographical** or **Nautical Mile** (or Knot) is about 2025 yards.

18. **A Tropical Year** is the time elapsed from the moment the sun leaves a star until it reaches it again. It equals 365 days, 6 hours, 9 minutes and 9 seconds.

19. A **Solar Year** is the time elapsed from the moment the sun leaves the vernal equinox till it reaches it again. It equals 365 days, 5 hours, 48 minutes and 48 seconds.

20. The **Anomalistic Year** is measured from the time the sun leaves the perihelion till it reaches it again. It equals 365 days, 6 hours, 13 minutes and 45.6 seconds.

21. A **Lunar Year** is 12 lunar months. It equals 354 days.

22. The **Equator** is a line passing around the globe midway between the poles.

NOTE.—Some teach that the word "line" used in the above definition is an imaginary line, yet a careful and thorough investigation will show that it is not an imaginary line, but real. All geometrical lines are real, according to definition. Investigate this matter and see if there has not been some fallacy in the definition both of the equator and also of small circles.

CHAPTER IV.

PERCENTAGE.

23. **Percentage** is the process of computation in which 100 is the basis of comparison.

24. **Per Cent** (*per*, by, *centum*, a hundred) means *by* or *on the hundred.*

25. The quantities used in Percentage are the Base, the Rate, the Percentage and the Amount or Difference.

26. The **Base** is the number on which the percentage is computed.

27. The **Rate** is the number of hundredths of the base which is to be taken.

28. The **Percentage** is the result obtained by taking a certain per cent of the base.

29. The **Amount** or **Difference** is the sum or difference of the base and percentage. They may both be embraced under the general term, **Proceeds**.

CASE I.

PROBLEM 59.

What is 12% of 475?

Solution.

(1) 100% = 475.
(2) 1% = $\frac{1}{100}$ of 475 = 4.75.
(3) 12% = 12 times 4.75 = 57.

∴ 12% of 475 = 57.

PROBLEM 60.

What is $\frac{5}{8}$% of $800?

Solution.

(1) 100% = $800.

(2) $1\% = \frac{1}{100}$ of \$800 = \$8.
(3) $\frac{5}{8}\% = \frac{5}{8}$ times 8 = \$5.

∴ $\frac{5}{8}\%$ of \$800 = \$5.

PROBLEM 61.

What is 10% of 20% of \$13.50?

Solution.

(1) 100% = \$13.50.
(2) $1\% = \frac{1}{100}$ of \$13.50 = \$.1350.
(3) 20% = 20 times \$.1350 = \$2.7.
(4) 100% = \$2.7.
(5) $1\% = \frac{1}{100}$ of \$2.7 = \$.027.
(6) 10% = 10 times \$.027 = \$.27.

∴ 10% of 20% of \$13.50 = \$.27.

PROBLEM 62.

A man contracts to supply dressed stone for a court house for \$119449, if the rough stone costs him 16 c. a cu. ft.; but if he can get it for 15 c. a cu. ft., he will deduct 3% from his bill: how many cu. ft. would be needed, and what does he charge for dressing a cu. ft.?

Solution.

(1) 100% = \$119449.
(2) $1\% = \frac{1}{100}$ of \$119449 = \$1194.49.
(3) 3% = 3 times \$1194.49 = \$3583.47, the deduction.
(4) \$.16 — \$.15 = \$.01 the deduction per cubic foot.
(5) \$3583.47 ÷ \$.01 = 358347, cubic feet.
(6) \$119449 = cost of 358347 cubic feet.
(7) \$119449 ÷ 358347 = \$.33⅓, cost of dressing 1 cubic foot.
(8) \$.33⅓ — \$.16 =\$.17⅓, cost of dressing per cubic foot.

∴ 358347 = number of cubic feet, \$.17⅓ = cost of dressing per cubic foot.

PROBLEM 63.

48% of brandy is alcohol: how much alcohol does a man swallow in 40 years, if he drinks a gill of brandy 3 times a day?

Solution.

(1) 365¼ = number of days in a year.
(2) 365¼ × 40 = 14610 days.
(3) 14610 × 3 gills = 43830 gills.
(4) 100% = 43830 gills.

(5) 1% = $\frac{1}{100}$ of 43830 gills = 438.3 gills.
(6) 48% = 48 times 438 8 gills = 21038.4 gills.

∴ He swallows 21038.4 gills, or 657 gal. 1 qt. 1 pt. 2.4 gi.

PROBLEM 64.

A man has $1200; he gave 30% to a son, 20% of the remainder to his daughter, and so divided the rest among four brothers that each after the first had $12 less than the preceding: how much did the last receive? (*Ray's Higher, p. 191, prob. 25.*)

Solution.

(1) 100% = $1200.
(2) 1% = $\frac{1}{100}$ of $1200 = $12.
(3) 30% = 30 times $12 = $360, son's share.
(4) $1200 — $360 = $840, remainder.
(5) 100% = $840.
(6) 1% = $\frac{1}{100}$ of $840 = $8.40.
(7) 20% = 20 times $8.40 = $168, daughter's share.
(8) $840 — $168 = $672, amount to be divided between the 4 brothers.
(9) 100% = 4th brother's share.
(10) 100% + $12 = 3d brother's share.
(11) 100% + $24 = 2d brother's share.
(12) 100% + $36 = 1st brother's share.
(13) 100% + (100% + $12) + (100% + $24) + (100%+$36) = 400% + $72, amount the brothers receive.
(14) 400% + $72 = $672
(15) 400% = $600.
(16) 1% = $\frac{1}{400}$ of $600 = $1.50.
(17) 100% = 100 times $1.50 = $150.

∴ The fourth brother received $150.

PROBLEM 65.

What number increased by 20% of 3.5, diminished by 12½% of 9.6, gives 3½? (*Ray's Higher, p. 191, prob. 26.*)

Solution.

(1) 100% = the number.
(2) 100% = 3.5.
(3) 1% = $\frac{1}{100}$ of 3.5 = .035.
(4) 20% = 20 times .035 = .7.
(5) 100% = 9.6.
(6) 1% = $\frac{1}{100}$ of 9.6 = .096.
(7) 12½% = 12½ times .096 = 1.2.

(8) $\therefore 100\% + .7 - 1.2 = 3.5$.
(9) $100\% = 4$.

$\therefore$ The number is 4.

CASE II.

PROBLEM 66.

\$14 is what % of \$175?

Solution.

(1) $\$175 = 100\%$.
(2) $\$1 = \frac{1}{175}$ of $100\% = \frac{100}{175}\%$.
(3) $\$14 = 14$ times $\frac{100}{175} = 8\%$.

$\therefore$ \$14 is 8% of \$175.

PROBLEM 67.

$\frac{2}{3}$ is what % of $\frac{3}{4}$?

Solution.

(1) $\frac{3}{4} = 100\%$.
(2) $\frac{1}{4} = \frac{1}{3}$ of $100\% = \frac{100}{3}$ or $33\frac{1}{3}\%$.
(3) $\frac{4}{4} = 4$ times $33\frac{1}{3}\% = 133\frac{1}{3}\%$.
(4) $\frac{2}{3}$ is $\frac{2}{3}$ of $133\frac{1}{3}\% = 88\frac{8}{9}\%$.

$\therefore \frac{2}{3}$ is $88\frac{8}{9}\%$ of $\frac{3}{4}$.

PROBLEM 68.

30% of the whole of an article is how many % of $\frac{2}{3}$ of it?

(*R. A., p. 192, prob. 26.*)

Solution.

(1) 100% = the whole article.
(2) $\frac{2}{3}$ of $100\% = 66\frac{2}{3}\%$, $\frac{2}{3}$ of the article.
(3) $66\frac{2}{3}\% = 100\%$.
(4) $1\% = 100\% \div 66\frac{2}{3} = 1\frac{1}{2}\%$.
(5) $30\% = 30$ times $1\frac{1}{2}\% = 45\%$.

$\therefore$ 30% of the whole article is 45% of $\frac{2}{3}$ of it.

PROBLEM 69.

25% of $\frac{2}{5}$ of an article is how many % of $\frac{3}{4}$ of it?

(*R. A., p. 191, prob. 21.*)

Solution.

(1) 100% = the whole article.
(2) $\frac{3}{4}$ of $100\% = 75\%$.
(3) 25% of ($\frac{2}{5}$ of 100%) $= 10\%$.
(4) $75\% = 100\%$ of itself.

(5) 1% = $\frac{1}{75}$ of 100% = $1\frac{1}{3}$%.
(6) 10% = 10 times $1\frac{1}{3}$% = $13\frac{1}{3}$%.

∴ 25% of $\frac{2}{5}$ of an article is $13\frac{1}{3}$% of $\frac{3}{4}$ of it.

PROBLEM 70.

If a miller takes 10 quarts of every bushel he grinds for toll, what % does he take for toll? (*B. N. A., p. 220.*)

Solution.

(1) 1 bu. = 32 qt.
(2) 32 qt. = 100%.
(3) 1 qt. = $\frac{1}{32}$ of 100 = $3\frac{1}{8}$%.
(4) 10 qt. = 10 times $3\frac{1}{8}$% = $31\frac{1}{4}$%.

∴ He takes $31\frac{1}{4}$% for toll.

CASE III.

PROBLEM 71.

60 is 20% of what number?

Solution.

(1) 100% = number required.
(2) 20% = 60.
(3) 1% = $\frac{1}{20}$ of 60 = 3.
(4) 100% = 100 times 3 = 300.

∴ 60 is 20 % of 300.

PROBLEM 72.

$\frac{3}{4}$ is 200% of what number?

Solution.

(1) 100% = required number.
(2) 200% = $\frac{3}{4}$.
(3) 1% = $\frac{1}{200}$ of $\frac{3}{4}$ = $\frac{3}{800}$.
(4) 100% = 100 times $\frac{3}{800}$ = $\frac{300}{800}$ or $\frac{3}{8}$.

∴ $\frac{3}{4}$ is 200% of $\frac{3}{8}$.

PROBLEM 73.

A man owning 80% of a mill, sold 50% of his share for $8800: at this rate, what was the value of the mill?

Solution.

(1) 100% = value of mill.
(2) 80% = his share.

(3) 50% of 80% = 40%, part sold.
(4) \$8800 = value of part sold.
(5) 40% = \$8800.
(6) 1% = $\frac{1}{40}$ of \$8800 = \$220.
(7) 100% = 100 times \$220 = \$22000.

∴ \$22000 = value of mill.

PROBLEM 74.

I pay \$13 a month for board, which is 20% of my salary: what is my salary? (*R. H. A., p. 194, prob. 20.*)

Solution.

(1) 100% = what I receive per month.
(2) 20% = \$13.
(3) 1% = $\frac{1}{20}$ of \$13 = \$.65.
(4) 100% = 100 times \$.65 = \$65, my monthly salary.
(5) 12 times \$65 = \$780, my salary.

∴ My salary is \$780.

CASE IV.

PROBLEM 75.

2576 bu. is 60% less than what? (*R. H. A., p. 196, prob. 4.*)

Solution.

(1) 100% = the number.
(2) 100% — 60% = 40%, the number decreased by 60%.
(3) 2567 bu. = the number decreased by 60%.
(4) ∴ 40% = 2576 bu.
(5) 1% = $\frac{1}{40}$ of 2576 bu. = 64.4 bu.
(8) 100% = 100 times 64.4 bu. = 6440 bu.

∴ 2576 bu. is 60% less than 6440 bu.

PROBLEM 76.

A horse cost \$160, which was 20% less than the cost of the carriage: what was the cost of the carriage? (*Wh. Com., p. 142.*)

Solution.

(1) 100% = cost of carriage.
(2) 100% — 20% = 80%, cost of horse.
(3) \$160 = cost of horse.
(4) 80% = \$160.
(5) 1% = $\frac{1}{80}$ of \$160 = \$2.

(6) 100% = 100 times $2 = $200.

∴ The carriage cost $200.

PROBLEM 77.

A school enrolls 230 boys, which is 15% more than the number of girls enrolled: how many pupils in the school? (*Wh. Com., p. 142.*)

Solution.

(1) 100% = the number of girls.
(2) 100% + 15% = 115%, the number of boys.
(3) 230 = number of boys.
(4) 115% = 230.
(5) 1% = $\frac{1}{115}$ of 230 = 2.
(6) 100% = 100 times 2 = 200, the number of girls.
(7) 230 + 200 = 430, the number enrolled.

∴ There are 430 pupils enrolled.

PROBLEM 78.

A coat cost $32; the trimming cost 70% less, and the making 50% less than the cloth: what did each cost?

Solution.

(1) 100% = value of cloth.
(2) 100% — 70% = 30%, cost of trimming.
(3) 100% — 50% = 50%, cost of making.
(4) 100% + 30% + 50% = 180%, cost of coat.
(5) $32 = cost of coat.
(6) 180% = $32.
(7) 1% = $\frac{1}{180}$ of $32 = $.1777$\frac{7}{9}$.
(8) 100% = 100 times $.1777$\frac{7}{9}$ = 17.77\frac{7}{9}$, cost of cloth.
(9) 30% = 30 times $.1777$\frac{7}{9}$ = 5.33\frac{1}{3}$, cost of trimming.
(10) 50% = 50 times $.1777$\frac{7}{9}$ =8.88\frac{8}{9}$, cost of making.

∴ The cloth cost 17.77\frac{7}{9}$, the trimming 5.33\frac{1}{3}$, the making $8 88$\frac{8}{9}$.

PROBLEM 79.

In a certain company the number of children was 45% of the number of women, the number of women 80% of the number of men, and the whole company was 432: how many of each?

Solution.

(1) 100% = number of men.
(2) 80% = number of women.
(3) 45% of 80 = 36%, number of children.

(4) $100\% + 80\% + 36\% = 216\%$, the number in the company.
(5) $432 =$ number in the company.
(6) $\therefore 216 = 432$.
(7) $1\% = \frac{1}{216}$ of $432 = 2$.
(8) $100\% = 100$ times $2 = 200$, the number of men.
(9) $80\% = 80$ times $2 = 160$, the number of women.
(10) $36\% = 36$ times $2 = 72$, the number of children.

$\therefore$ There were 200 men, 160 women and 72 children.

PROBLEM 80.

Our stock decreased $33\frac{1}{3}\%$, and again 20%; then it rose 20%, and again $33\frac{1}{3}\%$; we have thus lost $66: what was the stock at first?

Solution.

(1) $100\% =$ the original stock.
(2) $100\% - 33\frac{1}{3}\% = 66\frac{2}{3}\%$, stock after first decrease.
(3) 20% of $66\frac{2}{3}\% = 13\frac{1}{3}\%$, second decrease.
(4) $66\frac{2}{3}\% - 13\frac{1}{3}\% = 53\frac{1}{3}\%$, stock after second decrease.
(5) 20% of $53\frac{1}{3}\% = 10\frac{2}{3}\%$, first increase.
(6) $53\frac{1}{3}\% + 10\frac{2}{3}\% = 64\%$, stock after first increase.
(7) $33\frac{1}{3}\%$ of $64\% = 21\frac{1}{3}\%$, second increase.
(8) $64\% + 21\frac{1}{3}\% = 85\frac{1}{3}\%$, stock after second increase.
(9) $100\% - 85\frac{1}{3}\% = 14\frac{2}{3}\%$, whole loss.
(10) $\$66 =$ whole loss.
(11) $\therefore 14\frac{2}{3}\% = \$66$.
(12) $1\% = \$66 \div 14\frac{2}{3} = \$4\ 50$.
(13) $100\% = 100$ times $\$4.50 = \$450$, the original stock.

$\therefore$ The original stock was $450.

PROBLEM 81.

A brewery is worth 4% less than a tannery, and the tannery 16% more than a boat; the owner of the boat has traded it for 75% of the brewery, losing thus $103: what is the tannery worth?

Solution.

(1) $100\% =$ value of the boat.
(2) $100\% + 16\% = 116\%$, value of the tannery.
(3) 4% of $116\% = 4.64\%$.
(4) $116\% - 4.64\% = 111.36\%$, value of the brewery.
(5) 75% of $111.36\% = 83.52\%$, what the owner received for the boat.
(6) $100\% - 83.52\% = 16.48\%$, loss.

(7) \$103 = loss.
(8) ∴ 16.48% = \$103.
(9) 1% = $\frac{1}{16.48}$ of \$103 = \$6.25.
(10) 116% = 116 times \$6.25 = \$725.

∴ The tannery is worth \$725.

CHAPTER V.

TRADE DISCOUNT.

30. **Discount** is the deduction from the list or regular price.

31. A **Net Price** is a fixed price from which no discount is allowed.

32. A **List Price** is an established price, assumed by the seller as a basis upon which to calculate discounts.

33. Trade discounts are often taken off, as "10, 20, 6 and 5% off," meaning 10% off, 20% off, 6% off and 5% off of the remainder.

PROBLEM 82.

Sold 20 dozen feather dusters, giving the purchaser a discount of 10, 10 and 10%, his discounts amounting to \$325.20: how much was my price per dozen? (*R. 3d p., p. 209, prob. 5.*)

Solution.

(1) 100% = my price per dozen.
(2) 10% of 100% = 10%, 1st discount.
(3) 100% — 10% = 90%.
(4) 10% of 90% = 9%, 2d discount.
(5) 90% — 9% = 81%.
(6) 10% of 81% = 8.1%, 3d discount.
(7) 10% + 9% + 8.1% = 27.1%, amount of discounts.
(8) \$325.20 = amount of discounts.
(9) 27.1% = \$325.20.

(10) 1% = $\frac{1}{27.1}$ of \$325.20 = \$12.
(11) 100% = 100 times \$12 = \$1200.
(12) 20 dozen = \$1200.
(13) 1 dozen = $\frac{1}{20}$ of \$1200 = \$60.

∴ The price per box is \$60.

PROBLEM 83.

Bought 100 dozen stay bindings at 60 cents per dozen; for 40, 10 and 7½% off: what did I pay for them?

Solution.

(1) 60 cents = list price of 1 dozen.
(2) 100 dozen = 100 times \$.60 = \$60, list price of 100 dozen.
(3) 40% of \$60 = \$24, first discount.
(4) \$60 — \$24 = \$36, first net proceeds.
(5) 10% of \$36 = \$3.60, second discount.
(6) \$36 — \$3.60 = \$32.40, second net proceeds.
(7) 7½% of \$32.40 = \$2.43, third discount.
(8) \$32.40 — \$2.43 = \$29.97, cost.

∴ I paid \$29.97.

PROBLEM 84.

A retail dealer buys a case of slates containing 10 dozen for \$50 list, and gets 50, 10 and 10 off; paying for them in the usual time, he gets an additional 2%: what did he pay per dozen for the slates?

Solution.

(1) \$50 = the list.
(2) 50% of \$50 = \$25, first discount.
(3) \$50 — \$25 = \$25, first net proceeds.
(4) 10% of \$25 = \$2.50, second discount.
(5) \$25 — \$2.50 = \$22.50, second net proceeds.
(6) 10% of \$22.50 = \$2.25, third discount.
(7) \$22.50 — \$2.25 = \$20.25, third net proceeds.
(8) 2% of \$20.25 = \$.405, fourth discount.
(9) \$20.25 — \$.405 = \$19.845, cost of the 10 dozen slates.
(10) 1 dozen = $\frac{1}{10}$ of \$19.845 = \$1.9845.

∴ The slates cost \$1.9845 a dozen.

PROBLEM 85.

A bookseller purchases books from the publisher at 20% off the list price; if he retail them at the list price, what will be his per cent of profit?

Solution.

(1) 100% = list price.
(2) 20% = discount.
(3) 100% — 20% = 80%, cost.
(4) 100% = bookseller's selling price.
(5) 100% — 80% = 20%, gain.
(6) 80% = 20%.
(7) 1% = $\frac{1}{80}$ of 20% = $\frac{1}{4}$%.
(8) 100% = 100 times $\frac{1}{4}$% = 25%.

∴ The profit is 25%.

PROBLEM 86.

Sold a case of hats containing 3 dozen, on which I had received a discount of 10%, and made a profit of 12½%, or 37½ cents on each hat: what was the wholesale merchant's price per case?

Solution.

(1) 37½ cents = profit on one hat.
(2) $.37½ × 36 = $13.50, profit on 3 dozen hats.
(3) 100% = wholesale merchant's price per case.
(4) 10% = discount.
(5) 100% — 10% = 90%, cost.
(6) 12½% of 90% = 11¼%, profit.
(7) 11¼% = $13 50.
(8) 1% = $13 50 ÷ 11¼ = $1.20.
(9) 100% = 100 times $1.20 = $120, wholesale merchant's price per case.

∴ The price per case was $120.

PROBLEM 87.

A dealer in notions buys 60 gross shoestrings at 70 cents per gross, list, 50, 10 and 5% off; if he sells them at 20, 10 and 5% off list, what will be his profit?

Solution.

(1) 70 cents = list price per gross.
(2) $.70 × 60 = $42, list price of 60 gross.
(3) 50% of $42 = $21, 1st discount.
(4) $42 — $21 = $21, 1st net proceeds.
(5) 10% of $21 = $2.10, 2d discount.
(6) $21 — $2.10 = $1890, 2d net proceeds.
(7) 5% of $18.90 = $.945, 3d discount.
(8) $18.90 — $.945 = $17.955, cost.

Now by 2d condition—

(9) 20% of \$42 = \$8.40, 1st dis. by 2d condition.
(10) \$42 — \$8.40 = \$33.60, 1st net pro. by 2d condition.
(11) 10% of \$33.60 = \$3.36, 2d dis. by 2d condition.
(12) \$33.60 — \$3.36 = \$30.24, 2d net pro. by 2d condition.
(13) 5% of \$30.24 = \$1.512, 3d dis. by 2d condition.
(14) \$30.24 — \$1.512 = \$28.728, selling price.
(15) \$28.728 — \$17.955 = \$10.773, his profit.

∴ \$10.773 = his profit.

PROBLEM 88.

Bought 50 gross of rubber buttons for 25, 10 and 5% off; disposed of the lot for \$35.91, at a profit of 12%: what was the list price of the buttons per gross?

Solution.

(1) 100% = list price.
(2) 25% of 100% = 25%, 1st discount.
(3) 100% — 25% = 75%, 1st net proceeds.
(4) 10% of 75% = $7\frac{1}{2}$%, 2d dis.
(5) 75% — $7\frac{1}{2}$% = $67\frac{1}{2}$%, 2d net pro.
(6) 5% of $67\frac{1}{2}$% = 3.375%, 3d dis.
(7) $67\frac{1}{2}$% — 3.375% = 64.125%, cost.
(8) 12% of 64.125% = 7.595%, gain.
(9) 64.125% + 7.695% = 71.82%, selling price.
(10) \$35.91 = selling price.
(11) 71.82% = \$35.91.
(12) 1% = $\frac{1}{71.82}$ of \$35.91 = \$.50.
(13) 100% = 100 times \$.50 = \$50, list price of 50 gross.
(14) \$50 ÷ 50 = \$1, list price of 1 gross.

∴ \$1 = list price per gross.

PROBLEM 89.

A merchant imported wine at \$2.80 per gal.; 9% was lost by leakage: at what price per gallon must he sell the remainder to gain 30% on the cost of all? (*Putnam Co.*)

Solution.

(1) Let 100 = the number of gal. bought.
(2) 100 × \$2.80 = \$28, the cost of the wine.
(3) 30% of \$280 = \$84, gain. \$280 + \$84 = \$364, selling price.
(4) 9% of 100 gal. = 9 gal., amount lost by leakage.
(5) 100 gal. — 9 gal. = 91 gal., remaining.

∴ \$364 ÷ 91 = \$4, price at which he sells the remainder per gallon in order to gain 30% on the cost of all.

PROBLEM 90.

A merchant buys hats from the manufacturers at 25% off list; if he retails them at list, what will be his % of profit?

Solution.

(1) 100% = list price.
(2) 25% = discount. 100 — 25% = 75%, cost.
(3) 100% = merchant's selling price.
(4) ∴ 100% — 75% = 25%, gain.
(5) 75% = 100%.
(6) 1% = $\frac{1}{75}$ of 100% = $1\frac{1}{3}$%, and 25% = 25 times $1\frac{1}{3}$% = $33\frac{1}{3}$%, gain.

∴ $33\frac{1}{3}$% is his per cent of profit.

CHAPTER VI.

PROFIT AND LOSS.

34. **Profit** and **Loss** are terms which denote the gain or loss in business transactions.

35. **Cost** is the price paid for goods.

36. The **Selling Price** is the price received for goods.

37. The **Profit** is what the goods sell for more than they cost.

38. The **Loss** is what the goods sell for less than they cost.

PROBLEM 91.

Sold silk at \$1.35 per yard and lost 10%: at what price per yard would I have sold it to make a profit of $16\frac{2}{3}$%?

Solution.

I. (1) 100% = cost price; 10% = loss.
(2) 90% = selling price; \$1.35 = selling price.
(3) 90% = \$1.35.
(4) 1% = $\frac{1}{90}$ of \$1.35 = \$015.
(5) 100% = 100 times \$.015 = \$1.50, cost.

II. (1) 100% = \$1.50.
(2) 1% = $\frac{1}{100}$ of \$1.50 = \$.015.
(3) $16\frac{2}{3}$% = $16\frac{2}{3}$ times \$.015 = \$.25, gain.
(4) \$1.50 + \$.25 = \$1.75.

∴ \$1.75 = price sold to gain $16\frac{2}{3}$%.

PROBLEM 92.

A bookseller sells a grammar for \$1.00 which costs 80 cents: what is his gain per cent?

Solution.

(1) 80 cents = cost price.
(2) \$1.00 = selling price.
(3) \$1.00 — \$.80 = \$.20, gain.
(4) 80 cents = 100%.
(5) 1 cent = $\frac{1}{80}$ of 100% = $\frac{5}{4}$%.
(6) 20 cents = 20 times $\frac{5}{4}$% = 25%.

∴ 25% = his gain.

PROBLEM 93.

A merchant sold velvet at a profit of \$3 per yard and gained 20%: how much did it cost?

Solution.

(1) 100% = cost per yard.
(2) 20% = gain; \$3 = gain.
(3) 20% = \$3.
(4) 1% = $\frac{1}{20}$ of \$3 = \$.15.
(5) 100% = 100 times \$.15 = \$15, price of velvet.

∴ \$15 = cost of velvet.

PROBLEM 94.

A merchant sells goods at retail 30% above cost and at wholesale 12% less than the retail price: what is his gain per cent on goods sold at wholesale? (*Wh., p. 148.*)

Solution.

(1) 100% = cost.
(2) 130% = retail.
(3) 12% of 130% = 15.6%.
(4) 130% — 15.6% = 114.4%, wholesale price.
(5) 114.4% — 100% = 14.4%, gain at wholesale.

∴ He gained 14.4% at wholesale.

PROBLEM 95.

A butcher sold two beeves for $150 each; on the one he gained 25% and on the other he lost 25%: did he gain or lose and how much?

Solution.

(1) 100% = cost of the first.
(2) 125% = selling price of first.
(3) $150 = selling price of first.
(4) 125% = $150.
(5) 1% = $\frac{1}{125}$ of $150 = $1.20.
(6) 100% = 100 times $1.20 = $120, cost of first.
(7) $150 — $120 = $30, gain on the first.
(8) 100% = cost of the second.
(9) 75% = selling price of the second.
(10) $150 = selling price of second.
(11) 75% = $150.
(12) 1% = $\frac{1}{75}$ of $150 = $2.
(13) 100% = 100 times $2 = $200, cost of the second.
(14) $200 — $150 = $50, loss.
(15) $50 — $30 = $20, loss.

∴ He lost $20 in the transaction.

PROBLEM 96.

A merchant reduced the price of cloth 5 cents per yard, and thereby reduced his profit on the cloth from 10% to 8%: what was the cost of the cloth per yard?

Solution.

(1) 100% = cost.
(2) 10% = profit before deducting.
(3) 8% = profit after deducting.
(4) 10% — 8% = 2%, deduction.
(5) 5 cents = deduction.
(6) 2% = 5 cents.

(7) $1\% = \frac{1}{2}$ of 5 cents $= \frac{5}{2}$ cents.
(8) $100\% = 100$ times $\frac{5}{2}$ cents $= \$2.50$.

∴ The cost price was $2.50.

PROBLEM 97.

Sold $\frac{2}{3}$ of an acre of land for $2 more than $\frac{3}{4}$ of an acre cost, and made 15% on the part sold: find the cost of one acre.

Solution.

(1) $100\% =$ cost of one acre.
(2) $\frac{3}{4}$ of $100\% = 75\%$, the cost of $\frac{3}{4}$ of an acre.
(3) $75\% + \$2 =$ the selling price of $\frac{2}{3}$ of an acre.
(4) $(75\% + \$2) \div \frac{2}{3} = 112\frac{1}{2}\% + \$3$, selling price of one acre.
(5) Then, by the condition of the problem, $115\% =$ selling price of one acre.
(6) $115\% = 112\frac{1}{2}\% + \$3$.
(7) $2\frac{1}{2}\% = \$3$.
(8) $1\% = \$3 \div 2\frac{1}{2} = \$1.20$.
(9) $100\% = 100$ times $\$1.20 = \$120$, cost of one acre.

∴ $\$120 =$ cost of one acre.

PROBLEM 98.

A sold two wagons for $150. He gained 5% on the first and 6% on the second: how much did each cost if the second cost $10 more than the first?

Solution.

(1) $100\% =$ cost of first wagon.
(2) $5\% =$ gain.
(3) $100\% + 5\% = 105\%$, selling price of first.
(4) $100\% + \$10 =$ cost of 2d wagon.
(5) 6% of $(100\% + \$10) = 6\% + \$.60$, gain.
(6) $(100\% + \$10) + (6\% + \$.60) = 106\% + \$10.60$, selling price of 2d.
(7) $105\% + (106\% + \$10.60) =$ selling price of both.
(8) $\$150 =$ selling price of both wagons.
(9) $105\% + (106\% + \$10.60) = \$150$.
(10) $211\% = \$139.40$.
(11) $1\% = \frac{1}{211}$ of $\$139.40 = \$.66\frac{14}{211}$.
(12) $100\% = 100$ times $\$.66\frac{14}{211} = \$66\frac{14}{211}$, cost of 1st wagon.
(13) $\$66\frac{14}{211} + \$10 = \$76\frac{14}{211}$, cost of 2d wagon.

∴ The 1st wagon cost $\$66\frac{14}{211}$, the 2d $\$76\frac{14}{211}$.

NOTE.—The above is taken from a Wood county examination list, and it seemed to bother many old teachers.

PROBLEM 99.

A merchant marked a piece of carpet at 25% above cost, and then sold it at 20% less than marked price: did he gain or lose, and how much?

Solution.

(1) 100% = the cost.
(2) 125% = the marked price.
(3) 20% of 125% = 25%.
(4) 125% — 25% = 100%, selling price.
(5) 100% — 100% = 0.

∴ He neither gained nor lost.

PROBLEM 100.

I sold a team of horses for $700; on one I gained 20%, on the other I lost 20%; my total loss was $50: what was the cost of each?

Solution.

(1) 100% = cost of 1st horse, and 20% the gain.
(2) 120% = selling price of 1st horse.
(3) $750 — 100% = cost of 2d horse.
(4) 20% of ($750 — 100%) = $150 — 20%, loss.
(5) ($750 — 100%) — ($150 — 20%) = $600 — 80%, selling price of 2d horse.
(6) 120% + ($600 — 80%) = $700.
(7) 40% = $100.
(8) 1% = $\frac{1}{40}$ of $100 = $2.50.
(9) 100% = 100 times $2.50 = $250, cost of 1st.
(10) $750 — $250 = $500, cost of 2d.

∴ The 1st horse cost $220, the 2d $500.

PROBLEM 101.

Bought hams at 8 ct. a lb.; the wastage is 10%: how must I sell them to gain 30%? (*R. H. A., p. 200, prob. 9.*)

Solution.

(1) Let 100 = the number of pounds the hams weigh.
(2) 8 ct. = the cost of 1 pound.
(3) 100 lb. at 8 ct. = $8, cost.
(4) 10% of 100 lb. = 10 lb.

(5) 100 lb. — 10 lb. = 90 lb., remaining after wastage.
(6) 30% of \$8 = \$2.40, gain.
(7) \$8 + \$2.40 = \$10.40, selling price.
(8) 90 lb. = \$10.40.
(9) 1 lb. = $\frac{1}{90}$ of \$10.40 = 11$\frac{5}{9}$ cents.

∴ I sell them at 11$\frac{5}{9}$ cents per pound.

PROBLEM 102.

How must cloth costing \$4 a yard be marked that a merchant may deduct 15% from the marked price and still make 15% profit?

Solution.

I. (1) \$4 = cost price.
(2) 15% = gain.
(3) 15% of \$4 = \$.60, gain.
(4) \$4 + \$.60 = \$4.60, selling price.

II. (1) 100% = marked price.
(2) 15% = deduction.
(3) 100% — 15% = 85%, selling price.
(4) 85% = \$4.60.
(5) 1% = $\frac{1}{85}$ of \$4.60 = \$.05$\frac{7}{17}$.
(6) 100% = 100 times \$.05$\frac{7}{17}$ = \$5$\frac{7}{17}$.

∴ The marked price is \$5$\frac{7}{17}$.

PROBLEM 103.

A sold two horses for \$432; on the gray he gained 20% and on the bay he lost 20%: did he gain or lose, if $\frac{2}{3}$ of the cost of the gray equals $\frac{4}{5}$ of the cost of the bay?

Solution.

(1) $\frac{2}{3}$ of the cost of the gray = $\frac{4}{5}$ of the cost of the bay.
(2) $\frac{1}{3}$ of the cost of the gray = $\frac{1}{2}$ of $\frac{4}{5}$ = $\frac{4}{10}$ of the cost of the bay.
(3) $\frac{3}{3}$ the cost of the gray = 3 times $\frac{4}{10}$ of the cost of the bay = $\frac{12}{10}$ of the cost of the bay (cost of the gray in terms of the bay.)
(4) $\frac{12}{10}$ = the cost of the gray.
(5) $\frac{10}{10}$ = the cost of the bay,
(6) 20% or $\frac{1}{5}$ of $\frac{12}{10}$ = $\frac{12}{50}$, gain.
(7) $\frac{12}{10}$ + $\frac{12}{50}$ = $\frac{72}{50}$, selling price of the gray.
(8) 20% or $\frac{1}{5}$ of $\frac{10}{10}$ = $\frac{1}{5}$, loss.
(9) $\frac{10}{10}$ — $\frac{1}{5}$ = $\frac{8}{10}$, selling price of the bay.
(10) $\frac{72}{50}$ + $\frac{8}{10}$ = $\frac{112}{50}$, selling price of both.

(11) \$432 = selling price of both.
(12) $\frac{112}{50}$ = \$432.
(13) $\frac{1}{50} = \frac{1}{112}$ of \$432 = \$3$\frac{6}{7}$.
(14) $\frac{60}{50}$ ($\frac{12}{10}$ cost of gray) = 60 times \$3$\frac{6}{7}$ = \$231$\frac{3}{7}$, cost of the gray.
(15) $\frac{50}{50}$ ($\frac{10}{10}$ cost of bay) = 50 times \$3$\frac{6}{7}$ = \$192$\frac{6}{7}$, cost of the bay.
(16) \$231$\frac{3}{7}$ + \$192$\frac{6}{7}$ = \$424$\frac{2}{7}$, amount both cost.
(17) \$432 — \$424$\frac{2}{7}$ = \$7$\frac{5}{7}$, gain.

∴ He gained \$7$\frac{5}{7}$.

PROBLEM 104.

Sold two cows for \$210. On the first I gained 25%, and on the second I lost 25%: what did I gain if the second cost $\frac{2}{3}$ as much as the first?

Solution.

(1) Let $\frac{12}{12}$ represent the cost of first cow.
(2) $\frac{2}{3}$ of $\frac{12}{12} = \frac{8}{12}$, the cost of second cow.
(3) 25% of $\frac{12}{12} = \frac{3}{12}$, gain.
(4) $\frac{12}{12} + \frac{3}{12} = \frac{15}{12}$, selling price of first cow.
(5) 25% of $\frac{8}{12} = \frac{2}{12}$, loss.
(6) $\frac{8}{12} - \frac{2}{12} = \frac{6}{12}$, selling price of second cow.
(7) $\frac{15}{12} + \frac{6}{12} = \frac{21}{12}$, selling price of both cows.
(8) \$210 = selling price of both cows.
(9) $\frac{21}{12}$ = \$210.
(10) $\frac{1}{12} = \frac{1}{21}$ of \$210 = \$10.
(11) $\frac{12}{12}$ = 12 times \$10 = \$120, cost of first cow.
(12) $\frac{8}{12}$ = 8 times \$10 = \$80, cost of second cow.
(13) \$120 + \$80 = \$200, cost of both cows.
(14) \$210 — \$200 = \$10, gain.

∴ I gained \$10.

PROBLEM 105.

A stock of goods is marked 22$\frac{1}{2}$% advance of cost, but becoming damaged is sold at 20% discount on marked price, whereby a loss of \$1186.40 is sustained: what was the cost of the goods?

Solution.

(1) 100% = cost of the goods.
(2) 122$\frac{1}{2}$% = marked price.
(3) 20% of 122$\frac{1}{2}$% = 24$\frac{1}{2}$%, discount.
(4) 122$\frac{1}{2}$% — 24$\frac{1}{2}$% = 98%, selling price.

(5) 100% — 98% = 2%, loss.
(6) \$1186 40 = loss.
(7) 2% = \$1186.40.
(8) 1% = $\frac{1}{2}$ of \$1186.40 = \$593.20.
(9) 100% = 100 times \$593.20 = \$59320, cost of the goods.

∴ \$59320 = cost of the goods.

PROBLEM 106.

I sold two horses, receiving $14\frac{2}{7}$% less for the second than for the first; on the first I gained 25% and on the second I lost 20%; my whole loss was \$5: find the cost.

Solution.

(1) 100% = selling price of 1st horse.
(2) 100% = cost of 1st horse.
(3) 125% = selling price of 1st horse.
(4) 125% = 100%.
(5) 1% = $\frac{1}{125}$ of 100% = $\frac{4}{5}$%.
(6) 100% = 100 times $\frac{4}{5}$% = 80%, cost of 1st horse.
(7) 100% — 80% = 20%, gain.
(8) 100% = cost of 2d horse.
(9) 100% — $14\frac{2}{3}$% = $85\frac{1}{3}$%, selling price of 2d.
(10) 20% of 100% = 20%, loss.
(11) 100% — 20% = 80%, selling price.
(12) 80% = $85\frac{1}{3}$%.
(13) 1% = $\frac{1}{80}$ of $85\frac{1}{3}$% = $1.06\frac{2}{3}$%.
(14) 100% = 100 times $1.06\frac{2}{3}$% = $106\frac{2}{3}$%, cost of 2d.
(15) $106\frac{2}{3}$% — $85\frac{1}{3}$% = $21\frac{1}{3}$%, loss on 2d.
(16) $21\frac{1}{3}$% — 20% = $1\frac{1}{3}$%, whole loss.
(17) \$5 = whole loss.
(18) $1\frac{1}{3}$% = \$5.
(19) 1% = \$5 ÷ $1\frac{1}{3}$ = \$3.75.
(2o) 80% = 80 times \$3.75 = \$300, cost of 1st.
(21) $85\frac{1}{3}$% = $85\frac{1}{3}$ times \$3.75 = \$400, cost of 2d.

∴ The 1st horse cost \$300, the 2d \$400.

PROBLEM 107.

How must I mark goods costing \$8 per yard that I may deduct 40% from the asking price and still make 20%?

Solution.

I. (1) \$8 = the cost, and 20% the gain.

(2) 20% of \$8 = \$1.60, gain.
(3) \$8 + \$1.60 = \$9.60, selling price.

II. (1) 100% = asking price.
(2) 40% = the reduction.
(3) 100% — 40% = 60%, selling price.
(4) 60% = \$9.60.
(5) 1% = $\frac{1}{60}$ of \$9.60 = \$.16.
(6) 100% = 100 times \$.16 = \$16.

∴ I must mark it \$16.

PROBLEM 108.

Sold hogs at 10% above cost. Invested \$150 more than these proceeds in cattle which I sold at 20% loss. If I now have \$84 more than at first, find value of hogs?

Solution.

(1) 100% = cost of the hogs.
(2) 110% = selling price of the hogs.
(3) 110% + \$150 = cost of the cattle.
(4) 20% of (110 + \$150) = 22% + \$30 = loss.
(5) (110% + \$150) — (22% + \$30) = 88% + \$120.
(6) 100% + \$84 = 88% + \$120.
(7) 12% = \$36.
(8) 1% = $\frac{1}{12}$ of \$36 = \$3.
(9) 100% = 100 times \$3 = \$300.

∴ \$300 = value of hogs.

PROBLEM 109.

I sold a horse at a gain of 20%, and with this money I bought another, which was sold for \$166.50, losing $7\frac{1}{2}$%: find cost of first horse?

Solution.

(1) 100% = cost of 1st horse.
(2) 20% = the gain.
(3) 120% = selling price of 1st.
(4) 100% = cost of 2d horse.
(5) $7\frac{1}{2}$% = the loss.
(6) 100% — $7\frac{1}{2}$% = $92\frac{1}{2}$%, selling price of 2d.
(7) \$166.50 = selling price of 2d.
(8) $92\frac{1}{2}$% = \$166.50.
(9) 1% = \$166.50 ÷ $92\frac{1}{2}$ = \$1.80.
(10) 100% = 100 times \$1.80 = \$180, cost of 2d, also selling price of 1st.

(11) 120% = \$180.
(12) 1% = $\frac{1}{120}$ of \$180 = \$150.
(13) 100% = 100 times \$1.50 = \$150, cost of first horse.

∴ \$150 = cost of first horse.

PROBLEM 110.

If a cabinet maker sold a set of furniture for \$18.75 more than cost, and gained 30%, what would have been his rate per cent of gain or loss if he had sold the furniture for \$87.50?

Solution.

(1) 100% = cost of furniture.
(2) 30% = the advance.
(3) \$18.75 = the advance.
(4) 30% = \$18.75.
(5) 1% = $\frac{1}{30}$ of \$18.75 = \$.625.
(6) 100% = 100 times \$.625 = \$62.50, cost of furniture.
(7) \$87.50 — \$62.50 = \$25, gain.
(8) \$62 50 = 100%.
(9) \$1 = 100% ÷ 62.50 = 1$\frac{3}{5}$%.
(10) \$25 = 25 times 1$\frac{3}{5}$% = 40%, gain.

∴ His rate of gain would have been 40%.

PROBLEM 111.

Sold an equal quantity of two kinds of cloth, losing 3% on the first and gaining 5% on the second; the difference in the amount of sales was \$32.25: find total amount of sales if the first cost $\frac{3}{4}$ as much as the second.

Solution.

(1) 100% = cost of the 2d.
(2) $\frac{3}{4}$ of 100% = 75%, cost of the 1st.
(3) 3% of 75% = 2.25%, loss.
(4) 75% — 2.25% = 72$\frac{3}{4}$%, selling price of 1st.
(5) 5% of 100% = 5%, gain.
(6) 100% + 5% = 105%, selling price of 2d.
(7) 105% — 72$\frac{3}{4}$% = 32$\frac{1}{4}$%, difference of sales.
(8) \$32.25 = difference of sales.
(9) 32$\frac{1}{4}$% = \$32.25.
(10) 1% = \$32.25 ÷ 32$\frac{1}{4}$ = \$1.
(11) 72$\frac{3}{4}$% = 72$\frac{3}{4}$ times \$1 = \$72$\frac{3}{4}$, selling price of 1st.
(12) 105% = 105 times \$1 = \$105, selling price of 2d.

(13) \$72.75 + \$105 = \$177.75, total amount of sales.

∴ \$177.75 = total amount of sales.

PROBLEM 112.

Each of two men, A and B, desired to sell his horse to C. A asked a certain price and B 50% more. A then reduced his price 20%, and B his price 30%, at which C took both horses, paying \$150: what was each man's price?

Solution.

(1) 100% = A's asking price.
(2) 150% = B's asking price.
(3) 20% of 100% = 20%, reduction.
(4) 100% — 20% = 80%, A's selling price.
(5) 30% of 150% = 45%, reduction.
(6) 150% — 45% = 105%, B's selling price.
(7) 105% + 80% = 185%, whole selling price.
(8) \$150 = whole selling price.
(9) 185% = \$150.
(10) 1% = $\frac{1}{185}$ of \$150 = \$.81$\frac{3}{37}$.
(11) 100% = 100 times \$.81$\frac{3}{37}$ = \$81$\frac{3}{37}$, A's asking price.
(12) 150% = 150 times \$.81$\frac{3}{37}$ = \$121.62$\frac{6}{37}$, B's asking price.

∴ A's asking price is \$81$\frac{3}{37}$, and B's is \$121.62$\frac{6}{37}$.

PROBLEM 113.

The sale of a horse was 20% more than that of a cow. My whole gain was \$12. Find cost of each, if I gained 25% on the horse and lost 16$\frac{2}{3}$% on the cow.

Solution.

(1) 100% = selling price of the cow.
(2) 120% = selling price of the horse.
(3) 100% = cost of cow.
(4) 100% — 16$\frac{2}{3}$% = 83$\frac{1}{3}$%, selling price of cow.
(5) 83$\frac{1}{3}$% = 100%.
(6) 1% = 100% ÷ 83$\frac{1}{3}$ = 1.20%.
(7) 100% = 100 times 1.20% = 120%, cost of the cow.
(8) 120% — 100% = 20%, loss on the cow.
(9) 100% = cost of the horse.
(10) 100% + 25% = 125%, selling price of horse.
(11) 125% = 120%.
(12) 1% = $\frac{1}{125}$ of 120% = $\frac{24}{25}$%.

(13) 100% = 100 times $\frac{24}{25}$% = 96%, cost of the horse.
(14) 120% — 96% = 24%, gain on the horse.
(15) 24% — 20% = 4%, gain.
(16) $12 = gain.
(17) 4% = $12.
(18) 1% = $\frac{1}{4}$ of $12 = $3.
(19) 120% = 120 times $3 = $360, cost of cow.
(20) 96% = 96 times $3 = $288, cost of horse.

∴ The cow cost $360, and the horse $288.

PROBLEM 114.

Sold a cow for $69, gaining 15%; sold another cow for $36, and lost the same amount of money as gained upon the first: what was the rate of loss on last sale?

Solution.

(1) 100% = cost of 1st cow.
(2) 15% of 100% = 15%, gain.
(3) 100% + 15% = 115%, selling price of 1st cow.
(4) $69 = selling price of 1st cow.
(5) 115% = $69.
(6) 1% = $\frac{1}{115}$ of $69 = $.60.
(7) 100% = 100 times $.60 = $60, cost of 1st cow.
(8) $69 -- $60 = $9, gain on 1st cow.
(9) 100% = cost of 2d cow.
(10) $36 + $9 = $45, cost of 2d cow.
(11) $45 = 100%.
(12) $1 = $\frac{1}{45}$ of 100% = $2\frac{2}{9}$%.
(13) $9 = 9 times $2\frac{2}{9}$% = 20%, rate of loss on last sale.

∴ 20% was the rate of loss on last sale.

PROBLEM 115

A man bought a horse for a certain price. Now, if he sells him for $24, he will lose as much per cent as the horse cost: required the price of the horse.

Solution.

(1) Let x denote the price.

(2) $\frac{x}{100}$ = the rate of loss.

(3) $x \times \frac{x}{100} = \frac{x^2}{100}$, loss.

(4) $x - \frac{x^2}{100} = 24$.

(5) $x^2 - 100x = -2400$.
(6) $x^2 - 100x + 2500 = 100$.
(7) $x - 50 = \pm 10$.
(8) $x = 60$ or 40.

∴ The price of the horse was either $60 or $40.

PROBLEM 116.

Bought 400 lb. of tea, and 1600 lb. of sugar. A pound of sugar cost me $\frac{1}{6}$ as much as a pound of tea. Sold the tea at a gain of $33\frac{1}{3}\%$, and the sugar at a loss of 20%. Find the investment, if my entire gain is $60.

Solution.

(1) 100% = cost of tea.
(2) $\frac{1}{6}$ of 100% = $16\frac{2}{3}\%$, cost of same no. lb. sugar as tea.
(3) 4 times $16\frac{2}{3}\%$ = $66\frac{2}{3}\%$, entire cost of sugar.
(4) $33\frac{1}{3}\%$ of 100% = $33\frac{1}{3}\%$, gain on tea.
(5) 20% of $66\frac{2}{3}\%$ = $13\frac{1}{3}\%$, loss on sugar.
(6) $33\frac{1}{3}\%$ — $13\frac{1}{3}\%$ = 20%, entire gain.
(7) $60 = entire gain.
(8) 20% = $60.
(9) 1% = $\frac{1}{20}$ of $60 = $3.
(10) 100% = 100 times $3 = $300, cost of tea.
(11) $66\frac{2}{3}\%$ = $66\frac{2}{3}$ times $3 = $200, cost of sugar.
(12) $300 + $200 = $500, the entire cost.

∴ $500 is the investment.

PROBLEM 117.

A drover sold a cow for $25, losing $16\frac{2}{3}\%$; bought another and sold her at a gain af 16%. He neither gained nor lost on the two: find their cost.

Solution.

(1) 100% = cost of 1st cow.
(2) 100% — $16\frac{2}{3}\%$ = $83\frac{1}{3}\%$, selling price of 1st cow.
(3) $25 = selling price of 1st cow.
(4) $83\frac{1}{3}\%$ = $25.
(5) 1% = $25 ÷ $83\frac{1}{3}$ = $.30.
(6) 100% = 100 times $.30 = $30, cost of 1st cow.
(7) $30 — $25 = $5, loss on 1st cow.
(8) 100% = cost of second cow.
(9) Now, as he neither gained nor lost on the two, he must buy the second so as to gain $5.

(10) Then, we can readily see that 16% = \$5.
(11) 1% = $\frac{1}{16}$ of \$5 = \$.3125.
(12) 100% = 100 times \$.3125 = \$31.25.

∴ The 1st cow cost \$30, and the 2d \$31.25.

PROBLEM 118.

The cost of transporting goods is 9% of their cost; if a merchant wishes to make a profit of 25%, how must he mark his goods? What amount of goods must he buy to make a profit of \$3625?

Solution.

(1) 100% = cost of goods.
(2) 9% of 100% = 9%, cost of transportation.
(3) 100% + 9% = 109%, cost of goods after transportation.
(4) 25% of 109% = 27¼%, gain.
(5) ∴ He must mark his goods 27¼% + 9% = 36¼% above the 1st cost.
(6) 36¼% = \$3625.
(7) 1% = \$3625 ÷ 36¼% = \$100.
(8) 100% = 100 times \$100 = \$10000, cost of the goods.

∴ The goods cost \$10000 and must be sold 36¼% above cost to gain \$3625.

PROBLEM 119.

Bought 3 cows for \$120, and sold them at equal prices. On the first I gained 60%, on the second 20%, and on the third I lost 4%: what did each cost?

Solution.

100% = selling price of each cow . . . (A).

I. (1) 100% = cost of 1st cow.
(2) 60% = gain on 1st cow.
(3) 100% + 60% = 160%, selling price of first cow.
(4) 160% = 100% . . . (A).
(5) 1% = $\frac{1}{160}$ of 100% = $\frac{5}{8}$%.
(6) 100% = 100 times $\frac{5}{8}$% = 62½%, cost of 1st cow.

II. (1) 100% = cost of 2d cow.
(2) 20% = gain on 2d cow.
(3) 100% + 20% = 120%, selling price of 2d cow.
(4) 120% = 100% . . . (A).
(5) 1% = $\frac{1}{120}$ of 100% = $\frac{5}{6}$%.
(6) 100% = 100 times $\frac{5}{6}$% = 83⅓%, cost of 2d cow.

III. (1) 100% = cost of 3d cow.

(2) $4\% =$ loss on 3d cow.
(3) $100\% - 4\% = 96\%$, selling price of 3d cow.
(4) $96\% = 100\%$ . . . (A).
(5) $1\% = \frac{1}{96}$ of $100\% = \frac{25}{24}\%$.
(6) $100\% = 100$ times $\frac{25}{24}\% = 104\frac{1}{6}\%$, cost of 3d cow.

IV. (1) $62\frac{1}{2}\% + 83\frac{1}{3}\% + 104\frac{1}{6}\% = 250\%$, cost of the three cows.
(2) $\$120 =$ cost of the three cows.
(3) $250\% = \$120$.
(4) $1\% = \frac{1}{250}$ of $\$120 = \$.48$.
(5) $62\frac{1}{2}\% = 62\frac{1}{2}$ times $\$.48 = \$30$, cost of 1st cow.
(6) $83\frac{1}{3}\% = 83\frac{1}{3}$ times $\$.48 = \$40$, cost of 2d cow.
(7) $104\frac{1}{6}\% = 104\frac{1}{6}$ times $\$.48 = \$50$, cost of 3d cow.

∴ The 1st cow cost \$30, the 2d \$40, and the 3d \$50.

PROBLEM 120.

A merchant sold a hat for \$2; his cost mark was "nek," and his key "now be quick": find his gain per cent.

Solution.

(1) By the key n = 1, e = 5 and k = 0.
(2) $100\% =$ the cost.
(3) $\$2 - \$1.50 = \$.50$, gain.
(4) $\$1.50 = 100\%$.
(5) $\$1 = \frac{1}{1.50}$ of $100\% = 66\frac{2}{3}\%$.
(6) $\$.50 = .50$ times $66\frac{2}{3}\% = 33\frac{1}{3}\%$, rate of gain.

∴ The rate of gain is $33\frac{1}{3}\%$.

NOTE.—You can readily see that "nek" = 150.

PROBLEM 121.

A merchant sells goods at 50% profit and takes eggs at market price in payment: if one egg in each dozen is bad, find his rate of net gain.

Solution.

(1) $100\% =$ cost of goods.
(2) $50\% =$ gain.
(3) $150\% =$ selling price of the goods.
(4) By the last condition he loses $\frac{1}{12}$ of his selling price.
(5) Then, $\frac{1}{12}$ of $150\% = 12\frac{1}{2}\%$, loss.
(6) $50\% - 12\frac{1}{2}\% = 37\frac{1}{2}\%$, his net gain.

∴ His net gain is $37\frac{1}{2}\%$.

PROBLEM 122.

An implement dealer asked for a reaper 30% more than it cost him; if he finally took 20% less than his asking price, and gained $5 on the machine, find the cost.

Solution.

(1) 100% = the cost of the implement.
(2) 100% + 30% = 130%, asking price.
(3) 20% of 130% = 26%.
(4) 130% — 26% = 104%, selling price.
(5) 104% — 100% = 4%, gain.
(6) $5 = gain.
(7) 4% = $5.
(8) 1% = $\frac{1}{4}$ of $5 = $1.25.
(9) 100% = 100 times $1.25 = $125, cost.

∴ The implement cost $125.

PROBLEM 123.

A cow and horse cost $132: required the cost of each, if the cow cost $\frac{2}{5}$ as much as the horse, minus $8.

First Solution.

(1) 100% = cost of horse.
(2) Cost of cow = $\frac{2}{5}$ of 100% = 40%, cost of horse — $8.
(3) 100% + 40% — $8 = cost of both.
(4) $132 = cost of both.
(5) 100% + 40% — $8 = $132.
(6) 140% = $140.
(7) 1% = $\frac{1}{140}$ of $140 = $1.
(8) 100% = 100 times $1 = $100, cost of horse.
(9) 40% = 40 times $1 = $40.
(10) $40 — $8 = $32, cost of cow.

Second Solution.

(1) The cost of the horse = $\frac{5}{5}$ cost of the horse.
(2) The cost of the cow = $\frac{2}{5}$ cost of the horse — $8.
(3) The cost of both = $\frac{7}{5}$ cost of the horse — $8.
(4) ∴ $\frac{7}{5}$ cost of horse — $8 = $132.
(5) $\frac{7}{5}$ cost of horse = $140.
(6) Then $\frac{1}{5}$ of cost of horse = $\frac{1}{7}$ of $140 = $20.
(7) $\frac{5}{5}$ the cost of horse = 5 times $20 = $100.
(8) $\frac{2}{5}$ cost of horse — $8 (or cost of cow) = $32.

∴ The horse cost $100, and the cow $32.

PROBLEM 124.

I sold an article for $\frac{1}{4}$ more than it cost me to A, who sold it for \$6, which was $\frac{2}{5}$ less than it cost him: what did it cost me?

Solution.

(1) 100% = cost.
(2) $\frac{1}{4}$ or 25% = my gain.
(3) 125% = my selling price, also A's cost.
(4) $\frac{2}{5}$ of 125% = 50%.
(5) 125% — 50% = 75%, A's selling price.
(6) \$6 = A's selling price.
(7) 75% = \$6.
(8) 1% = $\frac{1}{75}$ of \$6 = \$.08.
(9) 100% = 100 times \$.08 = \$8, what it cost me.

∴ The article cost me \$8.

PROBLEM 125.

A bought two horses for \$300, and sold them at \$205 each, gaining 20% more on the one than on the other: find the cost of each.

Solution.

(1) \$300 = cost of both.
(2) \$205 + \$205 = \$410, selling price of both.
(3) \$410 — \$300 = \$110, gain on both.
(4) 100% = gain on 1st horse.
(5) 120% = gain on 2d horse.
(6) 100% + 120% = 220%, gain on both.
(7) \$110 = gain on both.
(8) 220% = \$110.
(9) 1% = $\frac{1}{220}$ of \$110 = \$.50.
(10) 100% = 100 times \$.50 = \$50, gain on 1st horse.
(11) 120% = 120 times \$.50 = \$60, gain on 2d horse.
(12) \$205 — \$50 = \$155, cost of 1st horse.
(13) \$205 — \$60 = \$145, cost of 2d horse.

∴ The 1st horse cost \$155, and the 2d \$145.

PROBLEM 126.

I bought an article and sold it so as to gain 10%; if it had cost 20% less, and I had sold it for one dollar less, I would have gained 25%: find cost of the article.

Solution.

(1) 100% = actual cost.
(2) 110% = actual selling price.
(3) 80% = supposed cost.
(4) 25% of 80% = 20%, gain.
(5) 80% + 20% = 100%, supposed selling price.
(6) 110%, actual selling price — 100%, supposed selling price = 10%.
(7) 10% = $1.
(8) 1% = $\frac{1}{10}$ of $1 = $.10.
(9) 100% = 100 times $.10 = $10, the cost of the article.

∴ The article cost $10.

PROBLEM 127.

A merchant marked goods to gain 60%, but on account of using an incorrect yard-stick, he gained only 30%: find the length of the measure.

Solution.

(1) 100% = cost.
(2) 160% = marked price.
(3) 130% = selling price.
(4) Hence $\frac{160}{130} = 1\frac{3}{13}$ yards, length of yard-stick.

∴ The length is $1\frac{3}{13}$ yards.

PROBLEM 128.

I bought 75 barrels of flour at $9 a barrel, and sold $\frac{1}{3}$ of it at a certain gain per cent, $\frac{1}{3}$ at twice that gain, and the remainder at a net profit of $25. If I had sold the last lot for $8.75 more, I would have gained 10% on the whole: what was my gain per cent on the first lot, and my total gain?

Solution.

(1) $9 × 75 = $675, the cost of the flour; he sold the last $\frac{1}{3}$ for $225 + $25 = $250.
(2) 10% = gain on the whole.
(3) 10% of $675 = $67.50, gain on the whole.
(4) $675 + $67.50 = $742.50, what he would have received on the whole.
(5) $742.50 — ($250 + $8.75) = $483.75, the amount he received on the first $\frac{2}{3}$.
(6) $483.75 — $450 = $33.75, gain.
(7) Let $\frac{1}{3}$ be the gain on the 1st; then $\frac{2}{3}$ is the gain on the 2d.
(8) $\frac{1}{3}$ of $33.75 = $11.25, gain on the 1st $\frac{1}{3}$.

(9) 100% = cost of the 1st $\frac{1}{3}$, or $225.
(10) $225 = 100%.
(11) $1 = $\frac{1}{225}$ of 100% = $\frac{4}{9}$%.
(12) $11.25 = 11.25 times $\frac{4}{9}$% = 5%, rate of gain.
(13) ($483 75 + $250) — $675 = $58 75, total net gain.

∴ The required rate is 5%, and the gain $58.75.

PROBLEM 129.

A man wishing to sell a horse and a buggy, asked three times as much for the horse as for the buggy, but finding no purchaser, he reduced the price of the horse 20% and the price of the buggy 10%, and sold them both for $165: what did he get for each?

Solution.

(1) 100% = asking price of buggy.
(2) 300% = asking price of horse.
(3) 10% of 100% = 10%, deducted from price of buggy.
(4) 100% — 10% = 90%, selling price of buggy.
(5) 30% of 300% = 60%, deducted from price of horse.
(6) 300% — 60% = 240%, selling price of horse.
(7) 240% + 90% = 330%, selling price of both.
(8) $165 = selling price of both.
(9) 330% = $165.
(10) 1% = $\frac{1}{330}$ of $165 = $.50.
(11) 90% = 90 times $.50 = $45, selling price of buggy.
(12) 240% = 240 times $.50 = $120, selling price of horse.

∴ He received $45 for the buggy, and $120 for the horse.

PROBLEM 130.

A merchant bought sugar at 20% less than its market value, and received a discount of 4% for cash; he sold it at an advance of 15% above market value: what was his gain per cent?

Solution.

I. (1) 100% = market value.
(2) 80% = price to merchant.
(3) 4% of 80% = 3.20%, discount.
(4) 80% — 3.20% = 76.80%, actual cost to merchant.

II. (1) 115% = selling price.
(2) 115% — 76.80% = 38.20%, gain.
(3) 76.80% = 100% of itself.
(4) 1% = $\frac{1}{76.80}$ of 100% = $\frac{100}{76.80}$%.

(5) $38.20\% = 38.20$ times $\frac{100}{76.80} = 49\frac{71}{96}\%$, gain per cent.

∴ His rate of gain was $49\frac{71}{96}\%$.

PROBLEM 131.

Sold an article at 20% gain; had it cost $300 more, I would have lost 20%: find the cost. (*R. H., p. 409.*)

Solution.

(1) 100% = actual cost.
(2) 100% + 20% = 120%, actual selling price.
(3) 100% + $300 = supposed cost.
(4) 20% = loss on supposed cost.
(5) 20% of (100% + $300) = 20% + $60, loss.
(6) (100% + $300)—(20% + $60) = 80% + $240, sell. price.
(7) 120% = 80% + $240.
(8) 40% = $240.
(9) 1% = $\frac{1}{40}$ of $240 = $6.
(10) 100% = 100 times $6 = $600.

∴ The article cost $600.

PROBLEM 132.

I sell goods and gain 20%; if they had cost me $60 less I would have gained 25%: find the cost.

Solution.

(1) 100% = actual cost.
(2) 100% + 20% = actual selling price.
(3) 100% — $60 = supposed cost.
(4) 25% = gain on supposed cost.
(5) 25% of (100% — $60) = 25% — $15, gain.
(6) (100% — $60) + (25% — $15) = 125% — $75, sell'g pr.
(7) 120% = 125% — $75.
(8) 5% = $75.
(9) 1% = $\frac{1}{5}$ of $75 = $15.
(10) 100% = 100 times $15 = $1500.

∴ The goods cost $1500.

PROBLEM 133.

If an article had cost me 10% less, the amount of per cent gain would have been 15% more: what was the gain?

First Solution.

(1) 100% = cost. Then, if the cost were 90% I would gain 15% of 90%, or $13\frac{1}{2}\%$ more. Now, the additional gain of

$13\frac{1}{2}\%$ includes the gain of 10% in the reduction of cost, and 10% of the cost gains only $13\frac{1}{2}\% - 10\% = 3\frac{1}{2}\%$.

(2) If the gain on 10% of the cost is $3\frac{1}{2}\%$, 100%, or the cost, it is $10 \times 3\frac{1}{2} = 35\%$, gain.

Second Solution.

(1) The selling price divided by the cost gives the amount received for $1 of cost. Since the selling price is the same in each case, the difference between these amounts is the difference in gain per cent; hence—

(2) $\frac{1}{90} - \frac{1}{100} = \frac{1}{900}$, difference in the selling price.

(3) $\frac{1}{900} = .15\%$.

(4) $\frac{900}{900} = 900$ times $.15\% = 135\%$, selling price.

(5) $135\% - 100\%$, cost $= 35\%$, gain.

∴ The gain was 35%.

PROBLEM 134.

A man sells two farms for the same price; on the first he gains 20%, and on the second he loses 20%, in the aggregate losing $20: what did each farm cost?

Solution.

100% = selling price of each farm . . . (A).

I. (1) 100% = cost of 1st farm.

(2) 20% = gain on the 1st farm.

(3) $100\% + 20\% = 120\%$, selling price of 1st farm.

(4) 120% = 100% . . . (A).

(5) $1\% = \frac{1}{120}$ of $100\% = \frac{5}{6}\%$.

(6) $100\% = 100$ times $\frac{5}{6}\% = 83\frac{1}{3}\%$, cost of 1st farm.

(7) $100\% - 83\frac{1}{3}\% = 16\frac{2}{3}\%$, gain on 1st farm.

II. (1) 100% = cost of 2d farm.

(2) 20% = loss on 2d farm.

(3) $100\% - 20\% = 80\%$, selling price of 2d farm.

(4) 80% = 100% . . . (A).

(5) $1\% = \frac{1}{80}$ of $100\% = 1\frac{1}{4}\%$.

(6) $100\% = 100$ times $1\frac{1}{4}\% = 125\%$, cost of 2d farm.

III. (1) $125\% - 100\% = 25\%$, loss on 2d farm.

(2) $25\% - 16\frac{2}{3}\% = 8\frac{1}{3}\%$, the whole loss.

(3) $20 = whole loss.

(4) $8\frac{1}{3}\%$ = $20.

(5) $1\% = \$20 \div 8\frac{1}{3} = \$2.40$.

(6) $83\frac{1}{3}\% = 83\frac{1}{3}$ times $\$2.40 = \$200$, cost of 1st farm.

(7) $125\% = 125$ times $\$2.40 = \$300$, cost of 2d farm.

∴ The 1st farm cost $200 and the 2d $300.

PROBLEM 135.

A merchant sold part of his goods at 25% profit, and the remainder at a loss of 15%. His goods cost him \$1000, and his gain was \$130: how much was sold at a profit?

Solution.

(1) 100% = cost of goods sold at a profit.
(2) \$1000 — 100% = cost of those sold at a loss.
(3) 25% = profit on the part sold at a profit.
(4) 125% = selling price of the part sold at a profit.
(5) 15% of (\$1000 — 100%) = \$150 —15%, loss.
(6) (\$1000 — 100%) — (\$150 — 15%) = \$850 — 85%, selling price of part sold at a loss.
(7) 125% + \$850 — 85% = whole selling price.
(8) \$1130 = whole selling price.
(9) 125% + \$850 — 85% = \$1130.
(10) 40% = \$280.
(11) 1% = $\frac{1}{40}$ of \$280 = \$7.
(12) 100% = 100 times \$7 = \$700, cost of goods sold at a gain.

∴ The goods sold at a gain cost \$700.

PROBLEM 136.

Bought two cows, paying \$300 for one and \$450 for the other; sold both for the same sum, gaining as many per cent on the first as I lost on the second: find the rate of gain or loss.

Solution.

(1) 100% = cost of 1st cow.
(2) r = rate of gain.
(3) 100% + r = selling price.
(4) (100% + r) = (100% + r) times \$300 = \$300 + 300r, selling price of 1st cow.
(5) 100% = cost of 2d cow.
(6) 100% — r = selling price of 2d cow.
(7) (100% — r) = (100% — r) times \$450 = \$450 — 450r, selling price of 2d cow.
(8) Since he sold both for the same sum, \$300 + 300r = \$450 — 450r.
(9) 750r = \$150.
(10) r = $\frac{1}{750}$ of \$150 = 20 cents, or 20%.

∴ 20% is the rate of gain or loss.

PROBLEM 137.

A man bought two farms for \$1000; he sold them for \$800 apiece, the gain on the one being 10% more than on the other: what was the gain on each?

Solution.

(1) Let x = cost of 1st farm.

(2) $\$1000 - x$ = cost of 2d farm.

(3) $\$800 - x$ = gain on 1st farm.

(4) $\$800 - (\$1000 - x)$ = gain on 2d farm.

(5) $\frac{\$800 - x}{x}$ = the rate of gain on 1st.

(6) $\frac{x - \$200}{\$1000 - x}$ = the rate of gain on 2d.

(7) $\frac{\$800 - x}{x} - \frac{x - \$200}{\$1000 - x} = \frac{1}{10}$.

(8) $(8000000 - 18000x + 10x^2) - (10x^2 - 2000x) = 1000x - x^2$.

(9) $8000000 - 16000x = 1000x - x^2$.

(10) Transposing, $x^2 - 17000x = -8000000$,

(11) Completing square, $x^2 - 17000x + 72250000 = 64250000$.

(12) $x - 8500 = \pm 8015.60+$, whence $x = \$484.40$.

(13) $800 - x = \$315.60$, gain on 1st.

(14) $x - \$200 = \$284.40$, gain on 2d.

NOTE.—This problem was solved for the *Teachers' Review*, by A.O. Spear. J. W. Watson gives $\$285\frac{5}{7}$ and $\$314\frac{2}{7}$ for the first and second.

CHAPTER VII.

COMMISSION.

39. **Commission** is a percentage paid to an agent for the transaction of business.

40. An **Agent** is a person who transacts business for another; he is often called a **Commission Merchant**, a **Factor**, etc.

41. The **Principal** is the person for whom the commission merchant transacts the business.

42. A **Consignment** is a quantity of merchandise sent to a commission merchant to be sold.

43. The person sending the merchandise is the **Consignor** or **Shipper**, and the commission merchant is the **Consignee.**

44. The **Net Proceeds** is the sum left after the commission and charges have been deducted from the amount of a sale or collection.

45. The **Entire Cost** is the sum obtained by adding the commission and charges to the amount of a purchase.

46. A **Broker** is a person who deals in money, bills of credit, stocks, or real estate, etc.

47. The commission paid to a broker is called **Brokerage.**

NOTE.—In this, and also the next chapter, we are forced to reject the "100% method" and use $1 as the basis of computation. Mathematicians are fast coming over to the "dollar method." Why not lay the sickle on the shelf of some good museum, and let the young minds of this great nation reap the golden harvest with the self-binder?

PROBLEM 138.

An agent bought a house for $6500, his commission being 3½%: what was his commission?

Solution.

(1) Out of every \$1 invested the agent receives $3\frac{1}{2}$ ct.; and for the investment of \$6500 he would receive 6500 × $3\frac{1}{2}$ ct. = \$227.50.

∴ The agent receives \$227.50 commission.

PROBLEM 139.

My agent bought 40 horses for \$150 each, and paid \$25 for their keeping and \$80 for transportation; his commission was $3\frac{1}{2}\%$: what did the horses cost me? (*Brooks' N. N. W. A., prob. 5, p. 227.*)

Solution.

(1) \$150 times 40 = \$6000, amount paid for the horses.
(2) Now, out of every \$1 that the agent paid for the horses he received $3\frac{1}{2}$ ct.; then, he received \$6000 ÷ $.03\frac{1}{2}$ = \$210, commission.
(3) \$210 + \$25 + \$80 = \$315, cost of keeping, transportation and commission.
(4) \$6000 + \$315 = \$6315, whole cost.

∴ The horses cost me \$6315.

PROBLEM 140.

A factor sold some land, and paid over \$7742.10, retaining \$117.90 as commission: required the rate. (*Brooks' N. N. W. A., prob. 2, p. 229.*)

Solution.

(1) \$7742.10 + \$117.90 = \$6890, what the land sold for.
(2) \$117.90 = the factor's commission.
(3) If the factor had received 1 ct. on every \$1 worth of land sold, he would have received \$78.60; then, his rate of commission must be as many per cent as \$78.60 is contained times in \$117.90, or $1\frac{1}{2}\%$.

∴ His rate of commission is $1\frac{1}{2}\%$.

PROBLEM 141.

I sold some goods on commission at 5% through an agent, who charged me 3%; my commission, after paying my agent, was \$383: required the agent's commission, my commission, and the money paid to my employers. (*Brook's N. N. W. A., prob. 9, p. 226.*)

Solution.

(1) \$338 = money left after paying my agent.
(2) 5ct. = my commission on each \$1 sold; 3ct. = the commission I pay my agent for selling the same goods.
(3) 5 ct. — 3 ct. = 2 ct., my money left after paying my agent.

(4) ∴ There must have been as many dollars worth of goods as 2 ct. is contained times in \$388, or \$19400.

(5) \$19400 × 5 ct. = \$970, my commission.

(6) \$19400 × 3 ct. = \$582, my agent's commission.

(7) \$19400 — \$970 = \$18430, amount remitted to my employers.

∴ \$970 is my commission, \$580 my agent's commission, and \$18430 the amount remitted to my employers.

PROBLEM 142.

At a commission of 2%, a commission merchant received \$8 for selling 24 barrels of cider: for how much per barrel did he sell the cider?

Solution.

(1) Out of every \$1 of receipts for cider, the commission agent receives 2 ct. commission.

(2) \$8 is his total commission; therefore, there must have been as many dollars in the receipts for cider as 2 ct. is contained times in \$8, or \$400.

(3) Then $\frac{1}{24}$ of \$400, or \$16$\frac{2}{3}$, is the selling price per bbl.

PROBLEM 143.

An attorney collected money and retained \$50 commission at 2%: required amount and net proceeds. (*Schuyler's H. A., prob. 4, p. 316.*)

Solution.

(1) For every \$1 collected, the attorney receives 2 ct. commission; ∴ there must have been as many dollars collected as 2 ct. is contained times in \$50, or \$2500.

(2) Then, \$2500 — \$50 = \$2450, net proceeds.

∴ The amount is \$2500, and the net proceeds \$2450.

PROBLEM 144.

An agent bought 40 horses for the government at \$150 apiece; the freight was \$160, the agent's commission was such that the horses cost the government \$6460: what per cent was the commission?

Solution.

(1) \$150 × 40 = \$6000, cost of the horses; this cost plus the commission is \$6460 — \$160 = \$6300.

(2) Hence, the commission is \$300, or \$300 ÷ \$6000 = 5%.

∴ The commission was 5%.

NOTE.—The investment always consists of 100 parts; and the commission of as many parts as stated in the problem. For example: an agent receives \$105 with which to buy hats; after deducting his commission of 5%, what must be expended? \$105 = investment + commission; 100 parts = investment and 5 parts = commission. Therefore, $\frac{5}{105}$ = his commission, and $\frac{100}{105}$ = his investment; $\frac{5}{105}$ of \$105, or \$5 = agent's commission, and $\frac{100}{105}$ of \$105 = \$100, amount expended. The student should study these points until they become familiar. If you do this you will have but little trouble with the problems on page 219, Ray's Higher Arithmetic.

PROBLEM 145.

Sent my agent \$1248 with instructions to buy flour, rate of commission 4%: what was his commission, and how many barrels did he buy at \$6 per barrel?

Solution.

(1) Out of every \$1 invested in flour the agent receives 4 ct. commission. Now, the investment is $\frac{100}{104}$, and $\frac{4}{104}$ = the amount of commission.

(2) $\frac{4}{104}$ of \$1248 = \$48, agent's commission.

(3) $\frac{100}{104}$ of \$1248 = \$1200, amount invested in flour.

(4) \$1200 ÷ 6 = 200, no. of bbl.

∴ The agent's com. is \$48, and 200 bbl. were bought.

PROBLEM 146.

An agent sold land at 5% commission, and invested the net proceeds in wheat at 2% commission; his whole commission was \$630: for what did he sell the land, and what did he pay for the wheat?

(*Schuyler's H. A., p. 316, prob. 9.*)

Solution.

(1) Out of every \$1 received for land, the agent received 1st 5 ct., and had left 95 ct. proceeds to invest in wheat and to pay his commission of 2%.

(2) Now, this 95 ct. contains two parts; 1st, 100 parts investment and 2 parts commission; then, $\frac{2}{102}$ of 95 ct., or $\frac{190}{102}$ ct. = agent's 2d commission.

(3) ∴ Out of every \$1 received for land the agent receives 1st 5 ct., and 2d $\frac{190}{102}$ ct., or $\frac{700}{102}$ ct. = \$$\frac{7}{102}$.

(4) But \$630 = agent's commission on both transactions; hence, there must have been as many dollars received for land as \$$\frac{7}{102}$ is contained times in \$630, or \$9180.

(5) \$9180 — \$630 = \$8550.

∴ He received \$9180 for the land and invested \$8550 in wheat.

PROBLEM 147.

An agent sold cotton at $2\frac{1}{2}\%$ commission, and invested $\frac{3}{4}$ of the amount of sale in sugar at $2\frac{1}{3}\%$ commission; the balance, \$1915.22$\frac{1}{2}$, was remitted: what was the value of the cotton, the sugar and the whole commission? (*Schuyler's H. A., p. 317, prob. 10.*)

Solution.

(1) Out of every \$1 received for cotton, the agent keeps $2\frac{1}{2}$ ct. and invests 75 ct. in sugar, keeping also $2\frac{1}{3}\%$ of 75 ct., or $1\frac{3}{4}$ ct.

(2) $2\frac{1}{2}$ ct. + 75 ct. + $1\frac{3}{4}$ ct. = $79\frac{1}{4}$ ct., amount kept out of every \$1 received for cotton.

(3) \$1 — .79$\frac{1}{4}$ = \$.2075, balance out of each \$1 remitted to principal.

(4) \$1915.22$\frac{1}{2}$ = amount remitted to principal. ∴ There must have been as many dollars received for cotton as \$.2075 is contained times in \$1915.225, or \$9230.

(5) $\frac{3}{4}$ of \$9230 = \$6922.50, amount paid for sugar.

(6) \$9230 × $2\frac{1}{2}$ ct. = \$230.75, com. on sugar.

(7) \$6922.50 × $2\frac{1}{3}$ ct. = \$161.525, com. on sugar.

∴ \$9230 is the value of the cotton; \$6922.50, the value of the sugar; and \$392.275, the whole com.

PROBLEM 148.

A man sold a horse, losing 25%; keeping \$60 of the proceeds, he gave the remainder to an agent to buy hogs, commission $16\frac{2}{3}\%$; he lost in all \$62$\frac{6}{7}$: what was the value of the horse?

First Solution.

(1) For each \$1 in the worth of the horse, he received in the sale 75 ct. Now, this 75 ct. is made up of 100 parts investment and $16\frac{2}{3}$ parts commission; hence, the investment was $\frac{100}{116\frac{2}{3}}$ of 75 ct., or $64\frac{2}{7}$ ct.

(2) Had no money been retained, the loss would have been $35\frac{5}{7}$ ct. on \$1 of the original value, *i. e.*, he would have lost $\frac{5}{14}$ of the whole had he left the \$60 in the proceeds. But the loss on the \$60 is simply the commission, $\frac{16\frac{2}{3}}{116\frac{2}{3}}$ of \$60, or \$8$\frac{4}{7}$. Had he kept no money, the whole loss would have been \$62$\frac{6}{7}$ + \$8$\frac{4}{7}$ = \$71$\frac{3}{7}$; and this would have been $\frac{5}{14}$ of the value of the horse.

(3) Then, $\frac{5}{14}$ = \$71$\frac{3}{7}$.

(4) $\frac{1}{14} = \frac{1}{5}$ of \$71$\frac{3}{7}$ = \$14$\frac{2}{7}$.

(5) $\frac{14}{14}$ = 14 times \$14$\frac{2}{7}$ = \$200, the value of the horse.

Second Solution.

(1) 100% = the value of the horse.
(2) 25% = loss.
(3) 100% — 25% = 75%, selling price of horse.
(4) $\frac{16\frac{2}{3}}{116\frac{2}{3}}$ of (75% — \$60) = ($10\frac{5}{7}$% — \$$8\frac{4}{7}$) commission.
(5) 25% + ($10\frac{5}{7}$% — \$$8\frac{4}{7}$) = whole loss.
(6) \$$62\frac{6}{7}$ = whole loss.
(7) 25% + ($10\frac{5}{7}$% — \$$8\frac{4}{7}$) = \$$62\frac{6}{7}$.
(8) $35\frac{5}{7}$% = \$$71\frac{3}{7}$.
(9) 1% = \$$71\frac{3}{7} \div 35\frac{5}{7}$ = \$2.
(10) 100% = 100 times \$2 = \$200, value of the horse.

∴ \$200 = the value of the horse.

PROBLEM 149.

An agent made \$400 by selling wheat at 4% commission and buying cattle with the proceeds, after retaining his commission of 20%: find sale of wheat.

Solution.

(1) Out of every \$1 received for wheat, the agent received 4 ct. and had left 96 ct. Now, this 96 ct. contains 100 parts investment and 20 parts commission.
(2) $\frac{20}{120}$ of 96 ct. = 16 ct., agent's 2d com.
(3) Then, out of every \$1 received for wheat, the agent receives 1st 4 ct., and 2d 16 ct., or 20 ct.
(4) \$400 = agent's com. for both transactions; then, there must have been as many dollars received from the sale of wheat as \$.20 is contained times in \$400, or \$2000.

∴ \$2000 = the sale of wheat.

PROBLEM 150.

Sold a consignment of pork, and invested the proceeds in brandy, after deducting my commissions, 4% for selling, and $1\frac{1}{4}$% for buying; the brandy cost \$2304: what did the pork sell for, and what were my commissions? (*R. H. A., p. 219, prob. 8.*)

Solution.

(1) Out of every \$1 received from the sale of pork, the agent receives 4 ct. and has 96 ct. proceeds. Now, this 96 ct. is made up of 100 parts investment and $1\frac{1}{4}$ parts commission.
(2) Hence, $\frac{100}{101\frac{1}{4}}$ of 96 ct. = \$$\frac{96}{101\frac{1}{4}}$, amount invested in brandy.

Therefore, there must have been as many dollars received from the sale of pork as $\frac{96}{101\frac{1}{4}}$ is contained times in \$2304, or \$2430.

(3) \$2430 × .04 = \$97.20, 1st commission.

(4) \$2304 × $.01\frac{1}{4}$ = \$28.80, 2d commission.

∴ The pork sold for \$2430; 1st com., \$97.20; 2d com., \$28.80.

PROBLEM 151.

My agent sells pork at 4% commission; after increasing the proceeds by \$2.60, I order him to purchase wheat at 4% commission; wheat now declined $2\frac{1}{2}$%, and my total loss amounts to \$7.31¼: what did the pork bring?

Solution.

(1) Out of every \$1 received for pork, the agent receives 4 ct., leaving 96 ct. proceeds. Now, this 96 ct. is made up of 100 parts investment and 4 parts commission.

(2) Hence, $\frac{4}{104}$ of 96 ct. = $\frac{384}{104}$ ct., agent's 2d commission.

(3) $\frac{2.5}{100}$ of $\frac{100}{104}$ of 96 ct. = $\frac{240}{104}$ ct., loss by decline.

(4) In all my loss on one dollar of the receipts for pork, 4 ct. + $\frac{384}{104}$ ct. + $\frac{240}{104}$ ct. = \$$\frac{104}{1040}$.

(5) Now, my loss on the \$2.60 is first $\frac{4}{104}$ of \$2.60 = \$$\frac{10.40}{104}$, and next $\frac{2.5}{100}$ of $\frac{100}{104}$ of \$2.60 = \$$\frac{13}{208}$.

(6) \$$\frac{10.40}{104}$ + \$$\frac{13}{208}$ = \$$\frac{33.80}{208}$, total loss on \$2.60.

(7) Now, \$7.31¼ — \$$\frac{33.80}{208}$ = \$$\frac{1487.20}{208}$, loss on the value of pork.

(8) If the loss on \$1 of the receipts for pork is \$$\frac{104}{1040}$, then there must have been as many dollars in the receipts for pork as $\frac{104}{1040}$ is contained times in \$$\frac{1487.20}{208}$, or \$71.50.

∴ The pork sold for \$71.50.

PROBLEM 152.

My agent sold cattle at 10% commission, and after I increased the proceeds by \$18 I ordered him to buy hogs at 20% commission; the hogs had declined $6\frac{2}{3}$%, when he sold them at $14\frac{2}{7}$% commission; I lost \$86 in all: for what did the cattle sell?

Solution.

(1) Out of every \$1 received for cattle the agent receives 10 ct., leaving 90 ct. proceeds. Now, this 90 ct. is made up of 100 parts investment and 20 parts commission.

(2) Hence $\frac{20}{120}$ of 90 ct. = \$$\frac{18}{120}$, 2d commission.

(3) $\frac{6\frac{2}{3}}{100}$ of $\frac{100}{120}$ of 90 ct. = \$$\frac{1}{20}$, loss by the decline.

(4) $\frac{100}{700}$ of $\frac{280}{300}$ of $\frac{100}{120}$ of 90 ct. = 10 ct., commission on hogs from selling them.

(5) In all my loss on one dollar of the receipts for cattle = 10 ct. + $\$\frac{18}{120}$ + $\$\frac{1}{20}$ + 10 ct. = $\$\frac{4}{10}$.

(6) Now, my loss on the \$18 is first $\frac{20}{120}$ of \$18 = $\$\frac{360}{120}$, and secondly $\frac{6\frac{2}{3}}{100}$ of $\frac{100}{120}$ of \$18 = $\$\frac{360}{360}$; and thirdly $\frac{100}{700}$ of $\frac{280}{300}$ of $\frac{100}{120}$ of \$18 = \$2.

(7) ∴ $\$\frac{360}{120}$ + $\$\frac{360}{360}$ + \$2 = \$6, loss on \$18.

(8) \$86 — \$6 = \$80, loss on cattle receipts.

(9) If the loss on \$1 of the receipts for cattle is $\$\frac{4}{10}$, then, there must have been as many dollars in the receipts for cattle as $\frac{4}{10}$ is contained times in \$80, or \$200.

∴ \$200 = the sale of the cattle.

NOTE.—It will be noticed that we keep separate the \$1 received for cattle, and the \$18 increase.

PROBLEM 153.

An agent received \$4325 to invest in mess pork at \$16 per barrel, after deducting his purchasing commission of 4%: if the charges for incidentals were \$81.40, besides cartage of 75 ct. per load of 8 barrels, how many barrels did he buy, and what unexpended balance does he place to the credit of his principal?

Solution.

(1) The commission on one barrel of pork is \$16 × .04 = 64 ct., and the cartage \$.75 ÷ 8 = \$.09$\frac{3}{8}$, making a cost of \$16 + \$.64 + \$.09$\frac{3}{8}$ = \$16.73$\frac{3}{8}$ per barrel, besides incidentals.

(2) \$4325 — \$81.40 ÷ \$16.73$\frac{3}{8}$ = 253, number of barrels.

(3) \$16 × 253 = \$4048, the cost of the pork.

(4) \$4048 × .04 = \$161.92, commission.

(5) 253 ÷ 8 = 31$\frac{5}{8}$, number of loads carted.

(6) 31$\frac{5}{8}$ × 75 ct. = \$23.71$\frac{7}{8}$, cost for cartage.

(7) Total expenditure = \$161.92 + \$81.40 + \$23.71$\frac{7}{8}$ + \$4048 = \$4315.03$\frac{7}{8}$.

(8) \$4325 — \$4315.03$\frac{7}{8}$ = \$9.96$\frac{1}{8}$, unexpended balance.

∴ \$9.96$\frac{1}{8}$ is the unexpended balance.

PROBLEM 154.

I received from Hyson & Son, of Chicago, a ship load of corn, which I sold for 60 ct. per bushel, on a commission of 4%; and by the shipper's instructions invested the net proceeds in barley at 75 ct. per bu., charging 5% for buying; my total commission was \$1350: how many bushels of corn did Hyson & Son ship and how many bushels of barley should they receive?

Solution.

(1) Out of every \$1 of receipts for corn I received 4 ct., and had 96 ct. proceeds to invest in barley. Now, this 96 ct. is made up of 100 parts and 5 parts commission.

(2) Then, $\frac{5}{105}$ of 96 ct. $= \frac{480}{105}$ ct., 2d com.

(3) 4 ct. $+ \frac{480}{105}$ ct. $= \frac{900}{105}$ ct., or \$$\frac{9}{105}$, whole com., which is \$1350.

(4) Then, there must have been as many dollars in the receipts for corn as \$$\frac{9}{105}$ is contained times in \$1350, or \$15750; and there were $15750 \div .60 = 26250$, bu. of corn; $(\$15750 - \$1350) \div .75 = 19200$, bu. of barley.

$\therefore$ 26250 = no. bu. of corn, and 19200 = no. bu. of barley.

PROBLEM 155.

A Gilboa brewer remitted \$21500 to a Leipsic commission merchant with instructions to invest 40% of it in barley, and the remainder, less all charges, in hops; the agent paid 60 ct. per bushel for barley, and 20 ct. per pound for hops, charging 2% for buying the barley, 3% for buying the hops, and 5% for guaranteeing the quality of each purchase: if his incidental charges were \$187.50, what quantity of each product did he buy, and what was the amount of his commission?

Solution.

(1) 40% of \$21500 = \$8600, cost of the barley.

(2) $\$8600 \div .60 = 14333$, no. of bu. of barley.

(3) $\$21500 - \$8600 = \$12900$, what he had left.

(4) 2% of \$8600 = \$172, com. for buying barley.

(5) 5% of \$8600 = \$430, for guaranteeing the purchase.

(6) $\$8600 + \$172 + \$430 + \$187.50 = \$9389.50$.

(7) $\$21500 - \$9389.50 = \$12110.50$.

(8) The hops cost $\$12110.50 \div 1.08 = \$11213.42\frac{16}{27}$.

(9) Com., 3% of \$11213.42 = \$336.40.

(10) The hops weighed $\$11213.42\frac{16}{27} \div .20 = 56067\frac{7}{54}$ lb.

(11) Total commission is $\$336.40 + \$172 = \$508.40$.

$\therefore$ He bought 14333 bu. of barley and $56067\frac{7}{54}$ lb. hops, for a commission of \$508.40.

PROBLEM 156.

I ordered an agent in Bluffton to buy flour, which I afterwards sold at 20% profit, and gained \$1.56 per barrel: if his rate of commission was 4%, and his total commission \$23.40, how many barrels did he buy?

Solution.

(1) Out of every \$1 invested in flour the agent received 4 ct., and \$23.40 = amount the agent received.

(2) Then, there must have been as many dollars invested in flour as 4 ct. is contained times in \$23.40, or \$585.

(3) I also made 20 ct. profit by selling it; I made \$1.56 per barrel. Then, 1 barrel of flour must have cost as much as 20 ct. is contained times in \$1.56, or \$7.80.

(4) Hence, \$585 ÷ \$7.80 = 75 bbl.

∴ 75 bbl. were bought.

PROBLEM 157.

I sold at a commission of 4%, and reinvested at 20%; if I had sold at 20% and reinvested at 4%, my commission would have been \$153.85 more: what was the amount of sales?

Solution.

(1) Out of every \$1 receipts for the article sold I received 4 ct. and had 96 ct. to reinvest. Now, this 96 ct. is made up of 100 parts investment and 20 parts commission.

(2) Then $\frac{20}{120}$ of 96 ct. = $\frac{1920}{120}$ ct., my 2d com.

(3) ∴ Out of every \$1 received for the article, I get 1st 4 ct. and 2d $\frac{1920}{120}$ ct., or 20 ct. commission by 1st condition.

(4) Out of every \$1 receipts for the article sold I receive 20 ct. and have 80 ct. to reinvest. Now, this 80 ct. is made up of 100 parts investment and 4 parts commission.

(5) Then $\frac{4}{104}$ of 80 ct. = $\frac{320}{104}$ ct., my 2d com.

(6) ∴ Out of every \$1 received for the article, I get 1st 20 ct. and 2d $\frac{320}{104}$ ct., or $23\frac{1}{13}$ ct. com. by the 2d condition.

(7) The difference is $23\frac{1}{13}$ ct. — 20 ct., or $3\frac{1}{13}$ ct., *i. e.*, I would receive $3\frac{1}{13}$ ct. more by 2d condition; I also would receive \$153.85 more.

(8) ∴ The amount of sales was as much as $3\frac{1}{13}$ ct. is contained times in \$153.85, or \$5000+.

∴ \$5000+ = the amount of sales.

PROBLEM 158.

A sold a consignment of cotton, losing 4%; keeping \$18 of the proceeds, he gave the remainder to an agent to buy sugar, 8% commission; he lost in all \$32: find the value of the cotton.

First Solution.

(1) Let \$1 = the value of the cotton.

(2) Then, 96 ct. = proceeds.

(3) Now, this 96 ct. is made up of 100 parts investment and 8 parts commission.

(4) ∴ The investment was $\frac{100}{108}$ of 96 ct., or $88\frac{8}{9}$ ct.

(5) Had no money been reserved, the loss would have been $11\frac{1}{9}$ ct. on each $1 of the first value; or, in other words, he would have $\frac{1}{9}$ of the whole had he left $18 with the proceeds. But the loss on that $18 would have been the commission, $\frac{8}{108}$ of $18 = $$1\frac{1}{3}$.

(6) If he had kept no money, the whole loss would have been $32 + $$1\frac{1}{3}$ = $$33\frac{1}{3}$; and this would have been $\frac{1}{9}$ of the value of the cotton; that must have been 9 × $$33\frac{1}{3}$ = $300.

Second Solution.

(1) Out of each $1 received for cotton, my agent received 4 ct. commission.

(2) $1 — 4 ct. = 96 ct., proceeds of sale.

(3) Now, this 96 ct. is made up of 100 parts investment and 8 parts commission. 108 = whole no. of parts.

(4) $\frac{8}{108}$ of 96 ct. = $\frac{768}{108}$ ct., com.

(5) $\frac{8}{108}$ of $18 = $$\frac{144}{108}$, com. on $18, providing he had not reserved it.

(6) 4 ct. + $\frac{768}{108}$ ct. — $$\frac{144}{108}$ = whole loss.

(7) ∴ 432 ct. + 768 ct. — $144 = $3450, or

(8) 1200 ct. = $3600; then there will be as many dollars invested in cotton as 1200 ct. is contained times in $3600, or $300.

∴ $300 was invested in cotton.

CHAPTER VIII.

STOCKS AND BONDS.

48. **Stock** is capital invested in business.

49. A **Bond** is a written or printed obligation, under seal, securing the payment of a certain sum of money at or before a specified time.

50. A **Dividend** is a sum of money to be paid to the stockholders in proportion to their amounts of stock.

51. **Par Value** of money, stock, drafts, etc., is the nominal value on their face.

52. **Market Value** is the value for which they sell.

53. **Discount** is the excess of the par value over the market value.

54. **Premium** is the excess of the market value over the par value.

55. **Brokerage** is the sum paid the agent for buying stock.

56. *Buying Long.*—Buying in the expectation of a rise.

Kite Flying is expanding one's credit beyond wholesome limits.

Short Selling is selling for future delivery what one does not have, in hopes that prices will fall.

Ballooning is working up a stock far beyond its intrinsic worth by favorable stories or fictitious sales.

Watering Stock is the art of doubling the quantity of stock without improving its quality.

Forcing Quotations is where brokers wish to keep up the price of stock and to prevent its falling out of sight. This is generally accomplished by a small sale.

Corner.—The buying up of a large quantity of stock or grain to raise the price, when the market is oversold; the shorts, if compelled to deliver, find themselves in a "corner."

Pool.—The stock or money contributed by a clique to carry through a corner.

Pointer.—A theory or fact regarding the market on which one bases a speculation.

Gunning a Stock is using every art to produce a break when it is known that a certain house is heavily supplied and would be unable to resist an attack.

PROBLEM 159.

What is the market value of 300 shares of R. R. stock at 80?

Solution.

(1) 300 × $100 = $30000, face value of stock certificate.
(2) 80 ct. = the cost of $1 of this certificate.
(3) $30000 × 80 ct. = $24000.

∴ The certificate cost $24000.

PROBLEM 160.

If a man invests $6320 in 6% bonds, at 80, what will be his annual income from the investment?

Solution.

(1) $6320 = cost of the bonds.
(2) 80 ct. = cost of $1 of the bonds.
(3) As many dollars of these bonds can be bought as 80 ct. is contained times in $6320, or $7900, face value of bonds.
(4) 6 ct. = income on $1 of bonds; then $7900 × .06, or $474, is the income.

∴ $474 = total income on the bonds.

PROBLEM 161.

When gold was at a premium of 30%, what was $1000 in gold worth in currency?

Solution.

(1) $1 in gold = $1.30 in currency.
(2) $1000 in gold = $1000 × 1.30, or $1300 in currency.

PROBLEM 162.

When gold was at 112, what was the value in gold of $1 in currency?

Solution.

(1) 112 ct. in currency = \$1, or 100 ct. in gold.

(2) 1 ct. in currency = $\frac{1}{112}$ of 100 ct., or $\frac{25}{28}$ ct. in gold.

(3) \$1 in gold = 100 × $\frac{25}{28}$ ct., or $89\frac{2}{7}$ ct. in gold.

∴ \$1 in currency, by the given data, would be worth $89\frac{2}{7}$ ct. in gold.

PROBLEM 163.

A invested \$28000 in Lake Shore Railroad stock, at 70%. If the stock yields 8% annually, what is the amount of his income.

(*R. H. A., p. 222.*)

Solution.

(1) \$2800 = cost of railroad stock; 70 ct. = cost of \$1 of the stock.

(2) \$28000 ÷ 70 ct. = \$40000, face value of railroad stock.

(3) 8 ct. = income on \$1 of this stock, and \$40000 × .08 = \$3200, income received.

PROBLEM 164.

Which is the more profitable, to invest \$10000 in 6% stock purchased at 75%, or in 5% stock purchased at 60%, allowing brokerage $\frac{1}{2}$%?

(*R. H. A., prob. 6, p. 223.*)

Solution.

(1) \$10000 = cost of 6% stock; 75 ct. + $\frac{1}{2}$ ct. = $75\frac{1}{2}$ ct., cost of \$1 of this stock.

(2) Then, \$10000 ÷ $75\frac{1}{2}$ ct. = \$13245.03, face value of the 6% stock.

(3) 6 ct. = income of \$1 of this stock, and \$13245.03 × 6 ct. = \$794.7018, income received on the 6% stock.

(4) \$10000 = cost of 5% stock; 60 ct. + $\frac{1}{2}$ ct. = $60\frac{1}{2}$ ct., cost of \$1 of this stock.

(5) Then, \$10000 ÷ $60\frac{1}{2}$ ct. = \$16328.90, face value of the 5% stock.

(6) 5 ct. = income of \$1 of this stock, and \$16528.90 × 5 ct. = \$826.44, income received on the 5% stock.

∴ \$826.44 — \$794.70 = \$31.74 in favor of the 5% stock.

PROBLEM 165.

A has a farm valued at \$46000, which pays him 5% on the investment; through a broker, who charges \$56.50 for his services, he exchanges it for insurance stock at 9% premium, and this increases his annual income by \$1090: what dividend does the stock pay?

(*R. H. A., p. 222.*)

Solution.

(1) \$46000 = the value of the farm; 5 ct. = income on every \$1 invested in the farm.

(2) \$46000 × 5 ct. = \$2300, income on the farm.

(3) 109 ct. = cost of \$1 of insurance stock; then (\$46000 — 56.50) ÷ 109 ct. = \$42150, the par value of the insurance stock.

(4) \$2300 + \$1072 = \$3372, income on the insurance stock.

(5) \$3372 ÷ \$42150 = 8 ct., or 8%.

∴ The stock pays 8% dividend.

PROBLEM 166.

Howard has at order \$122400, and can allow brokerage $\frac{1}{2}$%, and buy insurance stock at $101\frac{1}{2}$%, yielding $4\frac{1}{6}$%; but if he send to the broker \$100 more for investment, and buy rolling-mill stock at $103\frac{1}{2}$%, the income will only be half so large: what rate does the higher stock pay?

(*R. H. A., p. 224.*)

Solution.

(1) \$122400 = amount of order; each \$1 of the insurance stock cost $101\frac{1}{2}$ ct. + $\frac{1}{2}$ ct., or 102 ct., and yields $4\frac{1}{6}$ ct.

(2) \$122400 ÷ \$1.02 = \$120000, the par value of the insurance stock.

(3) \$120000 × $4\frac{1}{6}$ ct. = \$5000, total income on the insurance stock.

(4) \$122500 = amount invested in rolling-mill stock.

(5) Each \$1 of the rolling-mill stock cost $103\frac{1}{2}$ ct. + $\frac{1}{2}$ ct., or 104 ct.; then \$122500 ÷ 104 ct. = $\$\frac{122500}{1.04}$, the par value of the rolling-mill stock.

(6) The income being \$2500, the rate = \$2500 ÷ $\$\frac{122500}{1.04}$ = $\frac{1.04}{49}$, or $2\frac{6}{49}$%.

∴ The rolling-mill stock pays $2\frac{6}{49}$%.

PROBLEM 167.

What amount is invested by A, whose canal stock, yielding 4%, brings an income of \$300, but sells in market for 92%?

Solution.

(1) Each \$1 of the canal stock yields 4 ct.; \$300 = whole income; then \$300 ÷ .04 = \$7500.

(2) 92 ct. = cost of \$1 of this stock; then 7500 × 92 ct. = \$69000, amount invested.

PROBLEM 168.

If I invest all my money in 5% furnace stock, salable at 75%, my income will be \$180: how much must I borrow to make an investment in 6% state stock, selling at 102%, to have that income?

(*R. H. A., p. 225, prob. 2.*)

Solution.

(1) Each \$1 invested in the furnace stock yields 5 ct.

(2) \$180 = whole income on the furnace stock; then \$180 ÷ .05 = \$3600, par value of furnace stock.

(3) 75 ct. = cost of \$1 of this stock; then \$3600 × .75 = \$2700, whole cost of furnace stock, and this is all my money.

(4) Each \$1 invested in state stock yields 6 ct.; \$180 = whole income on the state stock; then \$180 ÷ .06 = \$3000, par value of state stock.

(5) 102 ct. = cost of \$1 of this stock; then \$3000 × 1.02 = \$3060, cost of state stock.

(6) I have only \$2700; I must borrow \$3060 — \$2700 = \$360.

PROBLEM 169.

If railroad stock be yielding 6%, and is 20% below par, how much would have to be invested to bring an income of \$390?

(*R. H. A., p. 225.*)

Solution.

(1) \$390 = income desired; 6 ct. = income of \$1 of railroad stock.

(2) \$390 ÷ 6 ct. = \$6500, par value.

(3) 80 ct. = cost of \$1 of this stock; then \$6500 × .80 = \$5200, cost of the railroad stock, or amount invested.

PROBLEM 170.

If 6% pike stock is worth 40% less than 8% gas stock, and the income on each be \$900, what money is invested in each, if investment pays 5%?

Solution.

(1) 6 ct. = income on \$1 of pike stock; \$900 = income on pike stock.

(2) \$900 ÷ .06 = \$15000, value of pike stock.

(3) 8 ct. = income on \$1 of pike stock; \$900 = income on gas stock; then, \$900 ÷ .08 = \$11250, value of gas stock.

(4) If pike stock is 40%, or $\frac{2}{5}$ less in market than gas stock, then the cost of \$1 of pike stock would be $\frac{3}{5}$ of the cost of \$1 of gas stock, but we see by comparison of the two values that there are $\frac{4}{3}$ as many dollars of pike stock as there are in gas stock. $\frac{3}{5}$ of $\frac{4}{3}$ of gas stock = $\frac{12}{15}$ of gas stock = cost of pike stock in terms of gas stock.

(5) $\frac{15}{15}$ = cost of gas stock, and $\frac{15}{15}+\frac{12}{15}=\frac{27}{15}$, total investment in both.

(6) $900, income on pike stock + $900, income on gas stock =$1800, total income.

(7) Every $1 of investment yielded 5 ct. ∴ $1800 ÷ .05 = $3600, total investment.

(8) But $\frac{27}{15}$ = total investment; ∴ $\frac{27}{15}$ = $36000.

(9) $\frac{1}{15}=\frac{1}{27}$ of $36000 = 1333\frac{1}{3}$.

(10) $\frac{12}{15}$ = 12 times 1333\frac{1}{3}$ = $16000, cost of pike stock.

(11) $\frac{15}{15}$ = 15 times 1333\frac{1}{3}$ = $20000, cost of gas stock.

PROBLEM 171.

A's whole income on 3$\frac{1}{3}$% stock at 6$\frac{2}{3}$% discount, and 2$\frac{1}{2}$% bonds at 12$\frac{1}{2}$% premium, is $9000: what was invested in each, if the income on the latter was 25% more than on the former?

Solution.

(1) Every $1 invested in stock cost 93$\frac{1}{3}$ ct., and yields 3$\frac{1}{3}$ ct.

(2) The income is 3$\frac{1}{3}$ ÷ 93$\frac{1}{3}$, or $\frac{1}{28}$ of the investment.

(3) Every $1 invested in bonds cost 112$\frac{1}{2}$ ct., and yields 2$\frac{1}{2}$ ct.; ∴ the income is 2$\frac{1}{2}$ ÷ 112$\frac{1}{2}$, or $\frac{1}{45}$ of the investm't.

(4) Since the latter income was 25%, or $\frac{1}{4}$ more than the former, then the former income is $\frac{4}{4}$ and the latter $\frac{5}{4}$.

(5) $\frac{4}{4}+\frac{5}{4}=\frac{9}{4}$, the whole income.

(6) $\frac{9}{4}$ = $9000.

(7) $\frac{1}{4}=\frac{1}{9}$ of $9000 = $1000.

(8) $\frac{4}{4}$ = 4 times $1000 = $4000, income of former.

(9) $\frac{5}{4}$ = 5 times $1000 = $5000, income of latter.

(10) $\frac{1}{28}$ = $4000.

(11) $\frac{28}{28}$ = 28 times $4000 = $112000, investment in stock.

(12) $\frac{1}{45}$ = $5000.

(13) $\frac{45}{45}$ = 45 times $5000 = $225000, investment in bonds.

PROBLEM 172.

I invest $3600 more in 7$\frac{1}{2}$% bank stock at 12$\frac{1}{2}$% premium, than in 8% canal stock at 4% discount; from the latter my income is 25% less than from the former: what is the face value of each stock and my whole income?

Solution.

(1) Every $1 invested in bank stock costs 112$\frac{1}{2}$ ct. and yields 7$\frac{1}{2}$ ct. ∴ the income is 7$\frac{1}{2}$ ÷ 112$\frac{1}{2}$, or $\frac{1}{15}$ of the investment.

(2) Every $1 invested in canal stock costs 96 ct., and yields 8 ct.; ∴ the income is $\frac{8}{96}$, or $\frac{1}{12}$ of the investment.

(3) Since the income from the latter is 25%, or $\frac{1}{4}$ less than the former, then the income from the latter is $\frac{3}{4}$ of the income from the former; $\frac{3}{4}$ of $\frac{1}{15}$ the former income = $\frac{3}{60}$ of the former.
(4) $\frac{1}{12}$ of the latter investment = $\frac{3}{60}$ of the former.
(5) $\frac{12}{12}$ of the latter = $\frac{36}{60}$ of the former.
(6) $\frac{60}{60}$ = former inv. — $\frac{36}{60}$ = $\frac{24}{60}$, diff. in investments.
(7) ∴ $\frac{24}{60}$ = \$3600.
(8) $\frac{1}{60}$ = $\frac{1}{24}$ of \$3600 = \$150.
(9) $\frac{60}{60}$ = 60 times \$150 = \$9000, inv. in bank stock.
(10) Also 36 times \$150 = \$5400, inv. in canal stock.
(11) $\frac{1}{15}$ of \$9000 = \$600, income from bank stock.
(12) $\frac{1}{12}$ of \$5400 = \$450, income from canal stock.
(13) The face value of the bank stock is \$9000 ÷ 1.12½ = \$8000, and that of canal stock is \$5400 ÷ .96 = \$5625.

PROBLEM 173.

For what must one buy a 5% stock to realize an income of 6% on his investment?

Solution.

(1) 5 ct. = income of \$1 of the stock; the income is to be 6% of the investment; this 5 ct. must be 6% of the cost of \$1 of stock.
(2) 6% of the cost of \$1 of stock = 5 ct.
(3) 1% of the cost of \$1 of stock = $\frac{1}{6}$ of 5 ct. or $\frac{5}{6}$ ct.
(4) 100% = 100 times $\frac{5}{6}$ ct. = 83⅓ ct., cost of \$1 of stock.

PROBLEM 174.

J. W. Burris, through his broker, buys N. Y. 6's at 107½, and twice as much in U. S. 5's of '82 at 98½, brokerage in each case ½%; his annual income from both is \$3348: how much was paid for each kind of stock?

Solution.

(1) 107½ ct. + ½ ct. = 108 ct., cost of \$1 of N. Y. 6's.
(2) \$1 invested will buy \$$\frac{100}{108}$ of N. Y. stock, yielding \$$\frac{6}{108}$, or \$$\frac{1}{18}$ per annum.
(3) 98½ ct. + ½ ct. = 99 ct., cost of \$1 of U. S. stock.
(4) Then, \$1 invested will buy \$$\frac{100}{99}$ U. S. 5's, and will yield \$$\frac{5}{99}$ per annum.
(5) \$2 invested will yield 2 times \$$\frac{5}{99}$, or \$$\frac{10}{99}$.
(6) \$$\frac{1}{18}$ + \$$\frac{10}{99}$ = \$$\frac{31}{198}$, income on \$3 of stock.
(7) $\frac{1}{3}$ of \$$\frac{31}{198}$ = \$$\frac{31}{594}$, average income on \$1 of the whole stock.

(8) \$3348, entire income $\div \frac{31}{594}$ = \$64152, whole am't inv.
(9) For every \$1 invested in N. Y. 6's, \$2 are invested in U. S. 5's; ∴ $\frac{1}{3}$ of \$64152 = \$21384, amount in N. Y. 6's.
(10) $\frac{2}{3}$ of \$64152 = \$42768, amount in U. S. 5's.

PROBLEM 175.

If 6% express stock is worth in market 20% more than 10% railroad stock, and the railroad stock 25% more than 12% telegraph stock, then at what rate was each bought and what was each investment, if the income from each is \$1200 and total investment pays $10\frac{10}{11}$%?

Solution.

(1) 6 ct. = income on \$1 of express stock; \$1200 = whole income on express stock; then, \$1200 ÷ .06, or \$20000 = entire value of express stock.
(2) 10 ct. = income on \$1 of railroad stock; \$1200 = entire income; then, \$1200 ÷ .10, or \$12000 = face value of railroad stock.
(3) 12 ct. = income on \$1 of telegraph stock; \$1200 = entire income; then, \$1200 ÷ .12, or \$10000 = face value of telegraph stock.
(4) If the railroad stock is 25%, or $\frac{1}{4}$, better than telegraph stock, then the cost of \$1 of railroad stock would be $\frac{5}{4}$ of the cost of \$1 of telegraph stock.
(5) We see by comparison of the two face values that there are $\frac{6}{5}$ as many dollars of railroad stock as there are of telegraph stock; ∴ $\frac{5}{4}$ of $\frac{6}{5}$, or $\frac{3}{2}$ of telegraph stock = cost of railroad stock in terms of telegraph stock.
(6) $\frac{2}{2}$ = cost of telegraph stock.
(7) Now, if express stock is 20%, or $\frac{1}{5}$, better than railroad stock, then the cost of \$1 of express stock would be $\frac{6}{4}$ of the cost of railroad stock; *i e.*, if the railroad stock were the basis of comparison; but we take the telegraph stock.
(8) By comparing the express stock and the telegraph stock there are $\frac{2}{1}$ as many dollars of express stock as there are of telegraph stock; ∴ $\frac{6}{4}$ of $\frac{2}{1} = \frac{6}{2}$ of telegraph stock = cost of express stock in terms of telegraph stock.
(9) $\frac{6}{2} + \frac{3}{2} + \frac{2}{2} = \frac{11}{2}$, or entire cost of the stocks.
(10) \$1200 = the income of each stock; hence, \$1200 × 3, or \$3600 = total income.
(11) Each \$1 invested yielded an income of $10\frac{10}{11}$ ct.; hence, \$3600 ÷ $10\frac{10}{11}$ ct. = \$33000, entire cost of investment; also, $\frac{11}{2}$ = entire cost of investment.
(12) $\frac{11}{2}$ = \$33000.

(13) $\frac{1}{2} = \frac{1}{11}$ of \$33000 = \$3000.
(14) $\frac{2}{2}$ = 2 times \$3000 = \$6000, cost of telegraph stock.
(15) $\frac{3}{2}$ = 3 times \$3000 = \$9000, cost of railroad stock.
(16) $\frac{6}{2}$ = 6 times \$3000 = \$18000, cost of express stock.
(17) The express stock was bought at \$18000 ÷ \$20000 = 90%.
(18) The railroad was bought at \$9000 ÷ \$12000 = 75%.
(19) The telegraph was bought at \$6000 ÷ \$10000 = 60%.

PROBLEM 176.

Invested in U. S. $4\frac{1}{2}$'s at 105, brokerage $\frac{1}{2}$%; $\frac{4}{5}$ as much in U. P. 6's at $119\frac{2}{3}$, brokerage $\frac{1}{3}$%; and 3 times as much in N. Y. 7's at $87\frac{1}{4}$, brokerage $\frac{1}{4}$%: if my entire income is \$1702, find my investment.

Solution.

(1) 105 ct. + $\frac{1}{2}$ ct. = $105\frac{1}{2}$ ct., total cost of \$1 of 1st.
(2) Then, \$1 invested will buy $\frac{100}{105.5}$ of U. S. $4\frac{1}{2}$'s, and will yield \$$\frac{4.5}{105.5}$, or \$$\frac{9}{211}$ per annum.
(3) $119\frac{2}{3}$ ct. + $\frac{1}{3}$ ct. = 120 ct., total cost of \$1 of 2d.
(4) Then, \$1 invested will buy \$$\frac{100}{120}$ of U. P. 6's, and will yield \$$\frac{6}{120}$ per annum.
(5) \$$\frac{4}{5}$ invested will yield $\frac{4}{5}$ of \$$\frac{6}{120}$, or \$$\frac{1}{25}$.
(6) $87\frac{1}{4}$ ct. + $\frac{1}{4}$ ct. = $87\frac{1}{2}$ ct., cost of \$1 of the 3d.
(7) Then, \$1 invested will buy \$$\frac{100}{87.5}$ of N. Y. 7's, and will yield \$$\frac{7}{87.5}$, or \$$\frac{14}{175}$ per annum.
(8) \$3 invested will yield 3 times \$$\frac{14}{175}$, or \$$\frac{6}{25}$.
(9) \$$\frac{9}{211}$ + \$$\frac{1}{4}$ + \$$\frac{6}{25}$ = \$$\frac{1702}{5275}$, income on \$$4\frac{4}{5}$ of stock.
(10) $\dfrac{1}{4\frac{4}{5}}$ of \$$\frac{1702}{5275}$ = $\dfrac{\$375\frac{7}{12}}{5275}$, av'ge income on \$1 of entire stock.
(11) \$1702, the total income ÷ $\dfrac{\$375\frac{7}{12}}{5275}$, the income on \$1 = \$25320, total amount invested.
(12) For every \$1 invested in U. S. $4\frac{1}{2}$'s, \$$\frac{4}{5}$ are invested in U. P. 6's, and \$3 invested in N. Y. 7's.
(13) ∴ $\frac{5}{24}$ of \$25320 = \$5275, amount invested in U. S. $4\frac{1}{2}$'s.
(14) $\frac{4}{5}$ of \$5275 = \$4220, amount invested in U. P. 6's.
(15) \$5275 × 3 = \$15825, amount invested in N. Y. 7's,
(16) ∴ \$25320 = the whole investment.

PROBLEM 177.

Suppose I sell \$15000 in U. S. bonds at $102\frac{1}{4}$, and invest part of the proceeds in $7\frac{1}{2}$ state stock at 106, realizing an income of \$720, brokerage in each case $\frac{1}{4}$%, and buy a house with the remainder: at what must I rent it per month to make 5% interest on my investment?

Solution.

(1) $1.02¼ — ¼ ct. = $1.02, amount received from the sale of $1 of stock.
(2) $15000 × 1.02 = $15300, received from the sale of the bonds.
(3) $720 = income on state stock, and 7½ ct. = income on $1 of state stock.
(4) $720 ÷ .075 = $9600, face value of state stock.
(5) $9600 × 1.06¼ = $10200, the cost of the state stock.
(6) Hence, the house cost $15300 — $10200, or $5100.
(7) 5 ct. = income on $1 for 1 year's rent.
(8) $5100 × .05 = $255, received for 1 year's rent.
(9) $255 ÷ 12 = $21.25, received per month.

PROBLEM 178.

A paid $1075 for U. S. 5–20 6% bonds at 7½% premium, interest payable in gold semi-annually: when the average premium on gold was 112%, did he make more or less than B, who invested an equal sum in railroad stock at 14% below par, which paid a semi-annual dividend of 4%?

Solution.

(1) $1.07½ = cost of $1 of U. S. 5–20's.
(2) $1075 ÷ 1.075 = $1000, face value of 5–20's, and $30 is the interest in gold, or $33.60 in currency.
(3) 86 ct. = cost of $1 of railroad stock.
(4) $1075 ÷ .86 = $1250, face value of railroad stock, and 4 % of $1250 = $50, interest on the stock.
(5) Hence, $50 — $33 60 = $16.40, what A makes less than B every 6 months.

PROBLEM 179.

I invested $2700 in stock at 25% discount, which pays 8% annual dividends: how much must I invest in stock at 4% discount and paying 10% annual dividends, to secure an equal income?

Solution.

(1) $2700 = cost of the 1st stock; 75 ct. = cost of $1 of this stock.
(2) $2700 ÷ .75, or $3600 = face value.
(3) 8 ct. = income on $1 of this stock; $3600 × .08, or $288 = income realized.
(4) 10 ct. = income on the 2d stock. We see that the incomes are equal; then—
(5) $288 ÷ .10 = $2880, the par value of the stock.

(6) 96 ct. = the cost of \$1 of this stock; then, \$2880 × .96 = \$2764.80, cost of the stock, or the investment.

PROBLEM 180.

I invest a certain sum in 6's at 85, and the same sum in 7's at 95, receiving \$5 a year more from the latter investment: how much do I invest in each?

Solution.

(1) 85 ct. = cost of \$1 of the 1st; then, \$1 invested will buy $\$\frac{100}{85}$ of 1st stock, and yield $\$\frac{6}{85}$ per annum.
(2) 95 ct. = cost of \$1 of 2d stock; then, \$1 invested will buy $\$\frac{100}{95}$ of 2d stock, and yield $\$\frac{7}{95}$ per annum.
(3) $\$\frac{7}{95} - \$\frac{6}{85} = \$\frac{25}{8075}$, difference in incomes of the two stocks.
(4) \$5 = difference in investments; then, $\$5 \div \$\frac{25}{8075} =$ \$1615, what I invest in each.
(5) $\frac{7}{95}$ of \$1615 = \$119, income of 2d stock.
(6) $\frac{6}{85}$ of \$1615 = \$114, income of 1st stock.

PROBLEM 181.

A man bought Michigan Central at 120, and sold at 124: what % of the investment did he gain?

Solution.

(1) \$1.20 = cost of \$1 of this stock, and \$1.24 = the selling price; \$1.24 — \$1.20 = 4ct., gain.
(2) Then, 4 ct. ÷ \$1.20 = $3\frac{1}{3}$%, gain on the investment.

PROBLEM 182.

I bought U. S. 4% bonds at $119\frac{7}{8}$%, brokerage $\frac{1}{8}$% additional, and derive from the purchase a quarterly income of \$300: how much did I invest?

Solution.

(1) \$300 = income for 3 months, and \$300 × 4 = \$1200, income for 1 year.
(2) 4 ct. = income of \$1 of these bonds; then, \$1200 ÷ .04 = \$30000, face value.
(3) $119\frac{7}{8} + \frac{1}{8}$ = \$1.20, cost of \$1 of this stock.
(4) \$30000 × \$1.20 = \$36000, the cost or investment.

PROBLEM 183.

What is my gain or loss, if I buy 112 shares of stock in a transportation company, at 17% premium, and, after receiving a dividend of 9%, sell it for 8% less than it cost me?

Solution.

(1) Let \$100 represent the cost of each share.
(2) \$100 × 112 = \$11200, face value of the stock.
(3) \$1.17 = the cost of each \$1 of this stock.
(4) \$11200 × \$1.17 = \$13104, the cost of the stock.
(5) \$11200 × .09 = \$1008, the dividend, and 92 ct., the selling price of this stock.
(6) Then, the amount realized from sale is \$13104 × .92 = \$12055.68.
(7) ∴ I sustained a loss of \$13104 — (\$12055.68 + \$1008) = \$40.32.

PROBLEM 184.

I hold 4% stock and at the end of the year I take my dividend in same stock at 20% discount; the par value of my stock is then \$10500: find dividend.

Solution.

(1) 4 ct. = income on \$1 of the stock, which dividend I take in the same stock.
(2) I get $\$\frac{4}{80}$ of the stock as dividend.
(3) \$1 + $\$\frac{4}{80}$, or $\$\frac{84}{80}$ = par value of \$1 of the stock.
(4) ∴ \$10500 ÷ $\$\frac{84}{80}$ = \$10000, the original value.
(5) $\frac{4}{80}$ of \$10000 = \$500, dividend of face value of the stock.

PROBLEM 185.

Bought stock at 20% discount and sold to gain $12\frac{1}{2}$% on my investment: if I invest proceeds again, at what discount must I buy to yield at next selling 35% on first investment?

Solution.

(1) Let \$100 represent the value of the stock; 80 ct. = the cost of \$1 of this stock.
(2) \$100 × 80 ct. = \$80, the cost of the stock.
(3) $\$1.12\frac{1}{2}$ = the selling price of \$1 of the stock; \$80 × $\$1.12\frac{1}{2}$ = \$90, selling price.
(4) (\$100 — \$90) ÷ \$100 = 10%, the discount at which it is sold.
(5) But at the second sale I am to make 35% profit on first investment, I must sell for \$80 × 1.35 = \$108.
(6) As the stock was sold at 10% discount, the face must be \$108 ÷ .90 = \$120.
(7) Then (\$120 — \$108) ÷ 120 = 10%, discount required.

PROBLEM 186.

How many shares of railroad stock (\$50 each) at $54\frac{1}{2}$, must be sold in order that the proceeds, invested in 6's at 95, may yield an income of \$750, not considering brokerage?

CHAPTER IX.

INSURANCE.

57. **Insurance** is an indemnity against loss. It is of two kinds: Property Insurance and Personal Insurance.

58. **Property Insurance** is security against loss by fire or transportation. Insuring anything is called "taking a risk."

59. **Fire Insurance** is security against loss by fire.

60. **Marine Insurance** is security against loss by navigation.

61. **The Policy** is the written agreement or contract between the insurers and the insured.

62. **Premium** is the sum charged for insurance; it is a certain rate per cent of the amount insured.

PROBLEM 187.

Insured a house for $2500, and furniture for $600, at $\frac{6}{10}$%: what was the premium? (*R. H. A., p. 230.*)

Solution.

(1) 100% = am't insured.
(2) $2500 + $600 = $3100.
(3) 100% = $3100.
(4) 1% = $\frac{1}{100}$ of $3100, or $31.
(5) $\frac{6}{10}$% = $\frac{6}{10}$ times $31 = $18.60, premium.

∴ $18.60 is the premium.

PROBLEM 188.

I insured property at 2%; reinsured $8000 at 1$\frac{3}{4}$%, and $10000 of it at 2$\frac{1}{8}$%: what was the amount, my share of the premium being $207.50?

Solution.

(1) 100% = amount insured; 2% = premium. $8000 = amount reinsured 1st time.
(2) 1$\frac{3}{4}$% of $8000 = $140, premium on $8000.

(3) $1\frac{1}{8}\%$ of \$10000 = \$212.56, premium on \$10000.
(4) \$207.50 = my share of the premium; then, \$140 + \$212.50 + \$207.50 = \$560, whole premium.
(5) 2% = premium = \$560.
(6) 1% = $\frac{1}{2}$ of \$560 = \$280.
(7) 100% = 100 times \$280 = \$28000.

∴ \$28000 = amount insured.

PROBLEM 189.

I paid \$180 for insuring my stock for $\frac{2}{3}$ of its value at 3%: what is the value of the stock?

Solution.

(1) 100% = value of $\frac{2}{3}$ of the stock.
(2) 3% = premium = \$180.
(3) 1% = $\frac{1}{3}$ of \$180, or \$60.
(4) 100% = 100 times \$60 = \$6000, value of $\frac{2}{3}$ of the stock.
(5) $\frac{2}{3}$ of the stock = \$6000.
(6) $\frac{1}{3}$ = $\frac{1}{2}$ of \$6000, or \$3000.
(7) $\frac{3}{3}$ the stock = 3 times \$3000 = \$9000.

∴ \$9000 = the value of the stock.

PROBLEM 190.

I insured a house at $1\frac{1}{2}\%$; reinsured $\frac{2}{5}$ of it at 2%, and $\frac{1}{4}$ of it at $2\frac{1}{2}\%$: what rate of insurance do I get on the remainder? (*R. H. A., p. 231.*)

Solution.

(1) 100% = risk; $1\frac{1}{2}\%$ = premium.
(2) $\frac{2}{5}$ of 100% = 40%, amount reinsured at 2%.
(3) 2% of 40% = .80%, amount paid out for reinsuring $\frac{2}{5}$ of the risk.
(4) $\frac{1}{4}$ of 100% = 25%, amount reinsured at $2\frac{1}{2}\%$.
(5) $2\frac{1}{2}\%$ of 25% = $.62\frac{1}{2}\%$, amount paid out for reinsuring $\frac{1}{4}$ of the risk.
(6) .80% + $.62\frac{1}{2}$ = $1.42\frac{1}{2}\%$, amount of premiums paid out.
(7) $1\frac{1}{2}\%$ — $1.42\frac{1}{2}\%$ = $.07\frac{1}{2}\%$, amount of premium left.
(8) 40% + 25% = 65%, amount reinsured.
(9) 100% — 65% = 35%, risk left.
(10) 35% = 100% of itself.
(11) 1% = $\frac{1}{35}$ of 100% = $\frac{100}{35}\%$.
(12) $.07\frac{1}{2}\%$ = $.07\frac{1}{2}$ times $\frac{100}{35}$ = $\frac{3}{14}\%$, rate of premium.

∴ $\frac{3}{14}\%$ = rate of insurance that I receive.

PROBLEM 191.

Took a risk at 2%: reinsured \$10000 of it at $2\frac{1}{8}$%, and \$8000 at $1\frac{3}{4}$%; my share of the premium was \$207.50: what sum was insured?

(*R. H. A., p. 231.*)

Solution.

(1) $2\frac{1}{8}$% of \$10000 = \$212.50, amount paid out on \$10000.
(2) $1\frac{3}{4}$% of \$8000 = \$140, amount paid out on \$8000.
(3) \$212.50 + \$140 = \$352.50, whole amount paid out.
(4) \$207.50 = amount I receive.
(5) \$352.50 + \$207.50 = \$560, premium on whole risk.
(6) 100% = risk.
(7) 2% = premium = \$560.
(8) 1% = $\frac{1}{2}$ of \$560 = \$280.
(9) 100% = 100 times \$280 = \$28000, risk.

∴ \$28000 = what was insured.

PROBLEM 192.

The Mutual Fire Insurance Company insured a building and its stock for $\frac{2}{3}$ of its value, charging $1\frac{3}{4}$%. The Union Insurance Company relieved them of $\frac{1}{4}$ of the risk, at $1\frac{1}{2}$%. The building and stock being destroyed by fire, the Union lost \$49000 less than the Mutual: what amount of money did the owners of the building and stock lose?

(*R. H. A., p. 232.*)

Solution.

(1) 100% = value of property insured.
(2) $\frac{2}{3}$ of 100% = $66\frac{2}{3}$, the whole risk.
(3) $1\frac{3}{4}$% of $66\frac{2}{3}$% = $1\frac{1}{6}$%, entire premium.
(4) $\frac{1}{4}$ of $66\frac{2}{3}$% = $16\frac{2}{3}$%, Union's risk.
(5) $66\frac{2}{3}$% — $16\frac{2}{3}$% = 50%, Mutual's risk.
(6) $1\frac{1}{2}$% of $16\frac{2}{3}$% = $\frac{1}{4}$%, Union's premium.
(7) $1\frac{1}{6}$% — $\frac{1}{4}$% = $\frac{11}{12}$%, Mutual's premium.
(8) 50% — $\frac{11}{12}$% = $49\frac{1}{12}$, loss of Mutual.
(9) $16\frac{2}{3}$% — $\frac{1}{4}$% = $16\frac{5}{12}$%, loss of Union.
(10) $49\frac{1}{12}$% — $16\frac{5}{12}$% = $32\frac{2}{3}$%, excess of Mutual's loss.
(11) $32\frac{2}{3}$% = \$49000.
(12) 1% = \$49000 ÷ $32\frac{2}{3}$ = \$1500.
(13) 100% — $66\frac{2}{3}$% + $1\frac{1}{6}$% = $34\frac{1}{2}$% =owners' loss.
(14) $34\frac{1}{2}$% = $34\frac{1}{2}$ times \$1500 = \$51750.

∴ \$51750 = owners' loss.

CHAPTER X.

I. INTEREST.

63. **Interest** is money paid for the use of money.

64. The **Principal** is the sum on which interest is charged.

65. The **Rate** of interest is the rate per cent on $1 for one year.

66. The **Time** is the period during which the money is on interest.

67. The **Amount** is the sum of the principal and the interest.

68. **Simple Interest** is interest on the principal only.

69. **Compound Interest** is interest on the principal and interest.

70. **Legal Interest** is interest at the rate fixed by law.

71. **Usury** is a rate of interest greater than the law allows.

NOTE.—In notes, contracts, mortgages, etc., when no rate is specified, the legal rate is understood,

72. The **Six Per Cent Method** is so called because the process is based upon that rate.

Rule.—*Multiply the number of years by the rate, take* ½ *of the number of months as cents, and* ⅙ *of the number of days as mills; their sum will be the interest of* $1 *for the given time at* 6%.

PROBLEM 193.

Required the interest of $380 for 3 yr. 4 mo. 12 da., at 6%.

Solution.

(1) The interest on $1 for 1 yr. is 6 ct., and for 3 yr. it is 3 times 6 ct., or $.18.

(2) For 4 mo. it is ½ of 4, or $.02.

(3) For 12 da. it is $\frac{1}{6}$ of 12, or \$.002.

(4) Adding, we have \$.202, which is the interest on \$1 for the given time and rate.

(5) On \$380 the interest is 380 times \$.202 = \$76.76.

PROBLEM 194.

What principal will in 7 yr. 4 mo., at 8%, amount to \$749.70?

Solution.

(1) The interest on \$1 for the given time and rate is \$.58$\frac{2}{3}$.

(2) \$1 + \$.58$\frac{2}{3}$ = \$1.58$\frac{2}{3}$, amount.

(3) \$749.70 ÷ 1.58$\frac{2}{3}$ = \$472.50, the required principal.

PROBLEM 195.

The sum of A's and B's money on interest for 4 yr. 6 mo., at 6%, gives \$5400 interest: how much money has each, if 3 times B's equals A's?

Solution.

(1) The interest on \$1 for the given time and rate is \$.27.

(2) If \$1 gives an interest of \$.27, to give \$5400 interest it will require as many dollars as .27 is contained times in \$5400, or \$20000.

(3) $\frac{4}{4}$ = A's and B's money = \$20000.

(4) $\frac{1}{4}$ of \$20000 = \$5000, B's money.

(5) $\frac{3}{4}$, A's money = 3 times \$5000 = \$15000.

∴ A's money is \$15000, B's is \$5000.

PROBLEM 196.

In what time will \$1800, at 4$\frac{1}{2}$%, give \$1247.40 interest?

Solution.

(1) The interest on \$1800 at 4$\frac{1}{2}$% for 1 yr. is \$81.

(2) If in 1 yr. the principal gives \$81 interest, to give \$1247.40 interest it will require as many times 1 yr. as \$81 is contained times in \$1247.42, which is 15$\frac{2}{5}$ yr., or 15 yr. 4 mo. 24 da.

Rule.—*Divide the given interest by the interest of the principal at the given rate for one year. If the amount is given, subtract the principal from the amount to find the interest; and then proceed as before.*

PROBLEM 197.

At what rate will \$13.25, in 8 yr. 10 mo. 18 da., give \$7.062$\frac{1}{2}$ interest?

Solution.

(1) The interest of $13.25 for 8 yr. 10 mo. 18 da. at 1% is $1.17704.

(2) If the principal in the given time at 1% gives $1.17704 interest, to give $7.0625 interest it will require as many times 1% as $1.17704 is contained times in $7.0625, which is 6%.

PROBLEM 198.

On a sum borrowed at 6% per annum and loaned at 8% per annum, I realized a gain of $31.20 in 3 mo. and 18 da: find amount loaned.

Solution.

(1) The interest on $1 for 3 mo. 18 da. at 6% is $.018.

(2) The interest on $1 for 3 mo. 18 da. at 8% is $.024.

(3) $.024 — $.018 = $.006, gain on $1.

(4) If $1 gives a gain of $.006, to give a gain of $31.20 it will require as many dollars as $.006 is contained times in $31.20, or $5200.

∴ $5200 = amount loaned.

PROBLEM 199.

The sum of $\frac{2}{3}$ of A's plus $\frac{1}{2}$ of B's money being on interest for 8 yr. at 6% gives $960 interest: what has each, if $\frac{1}{2}$ of B's is 3 times $\frac{2}{3}$ of A's?

Solution.

(1) The interest on $1 for 8 yr. at 6% is $.48.

(2) Then, to give $960 interest, it will require as many dollars as $.48 is contained times in $960, which is $2000, sum of $\frac{2}{3}$ of A's + $\frac{1}{2}$ of B's money.

(3) $\frac{2}{2} + \frac{1}{2} = \frac{3}{2}$; $2000 × $\frac{3}{2}$ = $3000, B's.

(4) $\frac{1}{2}$ of $3000 = $1500; $1500 ÷ 3 = $500.

(5) $\frac{2}{3}$ = $500; $\frac{1}{3} = \frac{1}{2}$ of $500 = $250.

(6) $\frac{3}{3}$ = 3 times $250 = $750, A's.

∴ A's money is $650, B's $3000.

PROBLEM 200.

A's money added to $\frac{2}{3}$ of B's, which is to A's as 2 to 3, being put on interest for 6 yr. at 4%, amounts to $744: what has each?

Solution.

(1) The interest on $1 for the given time and rate is $.24.

(2) $1 + $ 24 = $1.24, amount of $1.

(3) $744 ÷ $1 24 = $600, A's money + $\frac{2}{3}$ of B's.

(4) $\frac{3}{3}$ A's money + $\frac{2}{3}$ B's = $\frac{5}{3}$ = $600.

(5) $\frac{1}{3} = \frac{1}{5}$ of $600 = $120.

(6) $\frac{3}{3}$ = 3 times \$120 = \$360, A's money.
(7) $\frac{2}{3}$ of \$360 = \$240, which is as 2 : 3.
(8) Then, $\frac{2}{3}$ = \$240.
(9) $\frac{1}{3} = \frac{1}{2}$ of \$240 = \$120.
(10) $\frac{3}{3}$ = 3 times \$120 = \$360, B's money.

∴ A and B have each \$360.

PROBLEM 201.

$\frac{3}{5}$ of the cost of A's mill, increased by $\frac{4}{5}$ of the cost of his house for 2 yr. at 5% amounts to \$4950: what was the cost of each, if $\frac{3}{5}$ of the cost of the mill was only $\frac{2}{7}$ as much as $\frac{4}{5}$ of the cost of the house?

Solution.

(1) The interest on \$1 for the given time and rate is \$10.
(2) \$1 + \$.10 = \$1.10, amount of \$1.
(3) \$4950 ÷ \$1.10 = \$4500, $\frac{3}{5}$ of the cost of A's mill + $\frac{4}{5}$ of the cost of his house.
(4) $\frac{3}{5} = \frac{2}{7}$ of $\frac{4}{5}$, or $\frac{8}{35}$ of house.
(5) $\frac{4}{5} + \frac{8}{35} = \frac{36}{35} = \frac{3}{5}$ of the cost of mill + $\frac{4}{5}$ of the cost of the house.
(6) $\frac{36}{35}$ = \$4500.
(7) $\frac{1}{35} = \frac{1}{36}$ of \$4500, or \$125.
(8) $\frac{35}{35}$ = 35 times \$125 = \$4375, cost of house.
(9) $\frac{4}{5}$ of \$4375 = \$3500, $\frac{4}{5}$ of the cost of the house.
(10) $\frac{2}{7}$ of \$3500 = \$1000, $\frac{2}{7}$ of $\frac{4}{5}$ of the cost of the house.
(11) $\frac{3}{5}$ = \$1000.
(12) $\frac{1}{5} = \frac{1}{3}$ of \$1000 =\$333$\frac{1}{3}$.
(13) $\frac{5}{5}$ cost of mill = 5 times \$333$\frac{1}{3}$ = \$1666$\frac{2}{3}$, cost of the mill.

∴ The house cost \$4375, the mill \$1666$\frac{2}{3}$.

PROBLEM 202.

The interest on the sum of A's and B's money for 3 yr. and 9 mo. at 8%, is \$3213; $\frac{2}{3}$ of A's money is equal to $\frac{3}{4}$ of B's: how much has each?

Solution.

(1) The interest on \$1 for the given time and rate is 30 ct.
(2) If \$1 gives an interest of \$.30, to give \$3213 interest it will require as many dollars as \$.30 is contained times in \$3213, or \$10710.
(3) $\frac{2}{3}$ of A's money is equal to $\frac{3}{4}$ of B's.
(4) $\frac{1}{3}$ of A's = $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{8}$ of B's.
(5) $\frac{3}{3}$ A's = 3 times $\frac{3}{8} = \frac{9}{8}$ of B's.
(6) Then $\frac{8}{8}$ = B's, and $\frac{9}{8}$ A's.

(7) $\frac{8}{8} + \frac{9}{8} = \frac{17}{8}$, A's and B's money.
(8) $\frac{17}{8} = \$10710$.
(9) $\frac{1}{8} = \frac{1}{17}$ of $\$10710 = \$630$.
(10) $\frac{8}{8} = 8$ times $\$630 = \$5040$, B's money.
(11) $\frac{9}{8} = 9$ times $\$630 = \$5670$, A's money.

∴ A's money is \$5670, B's \$5040.

PROBLEM 203.

The amount of a certain principal, in a certain time, at 5%, is \$833, and the amount for the same time at 12%, is \$1047.20: required the principal and time.

Solution.

(1) 12% — 5% = 7%, the difference in rates of interest.
(2) Now, the principal and time are the same; the difference in amounts must have resulted from the difference in the rates of interest.
(3) The difference of the amounts equals the interest at 7%. \$1047.20 — \$833 = \$214.20, the difference in amounts, or 7% of the principal for the time.
(4) 7% = \$214.20.
(5) 1% = $\frac{1}{7}$ of \$214.20, or \$30.60.
(6) 5% of the principal for the time = 5 × \$30.60 = \$153.
(7) 12% of the principal for the time = 12 × \$30.60, or \$367.20.
(8) Now, \$833 — \$153, or \$1047.20 — \$367.20 = \$680, the principal.
(9) 5% of \$680 for 1 year = \$34.
(10) 12% of \$680 for 1 year = \$81.61; since the principal at 5% yields \$34 in 1 year, and at 12%, \$81.60 in 1 year, the money must have been loaned for as many years as \$34 is contained times in \$153, or \$81.60 is contained times in \$367.20, which is $4\frac{1}{2}$ yr., or 4 yr. and 6 mo.

∴ \$680 = the principal, and 4 yr. 6 mo. the time.

PROBLEM 204.

The amount of a certain principal for 4 yr., at a certain rate per cent, is \$3551, and for 19 yr., \$6929$\frac{3}{4}$: required the principal and rate.

Solution.

(1) 19 yr. — 4 yr. = 15 yr., the difference of time.
(2) The principal and the rate are the same, the difference of amounts, \$6929$\frac{3}{4}$ — \$3551, or \$3378$\frac{3}{4}$, is the interest on the principal for 15 yr.

(3) $\$3378\frac{3}{4} \div 15 = \$225.25$, the interest on the principal for 1 yr.

(4) Then $4 \times \$225.25 = \$901$, the interest on the principal for 4 yr.

(5) 19 times $\times \$225.25 = \$4279\frac{3}{4}$, the interest on the principal for 19 years.

(6) Now, $\$3551 - \$901$, or $\$6929\frac{3}{4} - \$4279\frac{3}{4} = \$2650$, the principal.

(7) \$2650, the principal loaned at 1% will earn \$26.50 in 1 yr.

(8) The principal at the required rate earned \$225.25 in 1 yr., then it must have drawn as many times 1% as \$26.50 is contained times in \$225.25, or $8\frac{1}{2}$.

$\therefore$ \$2650 = the principal, $8\frac{1}{2}\%$ the rate.

PROBLEM 205.

A owes B \$900, and has only \$350 cash on hand; he proposes to pay a part of this money on the debt, and to pay the interest in advance at 10%: for what sum was the note drawn?

First Solution.

(1) Let 100% be the face of the note.

(2) Then, he paid 20% interest, or he paid in all \$900 + 20% of face of note.

(3) He also paid \$300 + 100% of the face, and \$900 with 20% of the note = \$350 and the whole note.

(4) Subtracting 20% from each of the two equalities, we have 80% of note + \$350 = \$900, or 80% = \$550.

(5) $1\% = \frac{1}{80}$ of \$550 = \$6 875.

(6) 100% = 100 times \$6.875 = \$687.50, face of note.

Second Solution.

(1) If he had paid \$350 on the principal, there would have remained \$900 — \$350 = \$550 of the principal unpaid.

(2) By the condition of the problem the \$350 includes the interest on the unpaid balance of the principal for 2 yr. at 10%. Now, this unpaid balance must be \$550 + the interest on the unpaid balance for 2 yr. at 10%.

(3) Let 100 parts = the unpaid balance.

(4) $\frac{20}{100}$ will equal the interest and $\frac{80}{100}$ will represent the \$550.

(5) $\frac{80}{100} = \$550$.

(6) $\frac{1}{100} = \frac{1}{80}$ of \$550, or $\$\frac{550}{80}$.

(7) $\frac{100}{100}$, the unpaid balance = 100 times $\$\frac{550}{80} = \$687.50$.

$\therefore$ \$687.50 = face of new note.

PROBLEM 206.

By lending a sum of money at 4% and another sum at 5%, the total interest is \$10; if the rates are changed the interest is \$9.80: find the principal lent at each rate.

Solution.

(1) 5% — 4% = 1%, difference in rates.
(2) \$10 — \$9.80 = \$.20, difference.
(3) 1% = \$.20.
(4) \$.20 ÷ 1% = \$20, difference in amounts.
(5) (\$10 + \$9.80) ÷ (5% + 4%) = \$220, whole amount.
(6) \$220 — \$20 = \$200, now, this = both sums — \$20.
(7) ∴ the 1st sum must be $\frac{1}{2}$ of \$200, or \$100, and the 2d sum \$100 + \$20, or \$120.

PROBLEM 207.

The interest of $\frac{1}{2}$ of A's + $\frac{2}{3}$ of B's money for a certain time at 2% was to this sum as 9 to 250, and the amount of this interest for 25 times as long, at 10 times as great a per cent was \$180: what was their money, if A's money was to B's as 1 to 3?

Solution.

(1) Since the interest on the sum loaned was to that sum as 9 : 250, the interest was $\frac{9}{250}$ of that principal.
(2) The interest of any principal for one year at 2% = $\frac{1}{50}$ of it; then $\frac{9}{250} \div \frac{1}{50}$, or $\frac{9}{5}$ yr. (1 yr. 9 mo. 18 da.) is the time.
(3) 25 times this period is 45 yr., and 10 times the rate is 20%.
(4) The amount of \$1 for 45 yr. at 20% = \$10; hence, \$180 ÷ \$10, or \$18, is the interest on the part loaned at 2% for 1 yr. 9 mo. 18 da., which by the 1st condition is $\frac{9}{250}$ of the principal.
(5) Hence, \$18 ÷ $\frac{9}{250}$ = \$500 = $\frac{1}{2}$ A's + $\frac{2}{3}$ B's principal, or part loaned.
(6) $\frac{2}{3}$ of B's money = 2 times A's, because they are in the ratio of 1 to 3.
(7) Hence, \$500 is $2\frac{1}{2}$ times A's money.
(8) \$500 ÷ $2\frac{1}{2}$ = \$200, A's money, and \$200 × 3 = \$600, B's money.

∴ A's money is \$200, and B's \$600.

PROBLEM 208.

If $\frac{2}{5}$ of A's money equals $\frac{3}{4}$ of B's and $\frac{2}{3}$ of B's equals $\frac{3}{5}$ of C's, and the interest of all their money for 4 yr. 8 mo., at 6%, is \$13190; how much has each?

Solution.

(1) The interest on \$1 for the given time and rate is \$.28.
(2) Hence the principal is \$15190 ÷ .28 = \$54250.
(3) $\frac{2}{3}$ of A's money = $\frac{3}{4}$ of B's.
(4) $\frac{1}{3}$ of A's money = $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{8}$ of B's.
(5) $\frac{3}{3}$ A's money = 3 times $\frac{3}{8}$ = $\frac{9}{8}$ of B's.
(6) $\frac{8}{8}$ = B's money.
(7) $\frac{3}{5}$ of C's money = $\frac{2}{3}$ of B's money.
(8) $\frac{1}{5}$ of C's money = $\frac{1}{3}$ of $\frac{2}{3}$ = $\frac{2}{9}$ of B's.
(9) $\frac{5}{5}$ C's money = 5 times $\frac{2}{9}$ = $\frac{10}{9}$ of B's.
(10) $\frac{9}{8} + \frac{8}{8} + \frac{10}{9} = \frac{233}{72}$, their money = \$54250.
(11) $\frac{1}{72} = \frac{1}{233}$ of \$54250 = \$232$\frac{194}{233}$.
(12) $\frac{81}{72}$ = 81 times \$232$\frac{194}{233}$ = \$18859.44, A's money.
(13) $\frac{72}{72}$ = 72 times \$232$\frac{194}{233}$ = \$16663.95, B's money.
(14) $\frac{80}{72}$ = 80 times \$232$\frac{194}{233}$ = \$18626.61, C's money.

II. TRUE DISCOUNT.

73. **True Discount** is the difference between the amount of the debt and the present worth.

74. The **Present Worth** of a debt payable at a future time without interest, is such a sum as, being on interest for the time at a certain rate, will amount to the debt.

NOTE.—The true discount is the interest on the present worth for the time between the payment of the debt and the time it becomes due.

PROBLEM 209.

What is the present worth of \$1206, due 5 yr. 8 mo. hence without interest, money being worth 6%?

Solution.

(1) \$1 loaned at that time and rate amounts to \$1.34.
(2) One could pay as many dollars for a note of \$1206 as \$1.34 is contained times in \$1206, or \$900.

∴ \$900 is the present worth.

PROBLEM 210.

The interest on a sum of money for a certain time is \$300 and the true discount is \$240: find the sum of money.

Solution.

(1) The difference between the interest and true discount of any sum is always equal to the interest on the true discount for the same time and rate. ∴ \$300 — \$240 = \$60, the difference = the interest and \$240 the principal.

(2) \$60, the interest, is $\frac{60}{240}$, or $\frac{1}{4}$ of the principal.

(3) Then, $\frac{1}{4}$ of the principal = \$300, the interest.

(4) $\frac{4}{4}$, or the principal = 4 times \$300, or \$1200.

∴ \$1200 = the principal.

PROBLEM 211.

The true discount of a \$52 note, having 8 months to run, is \$4; and at the same rate the discount on a note of \$85 is \$5: find the time of the latter.

Solution.

(1) The sum on which discount is found is \$52 — \$4 = \$48.

(2) The interest on \$48 for 8 mo. at 1% is \$.32.

(3) \$4 ÷ .32 = $12\frac{1}{2}$%, rate of discount.

(4) On the other note \$85 — \$5 = \$80, the proceeds.

(5) $12\frac{1}{2}$% of \$80 = \$10, discount for one year.

(6) Hence, the time is 5 ÷ 10 = 6 mo.

∴ The time = 6 mo.

PROBLEM 212.

The true discount on a sum of money, at 6%, is \$50 more than the sum of the true discounts on half the sum at 8%, and on the other half at 4%: find the sum.

Solution.

(1) The true discount on any sum is equal to the present worth of its interest for the same time and rate.

(2) Hence, at 6% the true discount on the entire debt is $\frac{6}{100}$ of the debt; on half of the sum at 8% it is $\frac{4}{108}$; on the other half at 4% it is $\frac{2}{104}$ of it.

(3) ∴ $[\frac{6}{108} - (\frac{4}{108} + \frac{2}{104})] = \frac{25}{74412}$, or this equals \$50.

(4) Then, the sum is as many dollars as $\frac{25}{74412}$ is contained times in \$50, or \$148824.

∴ \$148824 is the required sum.

PROBLEM 213.

The sum of the simple interest, true present worth and true discount of a certain principal for a certain time is \$5000: find the amount.

Solution.

(1) The interest = Prt; the present worth = P ÷ (1 + rt); and the discount = P — P ÷ (1+rt) = Prt ÷ (1+rt).
(2) Then Prt + P ÷ (1 + rt) + Prt ÷ (1 + rt) = $5000.
(3) Now, P ÷ (1 + rt) + Prt ÷ (1 + rt) = P(1 + rt) ÷ (1 + rt) = P.
(4) ∴ We have Prt + P = $5000.
(5) But P(1 + rt) = A, amount, and A = $5000.

∴ $5000 is the amount required.

PROBLEM 214.

The interest of a certain sum is 20% of it, and the true discount is $20: find the sum.

Solution.

(1) 100% = sum.
(2) 100% ÷ 1.20 = 83⅓%, present worth.
(3) 100% — 83⅓% = 16⅔%, true discount.
(4) ∴ 16⅔% = $20.
(5) 1% = $20 ÷ 16⅔ = $1.20.
(6) 100% = 100 times $1.20, or $120.

∴ $120 = the sum.

PROBLEM 215.

The present worth is $100 more than the discount at 6%, and $95 more at 7%: find the debt and time.

Solution.

(1) Let P = debt, t = time, r = 6%, and r′ = 7%.
(2) Then the present worth at 6% is P ÷ (1 + rt) and at 7%, P ÷ (1 + r′t).
(3) P — [P ÷ (1 + rt)], or Prt ÷ (1 + rt) is the discount at 6%.
(4) Then [P ÷ (1 + rt)] — [Prt ÷ (1 + rt)] = $100.
(5) Whence t = (P — $100) ÷ (P ÷ $100)r . . . . (1).
(6) P — [P ÷ (1 + r′t)], or Pr′t ÷ (1 + r′t) is the discount at 7%.
(7) Hence, [P ÷ (1 + r′t)] — [Pr′t ÷ (1 + r′t)] = $95.
(8) t = (P — $95) ÷ (P + $95)r′ . . . . (2).
(9) Substituting in these equations the values of r and r′ and equating the values of t, we find P = $135.25 nearly, and from (1) t = 2.4969 yr.

∴ The debt is $135.25, and the time 2.4969 yr.

PROBLEM 216.

What sum is it whose true discount by simple interest for 4 yr. is $25 more at 6% than at 4% per annum?

Solution.

(1) The true discount on any sum is equal to the present worth of its interest for the same time and rate.
(2) Hence, the interest on $1 at 4% for 4 yr. = $.16.
(3) On $1 at 6% for 4 yr. = $.24. Then $25 $\div$ ($\frac{24}{124} - \frac{16}{116}$ = $449.50.

$\therefore$ $449.50 = the sum.

PROBLEM 217.

Two-thirds of A's money is $100 less than $\frac{1}{2}$ of B's; the present worth of A's for 1 yr. at 7% is $200 more than the simple interest of B's for 2 yr. at 8%: find the present worth of B's for 1 yr. 7 mo. and 19 da. at 9%.

Solution.

(1) If $\frac{2}{3}$ of A's money + $100 = $\frac{1}{2}$ of B's, then $\frac{4}{3}$ of A's + $200 = B's.
(2) The true present worth of A's money for 1 year at 7% is $\frac{100}{107}$ of A's money.
(3) The interest on B's money for 2 yr. at 8% is $\frac{4}{25}$ of ($\frac{4}{3}$ of A's + $200) = $\frac{16}{75}$ of A's money + $32.
(4) Hence, $\frac{100}{107}$ of A's money — $200 = $\frac{16}{75}$ of A's + $32.
(5) $\frac{100}{107} - \frac{16}{75}$ of A's = $200 + $32.
(6) A's is $232 $\div$ $\frac{5788}{8025}$ = $321.67.
(7) B's is $\frac{4}{3}$ of $321.67 + $200 = $628.89.
(8) $1.14725 is the amount of $1 for 1 yr. 7 mo. 19 da. at 9%.
(9) The present worth of $628.89 is $628.89 $\div$ $1.14725 = $548.17.

$\therefore$ $548.17 = present worth of B's money.

PROBLEM 218.

A note of $2000, dated July 4, 1876, due May 1, 1878, and bearing interest at 8%, was cancelled October 25, 1877, by payment of the present worth at 6%: what was the present worth at this date, and the discount? (*R. P., p. 259, prob. 15.*)

Solution.

(1) The interest on $1 from July 4, 1876, to May 1, 1878, is $.146.
(2) $2000 × $.146 = $292, interest on the note for the time.
(3) $2000 + $292 = $2292, amount of note.

(4) The interest on $1 from Oct. 25, 1877, to May 1, 1878, is $.031.

(5) $1 + $.031 = $1.031, the amount of $1 for time and rate.

(6) Then there would be as many dollars in the present worth of the note as $1.031 is contained times in $2292, or $2223.08.

(7) $2292 — $2223.08 = $68.92, the discount.

∴ The present worth was $2223.08, the discount $68.92.

PROBLEM 219.

The true discount of a sum for 6 mo. is $5, and the interest for the same time and rate is $5.25: find the debt.

Solution.

(1) The difference between the interest and true discount of any sum is always equal to the interest on the true discount for the same time and rate.

(2) $5.25 — $5 = $.25, the difference = the interest and $5 the principal.

(3) $.25, the interest, is $\frac{25}{500}$, or $\frac{1}{20}$ of the principal.

(4) $\frac{20}{20}$, or the principal, is 20 times $5.25, or $105.

(5) Suppose we were to find the rate, then we would find the interest on $105 for 6 mo. at 1%, or $.525.

(6) $5.25, the interest on the principal = $5.25 ÷ .525 = 10%.

∴ The rate is 10% and the debt $105.

III. BANK DISCOUNT.

75. A **Bank** is an incorporated institution which receives and loans and money.

76. **Bank Discount** is the interest on the face of the note for the time from the day of discount to the day of payment.

77. The **Proceeds** of a note is the sum received for it when discounted, and equals the face less the discount.

78. **Term of Discount** is the number of days from the time of discounting to the time of maturity of the note.

NOTE 1.—The difference between bank discount and true discount may be shown as follows: If I take a note to the bank promising to pay $108 at the end of 1 yr., to get it cashed, by true discount I would receive $100; but by bank discount, not counting days of grace, I would receive $108 minus the interest of $108 for 1 yr., *i. e.*, $108 — $8.64 = $99.36.

NOTE 2.—The discount of an interest bearing note is computed on the amount of the note at its maturity. Banks compute interest for the actual number of days a note has to run.

NOTE 3.—When no date of discount is given, the date of the note is taken as the date of discount.

PROBLEM 220.

A note of $200 is dated Aug. 25, payable in 60 days, and discounted at 6%: what is the proceeds?

Solution.

(1) We find the interest of $1 for 60 da. + 3 da., or 63 da., is $.0105, the bank discount.

(2) $200 × .0105 = $2.10, bank discount.

(3) $200 — $2.10 = $197.90, proceeds.

PROBLEM 221.

A note of $500 was given Jan. 1, 1895, at 8% interest, due in 6 mo.; it was discounted in bank March 1 at 10%: what was the proceeds of the note?

Solution.

(1) Counting forward, we have June 4, 1895, as the date of maturity; from March 1 to June 4 is 94 da., the time to run.

(2) As this is an interest bearing note, the sum to be discounted is the amount of the note.

(3) $\$.0406\frac{2}{3}$ = interest on $1 for 6 mo. 3 da.

(4) $\$500 \times .0406\frac{2}{3} = \$20\frac{1}{3}$, interest on the note.

(5) $\$520\frac{1}{3}$ is the sum to be discounted; $\$.02\frac{11}{18}$ is the bank discount on $1 for the time.

(6) $\$520\frac{1}{3} \times .97\frac{7}{18}$, the proceeds on $1 = $506.75, the proceeds of the note.

PROBLEM 222.

I had a 6% bond of $800, dated Jan. 1, 1896, due Jan. 1, 1897; on July 1, 1896, I sold the bond to Mr. Huntsman in such a way as to give him 8% on his investment; if Mr. Huntsman borrowed the money needed to pay the note, from a bank, at 10% for 90 da., what was the face of the bank note?

Solution.

(1) From Jan. 1, 1896, to Jan. 1, 1897, is 1 yr.

(2) $48 = interest on $800 for 1 yr. at 6%, and $848 = am't due Jan. 1, 1897.

(3) Amount of $1 at 8% for 6 mo. = $1.04.
(4) $848 ÷ 1.04 = $$815\frac{5}{13}$, paid for bond July 1, 1896.
(5) 93 da. = time to run; the proceeds of $1 for 93 da. at 10 % = $$.974\frac{1}{6}$.
(6) ∴ The face of the note must be as many dollars as $.974\frac{1}{6}$ is contained times in $$815\frac{5}{13}$, or $837.007.

PROBLEM 223.

A banker discounts a note at 8% per annum, thereby getting 9% per annum interest: how long does the note run?

Solution.

(1) The discount is reckoned on the face of the note, but the interest is estimated on the proceeds.
(2) Hence, 9%, or $\frac{9}{100}$ of the proceeds = 8%, or $\frac{2}{25}$ of the face.
(3) $\frac{1}{100}$ of the proceeds = $\frac{1}{9}$ of $\frac{2}{25}$ = $\frac{2}{225}$ of the proceeds.
(4) $\frac{100}{100}$, the proceeds = 100 times $\frac{2}{225}$ = $\frac{200}{225}$ of the face.
(5) The discount for the required time is $\frac{225}{225} - \frac{200}{225} = \frac{1}{9}$ of the face.
(6) ∴ The time is $\frac{1}{9} \div \frac{2}{25} = \frac{25}{18}$ yr., or 500 da.

PROBLEM 224.

A note dated February 19, 1876, payable January 1, 1877, and bearing 8% interest, was discounted October 12, 1876, at 6%; the proceeds was $1055.02: what was the face of the note? (*R. 3d P., p. 256.*)

Solution.

(1) If the note is nominally due Jan. 1, 1877, it will be legally due 3 days later, or Jan. 4, 1877.
(2) The interest on $1 from Feb. 19, 1876, to Jan. 4, 1877, is $.07; $1 + $.07 = $1.07, amount.
(3) From Oct. 12, 1876, to Jan. 4, 1877, is 84 da., time to run.
(4) $1.07 = amount of $1, or sum to be discounted; $.014 = bank discount on $1 for the time to run.
(5) $1.07 × .014 = $.01498, bank discount on $1.07.
(6) $1.07 — $.01498 = $1.05502, proceeds of $1 of the face of the note.
(7) Then there would be as many dollars in the face of the note as $1.05502 is contained times in $1055.02, or $1000.
∴ The face is $1000.

PROBLEM 225.

A commission merchant sold a consignment of cotton for $4500, receiving in payment a note, which yielded, on being discounted, $4475.25: what was the time of the note? (*Brooks' Arith.*)

Solution.

(1) \$4500 — \$4475.25 = \$24.75, the discount.

(2) The discount on \$1 for 1 day at 6% is $\$.000\frac{1}{6}$, and on \$4500 it is $\$4500 \times .000\frac{1}{6} = \$.75$.

(3) Hence, the note is discounted for as many days as \$.75 is contained times in \$24.75, or 33 days.

(4) ∴ The time is 33 da. — 3 da. or 30 da.

NOTE.—When we are to find the actual time, the days of grace should not be subtracted.

PROBLEM 226.

Mr. Herr buys goods to the amount of \$4000, and to pay for them gets his note for 60 days discounted at a bank; if the face is \$4042.45, what is the note? (*Brooks' Arith.*)

Solution.

(1) \$4042.45 — \$4000 = \$42.45, discount.

(2) The discount on \$4042.45 at 1% for 63 da. is \$7.074.

(3) Hence, the required rate is as many times 1% as \$7.074 is contained times in \$42.45, which is 6%.

SOME INTERESTING PROBLEMS.

PROBLEM 227.

A father left his four sons, whose ages were respectively 5, 9, 13, and 17 years, \$27500, to be divided in such a manner that the respective shares being placed out at 5% simple interest, shall amount to equal sums when they become 21 years of age: what were the shares? (*Putnam Co.*)

Solution.

(1) It will be 16, 12, 8 and 4 years respectively until they are of age.

(2) Now, the present worth of \$1 for 16 years at 5% is $\$.55\frac{5}{9}$.

(3) The present worth of \$1 for 12 yr. at 5% is $\$.62\frac{1}{2}$.

(4) The present worth of \$1 for 8 yr. at 5% is $\$71\frac{3}{7}$.

(5) The present worth of \$1 for 4 yr. at 5% is $\$.83\frac{1}{3}$.

(6) $\$.55\frac{5}{9} + \$.62\frac{1}{2} + \$.71\frac{3}{7} + \$.83\frac{1}{3} = \$2.72\frac{103}{126}$, the sum of the present worths.

(7) $\$27500 \div 2.72\frac{103}{126} = \$10080$.

(8) 1st son's share $= \$10080 \times .55\frac{5}{9} = \$5600$.

(9) 2d son's share $= \$10080 \times .62\frac{1}{2} = \$6300$.

(10) 3d son's share = $10080 × .71$\frac{3}{7}$ = $7200.
(11) 4th son's share = $10080 × .83$\frac{1}{3}$ = $8400.

PROBLEM 228.

A note of $312 given April 1, 1872, 8% from date, was settled July 1, 1874, the exact sum due being $304.98. Indorsed: April 1, 1878, $30.96; October 1, 1873, $——; April 1, 1874, $20.40. Restore the lost figures of second payment.

Solution.

(1) The new principal April 1, 1873, is ($312 × 1.08) — $30.96 = $306.

(2) The new principal April 1, 1874, is $304.98 ÷ $1.02 = $299.

(3) We should ascertain whether the payment Oct. 1 met the interest due. If it did, then we find the new principal for Oct. 1 and subtract it from the amount due on that date; but if the payment was less than the interest, we can readily restore the lost payment by subtracting the new principal April 1, 1874, $299, from the amount then due, less the payment, $20.40.

(4) The interest on $306 from April 1, 1873, to Oct. 1, at 8% is $306 × .04 = $12.24, and the amount due then would be $318.24.

(5) The new principal Oct. 1 would be ($299 + $20.40) ÷ 1.04 = $307.11.

(6) Hence, the lost payment could not be more than $318.24 ÷ 1.04 = $307.11.

(7) The lost payment could not be more than $318.24 — $307.11 = $11.13, which is less than the interest, $12.24, then due, and we must use the principal, $306, to April 1, 1874.

(8) The amount of $306 from April 1, 1873, to April 1, 1874, is $306 × $1.08 = $330.48.

(9) Subtracting the payment, we have $330.48 — $20.40 = $310.08, the new principal, had nothing been paid in the meantime.

(10) But we found the new principal to be $299, and the lost payment is $310.08 — $299 = $11.08.

∴ The missing figures are $11.08.

PROBLEM 229.

Burt owed in two accounts $487; neither was to draw interest till after due, one standing a year and the other two years. He paid both in 1 yr. 5 mo., finding the true discount of the second, at 6%, exactly equal to the interest of the first: what difference of time would the common rule have made?

Solution.

(1) One account drew interest 5 mo. and the other was discounted 7 mo. before due.

(2) Interest on \$1 for 5 mo. at 6% is $2\frac{1}{2}$ ct. = $\$\frac{1}{40}$.

(3) Interest on \$1 for 7 mo. at 6% is $3\frac{1}{2}$ ct.

(4) Hence, the 2d account was bought for $96\frac{1}{2}$ ct. on the dollar.

(5) The true discount on any sum is equal to the present worth of its interest for the same time and rate; hence, $3\frac{1}{2}$ ct. ÷ $1.03\frac{1}{2}$ = $\$\frac{7}{207}$.

(6) Now, the interest and discount are equal; ∴ $\frac{1}{40} = \frac{7}{207}$, and $\frac{40}{40} = 40 \times \frac{7}{207}$, or $\frac{280}{207}$.

(7) $\frac{280}{207} + \frac{207}{207} = \frac{487}{207}$ = \$487.

(8) $\frac{207}{207}$ = \$207, the 2d, and $\frac{280}{207}$ = \$280, the 1st account.

(9) \$280 × 1 = \$280, and \$207 × 2 = \$414.

(10) (\$280 + \$414) ÷ (\$280 + \$207) = 1 yr. 5 mo. 3 da.

(11) 1 yr. 5 mo. 3 da. — 1 yr. 5 mo. = 3 da.

∴ The difference is 3 days.

PROBLEM 230.

A Cincinnati manufacturer receives, April 18, an account of sales from New Orleans; net proceeds \$5284.67, due June 4–7. He advises his agent to discount the debt at 6%, and invest the proceeds in a 7 day bill on New York, interest off at 6%, at $\frac{1}{2}$% discount, and remit it to Cincinnati. The agent does this, April 26. The bill reaches Cincinnati May 3, and is sold at $\frac{1}{4}$% premium. What is the proceeds, and how much greater than if a bill had been drawn May 3, on New Orleans, due June 7, sold at $\frac{1}{8}$% premium, and interest off at 6%? (*R. H. A.*)

Solution.

(1) From April 26 to June 7 is 42 da.

(2) The discount on \$1 for 42 da. at 6% is \$.007.

(3) \$1 — \$.007 = \$.993, proceeds.

(4) \$5284.67 × .993 = \$5247.6773, amount invested in the bill on N. Y.

(5) 7 da. + 3 da. of grace = 10 da.

(6) $\frac{1}{2}$% = discount; 100% — $\frac{1}{2}$% = $99\frac{1}{2}$%.

(7) Bank discount of \$1 for 10 da. is $\frac{1}{6}$%.

(8) $99\frac{1}{2}$% — $\frac{1}{6}$% = $99\frac{1}{3}$%, or \$.$99\frac{1}{3}$.

(9) The N. Y. bill = \$5247.6773 ÷ .$99\frac{1}{3}$ = \$5282.896.

(10) In Cincinnati the bill sold at \$$1.00\frac{1}{4}$ on the dollar, or \$5282.896 × $1.00\frac{1}{4}$ = \$5296.10, proceeds.

(11) From May 3 to June 7 is 35 da.

(12) $\frac{1}{8}\%$ = premium; the bank discount on \$1 for 35 da. at 6% is \$.005$\frac{5}{6}$.

(13) Then, the New Orleans bill is worth \$5284.67 × (\$1 — \$.006$\frac{5}{6}$ + \$.00$\frac{1}{8}$) = \$5260.45.

(14) ∴ The gain = \$5296.10 — \$5260.45 = \$35.65.

PROBLEM 231.

What rate of income do I realize by purchasing United States 4% bonds at 105, if I sell them in six years at 104?

Solution.

(1) .04 × 6 = .24.

(2) 1.04 + .24 = 1.28, amount realized on bond.

(3) 1.28 — 1.05 = .23, amount gained in 6 yr.

(4) .23 ÷ 6 = .03$\frac{5}{6}$, amount gained in 1 yr.

(5) .03$\frac{5}{6}$ ÷ 1.05 = 3$\frac{41}{63}$%, rate of gain.

NOTE.—Solved by Prof. G. B. M. Zerr for the *Mathematical Monthly.*

PROBLEM 232.

A man bought a farm for \$5000, agreeing to pay principal and interest in five equal annual installments: what is the annual payment, including interest at 6%? (*Milne.*)

Solution.

(1) The interest on \$5000 for 1 yr. at 6% is \$300.

(2) (\$300 × 1.3382256) ÷ .3382256 = \$1186.08, one of the equal payments.

Rule.—*Multiply the interest on the debt for* 1 *year by the compound amount of* \$1 *for the given time and rate, and divide the product by the compound interest for the same time and rate.*

PROBLEM 233.

A miller buys a mill for \$6000, agreeing to pay for it in three equal annual payments, he paying 6% on the debt: what payment does he make?

Solution.

(1) The interest on \$6000 for 1 yr. at 6% is \$360.

(2) Hence, \$360 ÷ (1.191016 — 1) = \$1884.65, the part of the principal paid in 1st installment.

(3) Now, the compound amount of this for 3 yr. at 6% is \$1884.65 × 1.191016 = \$2244.65, the payment required.

Rule.—*To find the part of principal paid in first installment, divide the interest on the debt for* 1 *year by the compound interest for the given time.*

CHAPTER XI.

COUNTY EXAMINATION TESTS, AND OTHER PROBLEMS.

PROBLEM 234.

An agent sold a lot of cotton on commission of 4%; he invested the net proceeds in grain, after keeping his commission of 3%: if $1\frac{2}{5}$ times his commission is $3 more than $340, what was the value of the grain? (*Putnam Co.*)

Solution.

(1) $\frac{5}{5}$ = his commission.
(2) $\frac{5}{5} \times 1\frac{2}{5} = \frac{7}{5}$ = $3 more than $340, or $343.
(3) $\frac{1}{5} = \frac{1}{7}$ of $343 = $49.
(4) $\frac{5}{5}$, his commission = 5 times $49, or $245.
(5) Out of every $1 receipts of cotton the agent receives 4 ct. commission; 96 ct. = net proceeds.
(6) $\frac{3}{103}$ of 96 ct. = $\frac{288}{103}$ ct., 2d commission.
(7) 4 ct. + $\frac{288}{103}$ ct. = $\frac{700}{103}$ ct., or $\$\frac{7}{103}$, whole commission.
(8) $\$\frac{7}{103}$ = $245. Then, there must have been as many dollars of receipts for cotton as $\$\frac{7}{103}$ is contained times in $245, or $3605.
(9) Now, as $\$\frac{7}{103}$ is the whole commission on $1, $\frac{7}{103}$ of $3605 = $245, whole commission.
(10) $3605 — $245 = $3360.

∴ $3360 is the value of the grain.

PROBLEM 235.

A, B and C own a factory, a mill, a foundry respectively; the factory is worth 8% less than the mill, and the mill 32% more than the foundry; C has traded the foundry for 75% of the factory, thus losing $178.40: what is the value of the mill? (*Putnam Co.*)

Solution.

(1) 100% = value of the mill . . . . (A).
(2) 100% — 8% = 92%, value of the factory.
(3) 100% = value of the foundry.

(4) 132% = value of the mill in terms of the foundry.
(5) 132% = 100%, the value of the mill from . . . (A).
(6) 1% = $\frac{1}{132}$ of 100% = $\frac{25}{33}$%.
(7) 100% = 100 × $\frac{25}{33}$ = 75$\frac{25}{33}$%, value of the foundry.
(8) 75% of 92% = 69%, what C received for the foundry.
(9) 75$\frac{25}{33}$% — 69% = 6$\frac{25}{33}$%, what C lost in the trade.
(10) $178.40 = what he lost.
(11) 6$\frac{25}{33}$% = $178.40.
(12) 1% = $178.40 ÷ 6$\frac{25}{33}$ = $26.40.
(13) 100% = 100 times $26.40 = $2640.

∴ The mill is worth $2640.

PROBLEM 236.

A sold a sheep and lost 25%; if he had paid $1 more for it, he would have lost 40%: what did he pay for the sheep? (*Montgomery Co.*)

Solution.

(1) 100% = actual cost of the sheep.
(2) 25% of 100% = 25%, loss.
(3) 100% — 25% = 75%, selling price of the sheep.
(4) 100% + $1 = supposed cost of the sheep.
(5) 40% of (100% + $1) = 40% + $.40, loss.
(6) (100% + $1) — (40% + $.40) = 60% + $.60, selling price.
(7) 75% = 60% + $.60.
(8) 15% = $.60.
(9) 1% = $\frac{1}{15}$ of $.60 = $.04.
(10) 100% = 100 times $.04 = $4, the actual cost.

∴ He paid $4 for the sheep.

PROBLEM 237.

A, B and C can do a piece of work in 4 days, A and C in 8 days, B and C in 6 days: how long will it take each working alone? (*Wood Co.*)

Solution.

(1) 4 da. = time it takes A, B and C to do the work; $\frac{1}{4}$ = part they do in 1 da.
(2) 8 da. = time it takes A and C to do the work; $\frac{1}{8}$ = part they do in 1 da.
(3) 6 da. = time it takes B and C to do the work; $\frac{1}{6}$ = part they do in 1 da.
(4) $\frac{1}{4}$, part A, B and C do in 1 da. — $\frac{1}{8}$, part A and C do in 1 da. = $\frac{1}{8}$, part B does in 1 da.
(5) $\frac{8}{8}$ = part B does in $\frac{8}{8}$ ÷ $\frac{1}{8}$ = 8 da.

(6) $\frac{1}{4}$, part A, B and C do in 1 da. — $\frac{1}{6}$, part B and C do in 1 da. = $\frac{1}{12}$, part A does in 1 da.
(7) $\frac{12}{12}$ = part A does in $\frac{12}{12} \div \frac{1}{12}$ = 12 days.
(8) $\frac{1}{6}$, part B and C do in 1 da. — $\frac{1}{8}$, part B does in 1 da. = $\frac{1}{24}$, part C does in 1 da.
(9) $\frac{24}{24}$ = part C does in $\frac{24}{24} \div \frac{1}{24}$ = 24 da.

∴ A can do the work in 12 da., B in 8 da., and C in 24 da.

PROBLEM 238.

I bought an article and sold it so as to gain 10%; if it had cost 20% less, and I had sold it for one dollar less, I would have gained 25%: find the cost of the article. (*Noble Co.*)

Solution.

(1) 100% = actual cost.
(2) 110% = actual selling price.
(3) 80% = supposed cost.
(4) 25% of 80% = 20%, gain.
(5) 80% cost + 20 % gain = 100%, supposed selling price.
(6) But by the condition of the problem 110% — $1 = 100%.
(7) 10% = $1.
(8) 1% = $\frac{1}{10}$ of $1 = $.10.
(9) 100% = 100 times $.10 = $10, the actual cost.

∴ $10 is the cost of the article.

PROBLEM 239.

A banker had $1800, part of which he loaned at 6%, and the remainder at 5%, thus realizing an income of $100: find the amount loaned at 6%. (*Muskingum Co.*)

Solution.

(1) 100% = amount loaned at 6%.
(2) 6% of 100% = 6%, income on amount loaned at 6%.
(3) $1800 — 100% = amount loaned at 5%.
(4) 5% of ($1800 — 100%) = $90 — 5%, income on am't loaned at 5%.
(5) 6% — ($90 — 2%) = difference of incomes.
(6) 6% — ($90 — 2%) = $100.
(7) 1% = $10.
(8) 100% = 100 times $10 = $1000.

∴ $1000 = amount loaned at 6%.

PROBLEM 240.

Sold wheat at a loss of 5% and invested $25 more than the proceeds received from the wheat in corn, which I sold at a gain of 6%, making a net gain of $8.50: find the value of the wheat. (*Muskingum Co.*)

Solution.

(1) 100% = value of the wheat.
(2) 5% of 100% = 5%, loss.
(3) 100% — 5% = 95%, proceeds.
(4) 95% + $55 = amount invested in corn.
(5) 6% of (95% + $25) = 5.70% + $1.50, gain.
(6) 5.70% + $1.50 gain — 5% loss = $8.50, net gain.
(7) .70% = $7.00
(8) 1% = $\frac{1}{.70}$ of $7.00 = $10.
(9) 100% = 100 times $10 = $1000.

∴ $1000 = value of the wheat.

PROBLEM 241.

Paid $900 for a note due in 3 months, and sold it to a broker on the same day, taking bank discount at 6%, and cleared $84.50: find face of the note. (*Morrow Co.*)

Solution.

(1) It is evident that $900 + $84.50, or $984.50, is the net proceeds on the note.
(2) The net proceeds on $1 for 3 mo. 3 da. at 6% is $.9845.
(3) Then there would be as many dollars in the face of the note as $.9845 is contained times in $984.50, or $1000.

∴ $1000 is the face of the note.

PROBLEM 242.

If A had $60 more money, he could buy 30 oxen, or with $120 less he could buy only 15: how much money has he? (*York Co., Pa.*)

Solution.

(1) 100% = his money.
(2) (100% + $60) ÷ 30 = cost of 1 ox.
(3) (100% — $120) ÷ 15 = cost of 1 ox by second condition.
(4) [(100% + $60) ÷ 15] — [(100% — $120) ÷ 15] = 15 oxen = $180.
(5) $180 ÷ 15 = $12, or the cost of 1 ox.
(6) (100% + $60) ÷ 30 = $12, the cost of 1 ox.
(7) 100% = $300, A's money.

∴ A has $300.

PROBLEM 243.

On goods bought for \$4500, on 6 months credit, I was offered 5% off for cash; if money was worth 6%, how much did I lose by accepting the credit? (*Darke Co.*)

Solution.

(1) 5% of \$4500 = \$225, amount deducted for cash.
(2) \$4500 — \$225 = \$4275, amount required for cash.
(3) The interest on \$1 for 6 mo. at 6% is \$.03, and \$4275 is \$4275 × \$.03, or \$128.25.
(4) \$4275 + \$128.25 = \$4403.25.
(5) \$4500 — \$4403.25 = \$96.75, amount lost accepting the credit.

∴ I lost \$96.75.

PROBLEM 244.

Kerosene is bought at 50 ct. a gal., 10% of it is wasted: at what price must it be offered in order that the price may be discounted 10%, and yet 10% be made on the investment? (*Hancock Co.*)

Solution.

(1) Let 100 represent the number of gallons.
(2) 100 × 50 ct. = \$50, the cost of the kerosene.
(3) 10% = gain; 10% of \$50 = \$5, gain.
(4) \$50 + \$5 = \$55, selling price.
(5) 100% = marked price; 10% = deduction.
(6) 100% — 10% = 90%, selling price = \$55.
(7) 1% = $\frac{1}{90}$ of \$55 = \$$\frac{55}{90}$.
(8) 100% = 100 × \$$\frac{55}{90}$ = \$61$\frac{1}{9}$, marked price.
(9) 10% of 100 gallons = 10 gallons, amount wasted.
(10) 100 — 10 = 90 gallons, amount remaining.
(11) \$61$\frac{1}{9}$ ÷ 90 = 67$\frac{73}{81}$ ct., marked price per gallon.

∴ It must be offered at 67$\frac{73}{81}$ ct. per gallon.

PROBLEM 245.

Sold a horse and a cow for \$210; on the horse I gained 25%, and lost 25% on the cow: what was my gain, if the cow cost $\frac{2}{3}$ as much as the horse? (*Hardin Co.*)

Solution.

(1) 100% = cost of the horse; 25% = gain.
(2) 100% + 25% = 125%, selling price of the horse.
(3) $\frac{2}{3}$ of 100% = 66$\frac{2}{3}$%, cost of the cow.
(4) 25% of 66$\frac{2}{3}$% = 16$\frac{2}{3}$%, loss.
(5) 66$\frac{2}{3}$% — 16$\frac{2}{3}$% = 50%, selling price of the cow.

(6) 125% + 50% = 175%, selling price of both.
(7) \$210 = selling price of both.
(8) 175% = \$210.
(9) 1% = $\frac{1}{175}$ of \$210 = \$1.20
(10) 100% = 100 times \$1.20 = \$120, cost of the horse.
(11) $66\frac{2}{3}$% = $66\frac{2}{3}$ times \$1.20 = \$80, cost of the cow.
(12) \$120 + \$80 = \$200, cost of both.
(13) \$210 — \$200 = \$10, gain.

∴ My gain is \$10.

PROBLEM 246.

A broker charged me $2\frac{1}{4}$% for purchasing some uncurrent bank notes at 15% discount; three bills of \$20, \$50 and \$100 respectively, turned out to be worthless, but by selling the rest at par, I made \$85: what was the face of the notes? (*Logan Co.*)

Solution.

(1) 100% = face; 100% — $2\frac{1}{4}$% = $97\frac{3}{4}$%, the proceeds.
(2) 85% = the cost; $97\frac{3}{4}$% — 85% = $12\frac{3}{4}$%, gain.
(3) \$20 + \$50 + \$100 = \$170, loss.
(4) But we see that he made \$85 clear, then to make up the loss of \$170 he must gain \$170 + \$85, or \$255.
(5) $12\frac{3}{4}$% = \$255.
(6) 1% = \$255 ÷ $12\frac{3}{4}$ = \$20.
(7) 100% = 100 times \$20 = \$2000.

∴ \$2000 = the face.

PROBLEM 247.

At a certain time between 8 and 9 o'clock a boy stepped into the schoolroom, and noticed the minute hand between 9 and 10; he left, and on returning within an hour, he found the hour hand and minute hand had exchanged places: what time was it when he first entered the room, and how long was he gone? (*Hancock Co.*)

Solution.

(1) $\frac{2}{2}$ = distance minute hand moves, or the distance it was beyond the hour hand.
(2) $\frac{24}{2}$ = distance the minute hand moved while the hour hand traveled $\frac{2}{2}$.
(3) $\frac{24}{2}+\frac{2}{2}=\frac{26}{2}$, distance they both moved.
(4) ∴ $\frac{26}{2}$ = 60 min.
(5) $\frac{1}{2}=\frac{1}{26}$ of 60 = $\frac{60}{26}$, or $2\frac{4}{13}$ min.
(6) $\frac{2}{2}$ = 2 times $2\frac{4}{13}$ = $4\frac{8}{13}$ min., the distance the minute hand was in advance of the hour hand.

(7) $\frac{2}{2}$ = distance the hour hand moved past 8.
(8) $\frac{24}{2}$ = distance the minute hand moved.
(9) $\frac{24}{2}$ = 40 min. + $4\frac{8}{13}$ min. + $\frac{2}{2}$, or $44\frac{8}{13}$ min + $\frac{2}{2}$.
(10) $\frac{22}{2}$ = $44\frac{8}{13}$ min.
(11) $\frac{1}{2}$ = $\frac{1}{22}$ of $44\frac{8}{13}$ = $2\frac{4}{143}$ min.
(12) $\frac{24}{2}$ = 24 times $2\frac{4}{143}$ min. = $48\frac{96}{143}$ min., past 8.
(13) Now, if the hands changed places, the minute hand fell short $4\frac{8}{13}$ min. of being an hour.

∴ It was $48\frac{96}{143}$ minutes past 8 when he stepped into the room, and he was gone $55\frac{5}{13}$ minutes.

PROBLEM 248.

A and B wish to sell their horses to C; A asks a certain price, and B 50% more than A; A then reduces his price 20%, and B reduces his price 30%; C takes them both, paying $148: find the asking price of each. (*Scioto Co.*)

Solution.

(1) 100% = A's asking price.
(2) 150% = B's asking price.
(3) 20% of 100% = 20%, amount deducted from the asking price of the 1st horse.
(4) 100% — 20% = 80%, selling price of the 1st horse.
(5) 30% of 150% = 45%, amount deducted from the asking price.
(6) 150% — 45% = 105%, selling price of 2d horse.
(7) 105% + 80% = 185%, selling price of both horses.
(8) 185% = $148.
(9) 1% = $\frac{1}{185}$ of $145 = $.80.
(10) 100% = 100 times $.80 = $80, asking price of 1st horse.
(11) 150% = 150 times $.80 = $120, asking price of 2d horse.

∴ A first asked $80, B $120.

PROBLEM 249.

A and B are in partnership, A investing $400 and B $500: how much must A put in at the end of two months to entitle him to half of the year's profits?

Solution.

(1) A has $400 × 12 = $4800 for 1 month.
(2) B has $500 × 12 = $6000 for 1 month.
(3) ∴ A must contribute $6000 — $4800 = $1200 for 1 month.
(4) For 10 months he must contribute $1200 ÷ 10, or $120.

PROBLEM 250.

A sold goods which cost him \$300, at a certain rate of profit; B sold the same goods to C at the same rate of profit; C paid \$432 for them: what did B pay for them? (*Ross Co.*)

Solution.

(1) 100% = cost of goods to B.
(2) r = rate of profit to A and B.
(3) 100% + r = selling price of B's goods.
(4) (100% + r) times \$300 = \$300 + 300r, selling price, or B's cost.
(5) (100% + r) (\$300 + 300r) = \$300 + 600r + $300r^2$, B's selling price, or C's cost.
(6) \$432 = C's cost.
(7) ∴ \$300 + 600r + $300r^2$ = \$432.
(8) $r^2 + 2r = \frac{132}{300}$.
(9) r = .20, or 20%, A's and B's rate of gain.
(10) Then, \$300 × 20% = \$60, A's gain.
(11) \$300 + \$60 = \$360, A's selling price.

∴ B paid \$360.

PROBLEM 251.

A farmer sold two horses for \$810, receiving $\frac{4}{5}$ as much for the first as for the second; on the first he gained $33\frac{1}{3}\%$, on the second he lost $11\frac{1}{9}\%$: how much did he gain? (*Brown Co.*)

Solution.

(1) 100% = selling price of 2d horse.
(2) $\frac{4}{5}$ of 100% = 80%, selling price of 1st horse.
(3) 100% + 80% = 180%, selling price of both.
(4) 180% = \$810, selling price of both.
(5) 1% = $\frac{1}{180}$ of \$810 = \$4.50.
(6) 100% = 100 times \$4.50 = \$450, selling price of 2d horse.
(7) 80% = 80 times \$4.50 = \$360, selling price of 1st horse.
(8) 100% = cost of the 1st horse.
(9) 100% + $33\frac{1}{3}\%$ = 133%, selling price of 1st horse.
(10) $133\frac{1}{3}\%$ = \$360.
(11) 1% = \$360 ÷ $133\frac{1}{3}$ = \$2.70.
(12) 100% = 100 times \$2.70 = \$270, cost of 1st horse.
(13) 100% — $11\frac{1}{9}\%$ = $88\frac{8}{9}\%$, selling price of 2d horse.
(14) ∴ $88\frac{8}{9}\%$ = \$450.
(15) 1% = \$450 ÷ $88\frac{8}{9}$ = \$5.0625.

(16) 100% = 100 times \$5.0625 = \$506.25, cost of 2d horse.
(17) \$270 + \$506.25 = \$776.25, cost of both.
(18) \$810 — \$776.25 = \$33.75, gain.

∴ He gained \$33.75.

PROBLEM 252.

Bought a certain number of eggs at 2 for a cent, and as many more at 3 for a cent; sold them all at 5 for 2 cents, and lost four cents: how many eggs were there?

Solution.

(1) $\frac{1}{2}$ ct. = cost of 1 egg at 2 for a cent,
(2) $\frac{1}{3}$ ct. = cost of 1 egg at 3 for a cent.
(3) ($\frac{1}{2}$ ct. + $\frac{1}{3}$ ct.) ÷ 2 = $\frac{5}{12}$ ct., average cost of 1 egg.
(4) $\frac{2}{5}$ ct. = selling price of 1 egg.
(5) $\frac{5}{12}$ ct. — $\frac{2}{5}$ ct. = $\frac{1}{60}$ ct., average loss on each egg.
(6) Hence, to lose 4 ct., there were as many eggs sold as $\frac{1}{60}$ ct. is contained times in 4 ct., or 240.

∴ 20 dozen eggs were sold.

PROBLEM 253.

A note is drawn May 20 on 6 months time for \$840, bearing 6% interest; it is discounted at bank Sept 9, at 8%: find time to run and proceeds. (*Darke Co.*)

Solution.

(1) Counting forward, we have Nov. 23 the date of maturity.
(2) From Sept. 9 to Nov. 23 is 75 da., the time to run.
(3) As this is an interest bearing note, the sum to be discounted is the amount of the note.
(4) \$.0305 = interest on \$1 for 6 mo. and 3 da.
(5) \$840 × .0305 = \$25.62, interest on the note, and \$865.62 = sum to be discounted.
(6) \$.01$\frac{2}{3}$ = bank discount for the time and rate.
(7) \$865.62 × .01$\frac{2}{3}$ = \$14.427, bank discount.
(8) \$865.62 — \$14.427 = \$851.193, proceeds.

∴ The time to run is 75 da., and the proceeds \$851.193.

PROBLEM 254.

A banker had \$1800, part of which he loaned at 6%, and the remainder at 5%, thus realizing an income of \$100: find the amount loaned at 6%. (*Darke Co.*)

Solution.

(1) 100% = amount loaned at 6%.
(2) \$1800 — 100% = amount loaned at 5%.
(3) 6% of 100% = 6%, interest on 100%.
(4) 5% of (\$1800 — 100%) = \$90 — 5%, interest.
(5) 6% + \$90 — 5% = the whole income = \$100.
(6) 1% = \$10.
(7) 100% = 100 times \$10 = \$1000, amount loaned at 6%.
(8) \$1800 — \$1000 = \$800, amount loaned at 5%.

∴ He loaned \$1000 at 6%.

PROBLEM 255.

A grain dealer sold a quantity of rye and wheat for \$1320, gaining $33\frac{1}{3}$% on the rye and $12\frac{1}{8}$% on the wheat: what was his total gain, if he received 20% more for the wheat than for the rye? (*Putnam Co.*)

Solution.

(1) 100% = selling price of the rye.
(2) 120% = selling price of the wheat.
(3) 100% + 120% = 220%, whole selling price.
(4) 220% = \$1320.
(5) 1% = $\frac{1}{220}$ of \$1320 = \$6.
(6) 100% = 100 times \$6 = \$600, selling price of the rye.
(7) 120% = 120 times \$6 = \$720, selling price of the wheat.
(8) 100% = cost of the rye.
(9) $133\frac{1}{3}$% = selling price of the rye = \$600.
(10) 1% = \$600 ÷ $133\frac{1}{3}$ = \$4.50.
(11) 100% = 100 times \$4.50 = \$450, cost of the rye.
(12) 100% = cost of the wheat.
(13) $112\frac{1}{8}$% = selling price of the wheat = \$720.
(14) 1% = \$720 ÷ $112\frac{1}{8}$ = \$$6.42\frac{42}{299}$.
(15) 100% = 100 times \$$6.42\frac{42}{299}$, = \$$642\frac{42}{299}$, cost of the wheat.
(16) \$600 — \$450 = \$150, gain on the rye.
(17) \$720 — \$$642\frac{42}{299}$ = \$$77\frac{257}{299}$, gain on the wheat.
(18) \$150 + \$$77\frac{257}{299}$ = \$$227\frac{257}{299}$.

∴ \$$227\frac{257}{299}$ was his whole gain.

PROBLEM 256.

At what price per barrel shall an agent be ordered to buy potatoes at 2% commission that, after paying 7 ct. per bushel for transportation, they can be sold at \$1.76 per barrel and net 10% profit? (*Putnam Co.*)

Solution.

(1) 100% = cost.
(2) 110% = selling price.
(3) $1.76 = selling price.
(4) 1% = $\frac{1}{110}$ of $1.76 = $.0160.
(5) 100% = 100 times $.0160 = $1.60, cost to agent.
(6) $1.60 — $.07 = $1.53.
(7) 100 = number of parts investment.
(8) 2 = number of parts commission.
(9) 102 = whole number of parts.
(10) $\frac{100}{102}$ of $1.53 = $1.50.

∴ $1.50 = what the agent must pay per barrel.

PROBLEM 257.

A commission merchant received $4456.40 to invest in wheat, after deducting his commission of 2%, and drayage 50 cts. per load of 36 bu.: how much wheat did he purchase at $1.20 per bushel? (*Putnam Co.*)

Solution.

(1) 100 = number of parts invested in wheat, and 2 = number of parts commission.
(2) 100 + 2 = 102, whole number of parts.
(3) $4456.40 = whole number of parts.
(4) $\frac{100}{102}$ of $4456.40 = $4369.01, amount invested in wheat.
(5) $.50 ÷ 36 = $.01$\frac{7}{18}$, cost of drayage per bushel.
(6) ∴ $1.20 + $.01$\frac{7}{18}$ = 1.21\frac{7}{18}$, cost per bushel.
(7) $4369.01 ÷ 1.21$\frac{7}{18}$ = 3599+ bushels.

PROBLEM 258.

A manufacturer gained 35% on one kind of wares and 44% on another: if the cost of manufacturing each kind was the same, and the gain was $360 more on the higher priced wares, what were the total rates? (*Putnam Co.*)

Solution.

(1) 100% = cost of each kind of wares.
(2) 100% + 35% = 135%, selling price of the 1st kind.
(3) 100% + 44% = 144%, selling price of the 2d kind.
(4) 144% — 135% = 9%, gain on the 2d.
(5) $360 = gain on the 2d.
(6) 9% = $360.
(7) 1% = $\frac{1}{9}$ of $360 = $40.
(8) 100% = 100 times $40 = $4000, cost of each kind.

(9) 135% = 135 times \$40 = \$5400, selling price of 1st kind.
(10) 144% = 144 times \$40 = \$5760, selling price of 2d kind.

∴ The selling prices were \$5400 and \$5760 respectively.

PROBLEM 259.

A lady spent in one store ½ of all her money and \$1 more; in another, ½ of the remainder and \$1 more; in another, ½ of the remainder and \$1 more; in another, ½ of the remainder and \$1 more; and in another, ½ of the remainder and \$1 more; she then had nothing left: what sum had she at first? (*Hancock Co.*)

First Solution.

(1) Let x = her money at first.

(2) $\frac{x}{2} + 1$ = amount spent in 1st store.

(3) $x - \left(\frac{x}{2} + 1\right) = \frac{x}{2} - 1$, amount remaining.

(4) $\frac{x}{4} - \frac{1}{2} + 1$, or $\frac{x}{4} + \frac{1}{2}$ = amount spent in 2d store.

(5) $\frac{x}{2} - 1 - \left(\frac{x}{4} + \frac{1}{2}\right) = \frac{x}{4} - \frac{3}{2}$, amount remaining.

(6) $\frac{x}{8} - \frac{3}{4} + 1$, or $\frac{x}{8} + \frac{1}{4}$ = amount spent in 3d store.

(7) $\frac{x}{4} - \frac{3}{2} - \left(\frac{x}{8} + \frac{1}{4}\right) = \frac{x}{8} - \frac{7}{4}$, amount remaining.

(8) $\frac{x}{16} - \frac{7}{8} + 1$, or $\frac{x}{16} + \frac{1}{8}$ = amount spent in 4th store.

(9) $\frac{x}{8} - \frac{7}{4} - \left(\frac{x}{16} + \frac{1}{8}\right) = \frac{x}{16} - \frac{15}{8}$, amount remaining.

(10) Now, by the condition of the problem, $\frac{x}{16} = \frac{15}{8}$.

(11) x = \$30, amount she had at first.

Second Solution.

(1) Starting with the last amount, or amount she must have had on entering the 4th store, which must have been \$1 + \$1, or \$2; then, \$2 + \$1 = \$3; \$3 × 2 = \$6, amt. on entering 3d store.
(2) \$6 + \$1 = \$7; \$7 × 2 = \$14, amount on entering 2d store.
(3) \$14 + \$1 = \$15; \$15 × 2 = \$30, amount on entering 1st store.

PROBLEM 260.

If the sun crossed the equator March 20, at 8:25 A. M., central standard time, in what longitude did it cross?

Solution.

(1) Difference between standard and local time is 28 min.
(2) 8 hr. 25 min. + 28 min = 8 hr. 53 min.
(3) 8 hr. 53 min. = 133° 15′.
(4) We live in about 84° west longitude. ∴ 133° 15′ — 84° = 49° 15′ east longitude.

NOTE.—When you pass from one section into the next, the time becomes one hour slower if you are moving westward, one faster if you are moving eastward. Eastern time is that of the 75th meridian, Central time that of the 90th, Mountain time that of the 105th, Pacific time that of the 120th. See Art. 16. When it is noon at all places in the Eastern section, it is 11 a. m. at all places in the Central section, 10 a. m. at all places in the mountain section, and 9 a. m. at all places in the Pacific section. This neat and simple system is now in use all over the United States and Canada.

PROBLEM 261.

If the earth's rotation were reversed, how many days would we have in a year? (*Putnam Co.*)

Solution.

(1) If the earth's rotation were reversed we would have 367 days in a year. The earth would make about 4 minutes less than one complete rotation each day; this would make a gain of one day in the year.
(2) The earth makes 366 revolutions on its axis each year, and this would give us 367 days.

PROBLEM 262.

If the gain is $12\frac{1}{2}\%$ of the selling price: what is the rate of gain? (*Putnam Co.*)

Solution.

(1) Let 100% = selling price.
(2) $12\frac{1}{2}\%$ of $100\% = 12\frac{1}{2}\%$, gain.
(3) $100\% - 12\frac{1}{2}\% = 87\frac{1}{2}\%$, cost.
(4) Let 100% = cost.
(5) $87\frac{1}{2}\% = 100\%$.
(6) $1\% = 100\% \div 87\frac{1}{2} = \frac{100}{87.5}\%$.
(7) $12\frac{1}{2}\% = 12\frac{1}{2}$ times $\frac{100}{87.5}\%$, or $14\frac{2}{7}\%$.

∴ $14\frac{2}{7}\%$ is the rate of gain.

PROBLEM 263.

A owns the S. W. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$, the S. E. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$, the N. E. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$, and the N. W. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$ of a section of land. Show by a drawing the position of his farm in the section, and divide the remainder of the section into four farms of equal shape and area, giving their dimensions.

Solution.

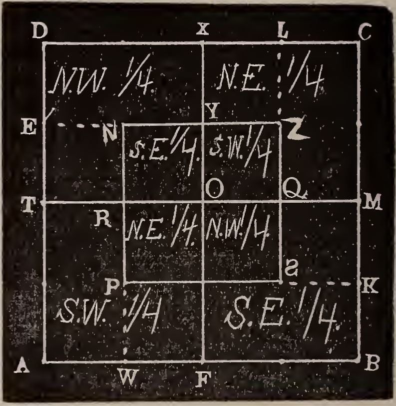

FIG. 1.

Let ABCD be the section of land, AB or AD = 320 rd.

Divide the section into quarters containing 160 A. each. Then TOXD contains 160 A., and ROYN contains 40 A.

It is obvious that A owns PSZN, or 160 A.

We have left the four rectangles WK, KL, LE and EW, each containing $\frac{1}{4}$ of (640 A. — 160 A.) = 120 A.

DC = 320 rd.; XC = $\frac{1}{2}$ of 320 rd., or 160 rd., and LC = $\frac{1}{2}$ of 160 rd., or 80 rd. LC = KB; then CK = 320 rd. — 80 rd. = 240 rd. ∴ The rectangles are 240 rd. long and 80 rd. wide.

PROBLEM 264.

What will it cost to fence the N. $\frac{1}{2}$ of the E. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$ of section 16, making 5 rectangular fields of equal area, at $2 per rod?

Solution.

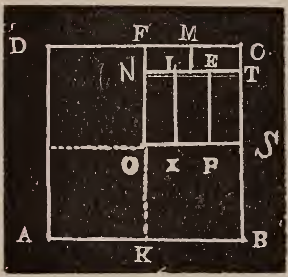

FIG. 2.

Let ABCD be the N. E. $\frac{1}{4}$ of section 16, FKBC = the E. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$ of section 16, and OSCF the N. $\frac{1}{2}$ of the E. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$.

OS, or OF = 80 rd., and the enclosed farm is 80 × 4 = 320 rd.

Then divide it into two parts by a fence 80 rd. long, running east and west, and 32 rd. south of north boundary; next divide the northern portion into two equal parts by a fence 32 rd. long, running north and south. Divide NOST into three equal parts by two fences (LX and EP) each 48 rd. long. This requires 320 + 80 + 32 + 48 × 2 = 528 rd. of fence.

∴ 528 rd. × \$2 = \$1056, the cost of fencing.

PROBLEM 265.

My agent sold produce at 2% commission; increasing the proceeds by \$42, I ordered him to purchase dry goods at 5% commission, which I sold at $3\frac{1}{2}$% loss, my whole loss, including commission, being \$63: find value of produce and dry goods. (*Putnam Co.*)

Solution.

(1) Out of every \$1 received for produce, my agent received 2 ct. commission, and secondly $\frac{5}{105}$ of 98 ct., or $\frac{490}{105}$ ct.

(2) I lost $3\frac{1}{2}$ ct., or $\frac{3.5}{100}$ of $\frac{100}{105}$ of 98 ct. = $\frac{343}{105}$ ct.

(3) In all, my loss on \$1 of produce = 2 ct. + $\frac{490}{105}$ ct. + $\frac{343}{105}$ ct., or $\$\frac{10.43}{105}$.

(4) My loss on \$42 is first $\frac{5}{105}$ of \$42 = $\$\frac{210}{105}$, and secondly $\frac{3.5}{100}$ of $\frac{100}{105}$ of \$42 = $\$\frac{147}{105}$.

(5) $\$\frac{210}{105} + \$\frac{147}{105} = \$\frac{357}{105}$, total loss on \$42.

(6) Then, \$63 — $\$\frac{357}{105} = \$\frac{6258}{105}$, loss on the produce.

(7) Now the loss on \$1 of the produce is $\$\frac{10.43}{105}$; then there are as many dollars in the value of produce as $\$\frac{10.43}{105}$ is contained times in $\$\frac{6258}{105}$, or \$600.

(8) 2% of \$600 = \$12, commission.

(9) \$600 — \$12 = \$588, net proceeds.

(10) \$588 + \$42 = \$630, amount to invest in goods at 5 ct. commission.

(11) Then, $\frac{100}{105}$ of \$630 = \$600, value of the goods.

PROBLEM 266.

An article cost \$6; at what price must it be marked so that the marked price may be reduced 22% and still 30% be gained?

Solution.

(1) \$6 = cost price.

(2) 30% = gain.

(3) 30% of \$6 = \$1.80.

(4) \$6 + \$1.80 = \$7.80, selling price.

(5) 100% = marked price.

(6) 22% = the deduction.

(7) 100% — 22% = 78%, selling price.

(8) 78% = \$7.80.

(9) 1% = $\frac{1}{78}$ of \$7.80 = \$.10.

(10) 100% = 100 times \$.10 = \$10, the marked price.

PROBLEM 267.

A boy was to receive 25 ct. per day for prompt attendance at school for a term of 80 days, but had to forfeit 75 ct. for each day absent; he received $15: find the number of days present. (*Putnam Co.*)

Solution.

(1) Had he attended the 80 days he would have received 80 times 25 ct., or $20.

(2) He lost by his absence $20 — $15, or $5.

(3) Each day absent he lost $.75 + $.25, or $1.

(4) Hence, to lose $5 he must have been absent as many days as $1 is contained times in $5, which is 5 days.

(5) ∴ 80 days — 5 days = 75 days, present.

PROBLEM 268.

A merchant sold ½ of his flour at 12% profit, ¼ at 10% profit, and ⅛ at 8% loss: how should he sell the remainder so as to gain 5% on the whole? (*J. B. Maurer.*)

Solution.

(1) Let 100% = cost.

(2) ½ of 100% = 50%, what he sold at 12% profit.

(3) 12% of 50% = 6%; 50% + 6% = 56%, what he received for the 50%.

(4) ¼ of 100% = 25%, what he sold at 10% profit.

(5) 10% of 25% = 2½%; 25% + 2½% = 27½%, what he received for the 25%.

(6) ⅛ of 100% = 12½%, what he sold at a loss of 8%.

(7) 8% of 12½% = 1%; 12½% — 1% = 11½%, what he received for the 12½%.

(8) 50% + 25% + 12½% = 87½%, amount sold.

(9) 100% — 87½% = 12½%, what remains to be sold.

(10) 56% + 27½% + 11½ = 95%, what he received for amount sold.

(11) 100% + 5% = 105%, what he asked for his flour, since he wanted to gain 5% on the whole.

(12) 105% — 95% = 10%, what he had to receive for the balance.

(13) In order to sell the remainder, 12½%, for 10%, he would sell it for 10% ÷ 12½% = 80% of the cost, or at a loss of 20%.

PROBLEM 269.

Ralph sold cotton and invested the proceeds in stock at 20% discount, receiving 4% commission in each transaction; sold this stock at 4% premium, commission 5%: what did the cotton sell for, my whole commission being $1780?

Solution.

(1) Out of each \$1 received for cotton, my agent received 4 ct. commission; 96 ct. = the proceeds.

(2) Now, this proceeds is made up of 100 parts investment and 4 parts commission.

(3) 104 = the whole number of parts; $\frac{100}{104}$ of 96 ct. = $92\frac{4}{13}$ ct., investment in stock.

(4) Since the stock was bought at 80 ct. and sold at 104 ct., the amount of sales on stock was ($92\frac{4}{13}$ ct. × 104) ÷ 80 = 120 ct., and the commission on this is 120 ct. × .05 = 6 ct.

(5) \$1 — $92\frac{4}{13}$ ct. + 6 ct. = $13\frac{9}{13}$ ct., whole commission on \$1 of the cotton.

(6) Hence, there will be as many dollars of receipts for cotton as $13\frac{9}{13}$ ct. is contained times in \$1780, or \$13000.

∴ The cotton sold for \$13000.

PROBLEM 270.

A broker bought 6% bonds at 20% premium, and kept them 5 years, when they were redeemed at par: what rate of interest did he make on the investment?

Solution.

(1) Let \$100 = the bond.

(2) The interest on \$100 for 5 years at 6% = \$30.

(3) Then, \$120 = cost of the bond, and

(4) \$130 — \$120 = \$10, the gain.

(5) \$10 ÷ \$120 = $8\frac{1}{3}$%.

(6) $8\frac{1}{3}$% ÷ 5 = $1\frac{2}{3}$%.

∴ He made $1\frac{2}{3}$%.

PROBLEM 271.

A sells a horse to B at a gain, and B to C at the same rate of gain for \$16; if B had sold for \$10, his loss would have been half what he now gains: find what A paid.

Solution.

(1) It is obvious that as often as he loses \$2 he gains \$4.

(2) The difference between \$16 and \$10, or \$6 = the loss + the gain.

(3) \$6 is $1\frac{1}{2}$ times B's gain; then, \$6 ÷ $1\frac{1}{2}$ = \$4, B's gain.

(4) \$16 — \$4 = \$12, B's cost.

(5) 100% = B's cost.

(6) \$12 = 100%, and \$1 = $\frac{1}{12}$ of 100% = $\frac{100}{12}$%.

(7) \$4 = 4 times $\frac{100}{12}$% = $33\frac{1}{3}$%, A and B's gain.
(8) 100% = A's cost.
(9) $133\frac{1}{3}$% = A's selling price.
(10) $133\frac{1}{3}$% = \$12, A's selling price.
(11) 1% = \$12 ÷ $133\frac{1}{3}$ = \$.09.
(12) 100% = 100 times \$.09 = \$9.

∴ A paid \$9.

PROBLEM 272.

A sold some cotton by his agent, giving 4% commission, and invested his proceeds in pike stock 20% below par. Having waited for a favorable turn, he has, this morning, sold his stock so as to gain upon it a sum equal to 14% of his cotton. Allowing me a rate of commission just $\frac{4}{5}$ of the present discount of pike stock, he has advised me to invest for him in a note which will, in 8 mo., at 6% bring \$$268.12\frac{1}{2}$: how much will be his whole gain upon the value of his cotton?

Solution.

(1) 100% — 4% = amount invested in stock, the stock is worth 80%.
(2) Now, if the gain on 96% is 14%, on 80% the gain is $\frac{80}{96}$ of 14%, or $11\frac{2}{3}$%.
(3) Value of stock = 80% + $11\frac{2}{3}$% = $91\frac{2}{3}$%.
(4) 100% — $91\frac{2}{3}$ = $8\frac{1}{3}$% discount.
(5) $\frac{4}{5}$ of $8\frac{1}{3}$% = $6\frac{2}{3}$%, my rate of commission.
(6) Out of every \$$1.06\frac{2}{3}$ received for stock, \$1 is invested in the note, or $\frac{106\frac{2}{3}}{100} = \frac{16}{15}$ of the note = amount received for the stock.
(7) The present worth of \$$268.12\frac{1}{2}$ for 8 mo. at 6% = \$$268.12\frac{1}{2}$, which divided by 1.04 = \$$257.81\frac{1}{4}$.
(8) ∴ The stock brought $\frac{16}{15}$ of \$$257.81\frac{1}{4}$ = \$275.
(9) It cost $\frac{80}{91\frac{2}{3}}$ of \$275 = \$240.
(10) Hence, the cotton was sold for \$240 ÷ .96 = \$250, and the whole gain is \$$268.12\frac{1}{2}$ — \$250 = \$$18.12\frac{1}{2}$.

PROBLEM 273.

Suppose 10% railroad stock is 20% better in market than 5% canal stock, how much money is invested in each if my income from each investment is \$600, and the whole investment pays 5%?

Solution.

(1) 10 ct. = income on \$1 of railroad stock; \$600 = total income on railroad stock.

(2) $600 ÷ .10 = $6000, face value of railroad stock.
(3) 5 ct. = income on $1 of canal stock; $600 = total income on canal stock.
(4) $600 ÷ .05 = $12000, face value of canal stock.
(5) If railroad stock is 20%, or $\frac{1}{5}$ better in the market than railroad stock, then the cost of $1 of railroad stock is $\frac{6}{5}$ of the cost of $1 of canal stock; and if there were just so many dollars in face value of railroad stock as there are in face value of canal stock, then its cost would be $\frac{6}{5}$ of the cost of canal stock.
(6) But a comparison of the two shows that there are only half as many dollars of railroad stock as there are of canal stock; hence, $\frac{6}{5}$ of $\frac{1}{2}$ of canal stock = $\frac{6}{10}$ of canal stock = cost of railroad stock expressed in terms of canal stock.
(7) $\frac{10}{10}$ = cost of canal stock, and $\frac{10}{10} + \frac{6}{10} = \frac{16}{10}$, or cost of both.
(8) $600, income on railroad stock + $600, income on canal stock = $1200, total income.
(9) Each $1 of investment yields an income of 5 ct.
(10) $1200 ÷ 5 ct., or $24000 = total cost, or investment.
(11) ∴ $\frac{16}{10}$ = $24000.
(12) $\frac{1}{10} = \frac{1}{16}$ of $24000 = $1500.
(13) $\frac{6}{10}$ = 6 times $1500 = $9000, cost of railroad stock.
(14) $\frac{10}{10}$ = 10 times $1500 = $15000, cost of canal stock.

PROBLEM 274.

If a man can make 5%, payable annually, on his money, what can he afford to pay for a 100-dollar 4% bond that is due in 2 years?

(*Putnam Co.*)

Solution.

(1) Amount of $1 in 2 yr. at 5%, payable annually, is $1.1025.
(2) The interest on the 100-dollar bond for 2 yr. at 4% is $8, and $108 is the amount of the bond.
(3) Hence, he can pay $108 ÷ 1.1025, or $97.969+ for the bond.

PROBLEM 275.

The difference between the true discount and the bank discount of a note due in 60 days, at 6%, is 24 cents: what is the face of the note?

(*Putnam Co.*)

Solution.

(1) True discount on $1 is $1 — ($1 ÷ 1.01) = $.00$\frac{100}{101}$.
(2) The bank discount on $1 for the time and rate is $.01.

(3) Now, \$ 01 — \$.00$\frac{100}{101}$ = \$\$.00$\frac{1}{101}$, the difference.
(4) \$24 ÷ \$.00$\frac{1}{101}$ = \$2424, the face of the note.

PROBLEM 276.

A dealer in stock bought 1270 head of stock, and $\frac{3}{4}$ of the number of horses is to $\frac{4}{5}$ of the number of cows as $\frac{5}{6}$ is to $\frac{7}{8}$: find the number of each. (*Putnam Co.*)

Solution.

(1) $\frac{3}{4}$ no. of horses : $\frac{4}{5}$ no. of cows :: $\frac{5}{6}$: $\frac{7}{8}$, or $\frac{21}{32}$ no. of horses = $\frac{20}{30}$ no. of cows.
(2) $\frac{1}{32}$ no. of horses = $\frac{1}{21}$ of $\frac{20}{30}$ = $\frac{20}{630}$ no. of cows.
(3) $\frac{32}{32}$, the number of horses = 32 × $\frac{20}{630}$ = $\frac{640}{630}$, no of cows. (Number of horses in terms of cows.)
(4) $\frac{630}{630}$ = no. of cows, or there are 64 horses to 63 cows.
(5) 64 + 63 = 127; $\frac{63}{127}$ of 1270 = 630 cows.
(6) $\frac{64}{127}$ of 1270 = 640 horses.

∴ He bought 630 cows and 640 horses.

PROBLEM 277.

If an article had cost me 20% more, my rate of gain would have been 25% less: find the rate of gain. (*Putnam Co.*)

Solution.

(1) The selling price divided by the cost gives the amount received for \$1 of cost.
(2) Since the selling price is the same in each case, the difference between the amounts is the difference in the gain %.
(3) Hence, $\frac{1}{100}$ — $\frac{1}{120}$ = $\frac{1}{600}$, the difference in the selling price.
(4) $\frac{1}{600}$ = $\frac{25}{100}$, or $\frac{1}{6}$ of selling price is 25%, and $\frac{6}{6}$ = 6 times 25% = 150%, the selling price.
(5) ∴ 150% — 100%, the cost = 50%, gain.

PROBLEM 278.

If my cost price had been 20% less, my rate of loss would have been 15% less: what was my rate of loss? (*Little Rock, Ark.*)

Solution.

(1) Solution is the same as the above; hence, $\frac{1}{80}$ — $\frac{1}{100}$ = $\frac{1}{400}$, the difference in selling price.
(2) $\frac{1}{400}$ = $\frac{15}{100}$, or $\frac{1}{4}$ of selling price is 15%, and $\frac{4}{4}$ = 4 times 15% = 60%, the selling price.
(3) 100%, the cost — 60%, the selling price = 40%, loss.

PROBLEM 279.

Bought rice at $6\frac{1}{2}$ cents a pound; waste by transportation and retailing was 5%; interest on first cost to time of sale was 2%: how much must be asked per pound to gain 20? (*N. Y. list.*)

Solution.

(1) Let 100 = number of pounds bought.
(2) $100 \times 6\frac{1}{2}$ ct. = \$6.50, cost of the rice.
(3) Interest at 2% = \$.13; \$6.50 + \$.13 = \$6.63, the whole cost.
(4) 125% of \$6.63 = $\$8.28\frac{3}{4}$, amount he must charge.
(5) 5% of 100 lb. = 5 lb., and 100 lb. — 5 lb. = 95 lb.
(6) ∴ $\$8.28\frac{3}{4} \div 95 = 8\frac{55}{76}$ cts.

∴ I must ask $8\frac{55}{76}$ ct. per pound.

PROBLEM 280.

When U. S. bonds are quoted in London at $108\frac{3}{4}$ and in Philadelphia at $112\frac{1}{4}$, exchange $\$4.89\frac{1}{2}$, gold quoted at 107, how much more is a \$1000 U. S. bond worth in London than in Philadelphia? (*Putnam Co.*)

Solution.

(1) $\$1000 \times 1.12\frac{1}{4} = \$1122.50$, price in Philadelphia.
(2) $\$1000 \times 1.08\frac{3}{4} = \$1087.50$, price in London.
(3) \$1 of London gold is worth \$1.07 of Philadelphia currency. ∴ $\$1087.50 \times 1.07 = \$1163.62\frac{1}{2}$, price of London bond in U. S. currency.
(4) $\$1163.62\frac{1}{2} - \$1122.50 = \$41.12\frac{1}{2}$, what the London bond cost an American more than the Philadelphia bond.
(5) To find the difference in cost to an Englishman in London, we proceed as follows:
(6) $\$1000 \times 1.12\frac{1}{4} = \$1122.50$.
(7) $\$1122.50 \div 1.07 = \$1049.06\frac{58}{107}$, price of the Philadelphia bond in English gold.
(8) $\$1000 \times 1.08\frac{3}{4} = \$1087.50$, and $\$1087.50 - \$1049.06\frac{58}{107} = \$38.43\frac{49}{107}$.
(9) $\$38.43\frac{49}{107} \div \$4.89\frac{1}{2}$ = £7 17s. .433d.

(*Prof. J. B. M. Zerr, Ph. D.*)

PROBLEM 281.

A Texan farmer owns 5169 cattle; there are 3 times as many horses as cows, plus 569, and 4 times as many cows as sheep, minus 126: how many has he of each? (*Brooks' Arith.*)

Solution.

(1) It is obvious that the number of cattle when there are 4 times as many cows as sheep and 3 times as many horses as cows = 5169 + 126 — 569 = 4726.

(2) Now, as often as he takes 1 sheep, he takes 4 cows and 12 horses, or 17 in all.

(3) ∴ 4726 ÷ 17 = 278, no. of lots of 1 sheep, 4 cows, 12 horses.

(4) ∴ 278 × 1 = 278, number of sheep.

(5) 278 × 4 — 126 = 986, cows.

(6) 278 × 12 + 569 = 3891, horses.

PROBLEM 282.

I owe A $100 due in 2 years, and $200 due in 4 years: when will the payment of $400 equitably discharge the debt, money being worth 3%?

Solution.

(1) $100 on interest for 2 yr. at 6% = $12.

(2) $200 on interest for 4 yr. at 6% = $48.

(3) $400 on interest for 1 yr. at 3% = $12.

(4) ($12 + $48) ÷ $12, the interest on $400 for 1 yr. at 3% = 5 years.

∴ 5 years is the required time.

PROBLEM 283.

Fritz owes James $200 due in 2 years, and James owes Fritz $100 due in 4 years: when can Fritz pay James $100 to settle the account equitably, money being worth 6%?

Solution.

(1) The present value of $200 due in 2 yr. is $200 ÷ 1.12 = $178.571.

(2) The present value of $100 due in 4 yr. is $100 ÷ 1.24 = $80.645.

(3) $178 571 — $80.645 = $97.926, amount due James now.

(4) $97.926 on interest at 6% yields $5.876 in a year,

(5) $100 — $97.926 = $2.074, the interest which must accumulate in order that it may equal $100.

(6) ∴ $2.074 will accumulate in $2.074 ÷ 5.876, or .3529 yr.

PROBLEM 284.

Mr. Merchant sells 20% above cost, with weights and measures 12½% "short," allows a discount of $5 on every bill of $50, and loses 5% of his sales as "bad debts." Find his rate % of net profit or net loss, 1 cent in every $1 of sales proving counterfeit, and collection charges being 2½%. (*Mathematical Monthly.*)

Solution.

(1) 100% — 12½% = 87½%, what he sells for 120%.

(2) 120% ÷ 87½% = 137½%, what he gets for 100%.

(3) \$5 on \$50 is 10%.
(4) $10\% + 5\% + 1\% + 2\frac{1}{2}\% = 18\frac{1}{2}\%$, what he loses.
(5) $137\frac{1}{2}\%$ of $18\frac{1}{2}\% = 25\frac{13}{35}\%$.
(6) $137\frac{1}{2}\% - 25\frac{13}{35}\% = 111\frac{27}{35}$.

$\therefore$ He gains $11\frac{27}{35}\%$. (*G. B. M. Zerr.*

PROBLEM 285.

Bought an article and sold it for 3% less than it cost me; bought it back, paying 3% more than I sold it for; I lost \$12 by the transaction: what did the article cost at first?

Solution.

(1) 100% = the cost of the article.
(2) 97% = the selling price.
(3) 3% of 97% = 2.91%.
(4) 3% + 2.91% = 5.91%, loss.
(5) $\therefore$ 5.91% = \$12.
(6) $1\% = \frac{1}{5.91}$ of \$12 = \$2.0305.
(7) 100% = 100 times \$2.0305 = \$203.05, cost of the article.

PROBLEM 286.

A sells goods which cost him \$160, at a certain rate of gain; B sells to C at the same rate of gain; C pays \$360 for the goods: find the rate of gain for A and B.

Solution.

(1) 100% = cost of goods.
(2) r = rate of gain.
(3) 100% + r = selling price.
(4) 100% + r = 100% + r times \$160 = \$160 + 160r, A's selling price and also B's cost.
(5) 100% = cost of goods by 2d condition.
(6) 100% + r = selling price.
(7) 100% + r = 100% + r times \$160 + 160r = $160r^2$ + 320r + \$160 = selling price, or C's cost.
(8) $160r^2 + 320r + \$160 = \$360$.
(9) $4r^2 + 8r + \$4 = \$9$.
(10) $r^2 + 2r = \$\frac{5}{4}$.
(11) r = \$.50, or 50%.

$\therefore$ A's and B's rate of gain is 50%.

PROBLEM 287.

A man left his property to three sons, to A $\frac{1}{3}$ wanting \$180, to B $\frac{1}{4}$, and to C the rest, which was \$590 less than A and B received: what was the estate? (*Teachers' Review.*)

Solution.

(1) 100% = the estate.
(2) $\frac{1}{3}$ of 100% = $33\frac{1}{3}$%, A's share + \$180.
(3) $33\frac{1}{3}$% — \$180 = A's share.
(4) $\frac{1}{4}$ of 100% = 25%, B's share.
(5) $33\frac{1}{3}$% — \$180 + 25%, or $58\frac{1}{3}$% — \$180 = A's and B's.
(6) 100% — ($58\frac{1}{3}$% — \$180) = $41\frac{2}{3}$% + \$180, C's money.
(7) ($58\frac{1}{3}$% — \$180) — ($41\frac{2}{3}$% + \$180) = $16\frac{2}{3}$% — \$360, or what A and B have more than C = \$590.
(8) $16\frac{2}{3}$% = \$950.
(9) 1% = \$950 ÷ $16\frac{2}{3}$ = \$57.
(10) 100% = 100 times \$57 = \$5700.

∴ The estate was worth \$5700.

PROBLEM 288.

My loss on the sale of one article is 25%, and my gain on another is 20%; the difference in the cost of the two articles is \$20, and my loss is equal to my gain: what was the price of each article?

Solution.

(1) 100% = cost of one article.
(2) 100% + \$20 = cost of the other.
(3) 25% of 100% = 25%, loss on one article.
(4) 20% of (100% + \$20) = 20% + \$4, gain on the other.
(5) 25% = 20% + \$4.
(6) 5% = \$4; 1% = \$.80.
(7) 100% = 100 times \$.80 = \$80, cost of one article.
(8) 100% + \$20 = \$100, cost of the other.

PROBLEM 289.

If 5 acres of grass, together with what grows on it during the time of grazing, keep 20 oxen 10 weeks, and 8 acres keep 29 oxen 16 weeks, how many oxen will 15 acres keep for 6 weeks?

Solution.

(1) Suppose each ox eats 100 lb. of grass in a week.
(2) Then, (20 × 10 × 100 lb.) ÷ 5 = 4000 lb., quantity of grass on 1 A. in 10 weeks.
(3) (29 × 16 × 100 lb.) ÷ 8 = 5800 lb., quantity of grass on 1 A. in 16 weeks.
(4) (5800 lb. — 4000 lb.) ÷ (16 — 10) = 300 lb., grown on 1 A. in 1 week.
(5) 300 lb. × 10 = 3000 lb., grown on 1 A. in 10 weeks.
(6) 4000 lb. — 3000 lb. = 1000 lb., original quantity on 1 A.

(7) 1000 lb. $\times$ 15 = 15000 lb., original quantity on 15 A.
(8) 300 lb. $\times$ 15 $\times$ 6 = 27000 lb., grown on 15 A. in 6 weeks.
(9) 15000 lb. + 27000 lb. = 42000 lb., amount to be eaten from 15 A. in 6 weeks.
(10) 100 lb. $\times$ 6 = 600 lb., what 1 ox eats in 6 weeks.
(11) 42000 $\div$ 600 = 70.

$\therefore$ 15 acres will keep 70 oxen 6 weeks.

PROBLEM 290.

A and B found a sum of money; A takes $1000 and $\frac{2}{13}$ of the remainder for his half: how much did they find?

Solution.

(1) Let 100% represent the sum.
(2) 100% — $1000 = the remainder.
(3) $\frac{2}{13}$ of (100% — $1000) = $\frac{200}{13}$% — $\$\frac{2000}{13}$.
(4) $1000 + $\frac{200}{13}$% — $\$\frac{2000}{13}$ = 50%.
(5) Then, $1300 + 200% — $2000 = 650%.
(6) 450% = $11000.
(7) 1% = $\frac{1}{450}$ of $11000 = 24.4444\frac{4}{9}$.
(8) 100% = 100 times 24.4444\frac{4}{9}$ = 2444.44\frac{4}{9}$.

$\therefore$ They found 2444.44\frac{4}{9}$.

PROBLEM 291.

A man bought a farm for $3448.10, and agreed to pay principal and interest in 4 equal annual installments: how much was the annual payment, interest being 5%? (*Putnam Co.*)

Solution.

(1) Evidently the payment exceeded the interest due, or the principal would not have been reduced.
(2) Now, for each $1 paid on the principal the 1st time, there was paid $1.05 the 2d time, $1.1025 the 3d time, and $1.157625 the 4th time.
(3) $\therefore$ $4.310125 was paid in the 4 payments as often as $1 was paid in the 1st.
(4) But $3448.10 was paid on the principal in the 4 payments.
(5) Then, the amount paid on the principal the 1st time is $3448.10 $\div$ $4.310125, or $800.
(6) The 1st interest is $3448.10 $\times$ 5%, or $172.405.
(7) $800 + $172.405 = $972.405, the 1st payment.

$\therefore$ The payments are each $972.405.

PROBLEM 292.

An interest bearing note dated August 1, 1892, was discounted at 90 days at 8%; the face of the note was \$750, and the proceeds \$759.982: what was the date of discount?

Solution.

(1) 90 da. + 3 da. of grace = 93 da.
(2) Interest on \$1 for 93 da. at 8% = \$.0206$\frac{2}{3}$.
(3) \$1 — \$.0206$\frac{2}{3}$ = \$.9793$\frac{1}{3}$, proceeds of \$1.
(4) \$759.982 ÷ .9793$\frac{1}{3}$ = \$776.0197, the amount of the note at the end of the 90 da.
(5) \$776.0197 — \$750 = \$26.0197, interest.
(6) Interest on \$750 for 1 yr. at 6% = \$45.
(7) \$26.0197 ÷ \$45 = the time, 6 mo. 28 da.
(8) 6 mo. 28 da. — 90 da. = 3 mo. 28 da.
(9) Aug. 1, 1892 + 3 mo. 28 da. = Nov. 29, 1892.

∴ The note was discounted Nov. 29, 1892.

NOTE.—Solved by Prof. G. B. M. Zerr for the *A. M. Monthly.*

PROBLEM 293.

A company of fifty persons engage dinner at a hotel, but before paying the bill 10 of the persons withdrew, by which each person's bill was increased 25 ct.: what was the bill?

Solution.

(1) If the expense of one is increased \$.25, the expense of 50 is 50 times 25 ct., or \$12.50, which the 10 men should have paid.
(2) One man should have paid $\frac{1}{10}$ of \$12.50, or \$1.25.
(3) 50 — 10 or 40 times \$1.25 = \$50.

∴ The bill was \$50.

PROBLEM 294.

Sixteen men hire a coach for a certain sum, but by taking in 4 more persons the expense of each is diminished \$$\frac{1}{8}$: what did they pay for the coach?

Solution.

(1) If the expense of one is diminished \$$\frac{1}{8}$, the expense of 16 is diminished 16 times \$$\frac{1}{8}$, which is \$2, which the 4 men pay.
(2) If 4 men pay \$2, one man pays $\frac{1}{4}$ of \$2, or \$$\frac{1}{2}$.
(3) 16 + 4, or 20 men, pay 20 times \$$\frac{1}{2}$, which is \$10.

∴ They pay \$10 for the coach.

BOAT PROBLEMS.

PROBLEM 295.

A vessel sails 15 miles an hour in still water; going up stream it requires 72 minutes to sail 15 miles: how long will it require to go down stream 15 miles?

Solution.

(1) Rate up stream is $15 \div \frac{6}{5}$ hr. $= 12\frac{1}{2}$ mi. an hr.

(2) The rate of current is 15 mi. — $12\frac{1}{2}$ mi. $= 2\frac{1}{2}$ mi. per hr.

(3) Then, the rate down stream is 15 mi. $+ 2\frac{1}{2}$ mi., or $17\frac{1}{2}$ mi. an hour, and it will take $15 \div 17\frac{1}{2} \times 60 = 51\frac{3}{7}$ min.

∴ The vessel goes 15 mi. down stream in $51\frac{3}{7}$ min.

PROBLEM 296.

I can row up stream in 64 minutes and back again in 60 minutes: determine the distance, the rate of current being $\frac{1}{2}$ mile per hour.

Solution.

(1) In 1 minute I can row up stream $\frac{1}{64}$ of the distance, and $\frac{1}{60}$ of the distance back again in the same time.

(2) Hence, twice the distance the current moves in a minute is $\frac{1}{60} - \frac{1}{64} = \frac{1}{960}$ of the distance.

(3) It is true that the rate of the current will be half the difference of the rates of sailing down and up; that is, $\frac{1}{2}$ of $\frac{1}{960}$, or $\frac{1}{1920}$ of the distance the current moves in 1 minute.

(4) The current in 1 hour moves $\frac{1}{1920} \times 60 = \frac{60}{1920}$, or $\frac{1}{32}$ of the distance, or $\frac{1}{2}$ a mile.

(5) $\frac{32}{32}$, the distance $= 32 \times \frac{1}{2}$ of a mile, or 16 miles.

∴ The distance is 16 miles.

PROBLEM 297.

If a boat goes down stream 42 miles and back in 20 hours, going 14 miles down in the same time that it goes 6 miles up, what is the rate in still water, and what is the rate of current? (*The Teachers' Review.*)

Solution.

(1) The time down stream is to the time up stream as $\frac{1}{14}$ is to $\frac{1}{6}$, or as 6 to 14.

(2) Hence, it takes $\frac{3}{10}$ of 20 hr., or 6 hr., to go down and $\frac{7}{10}$ of 20 hr., or 14 hr., to come up.

(3) The rate down stream is $42 \div 6 = 7$ mi. an hr., and the rate up, $42 \div 14 = 3$ mi. an hour.

(4) The rate of current must be (7 — 3) ÷ 2 = 2 mi. an hr., and the rate of sailing in still water is 7 — 2 = 5 mi. per hr.

PROBLEM 298.

Frank would row a boat 7 miles with the tide in the same time he would row 5 miles against it; but if the current ran half a mile an hour more, he would row twice as rapidly with the tide as against it: find his rate in miles per hour in still water.

Solution.

(1) 7 mi. = rate per hr. with the tide.
(2) 5 mi. = rate of rowing against the tide.
(3) (7 mi. — 5 mi.) ÷ 2 = 1 mi., rate of tide.
(4) 7 mi. — 1 mi. = 6 mi., the rate of rowing.
(5) 1 mi. ÷ 6 mi. = $\frac{1}{6}$; ∴ the rate of the tide is $\frac{1}{6}$ of the rowing rate.
(6) Then, by the second condition he rows 2 mi. with the tide and 1 mi. against it.
(7) (2 mi. — 1 mi.) ÷ 2 = $\frac{1}{2}$ mi., rate of tide.
(8) 2 mi. — $\frac{1}{2}$ mi. = $1\frac{1}{2}$ mi., rate of rowing.
(9) $\frac{1}{2}$ mi. ÷ $1\frac{1}{2}$ mi. = $\frac{1}{3}$; ∴ the rate of tide is $\frac{1}{3}$ of the rowing rate.
(10) Hence, the additional $\frac{1}{2}$ mile per hr. is $\frac{1}{3}$ — $\frac{1}{6}$, or $\frac{1}{6}$ of rowing rate.
(11) ∴ $\frac{1}{6}$ = $\frac{1}{2}$ mi.; $\frac{6}{6}$, the rowing rate = 6 times $\frac{1}{2}$ = 3 mi.

∴ The rate of rowing is 3 mi. per hr.

CHAPTER XII.

MENSURATION.

79. Mensuration is that branch of applied geometry which gives the rules for finding the lengths of lines, the areas of surfaces, and the volumes of solids.

80. A position in space that is without magnitude is called a **Point.**

81. A **Line** is the path of a point in motion. Lines are represented upon paper or blackboard by marks made by pen or crayon. The point of the pen or crayon leaves a visible mark. This mark has breadth and occupies some of the surface upon which it is drawn, and is called a *physical line.* If we continually diminish the breadth of the physical line we make it approximate to the geometric line.

82. A **Straight Line** is a line traced by a point which does not change its direction of motion.

83. A **Curved Line** is the path of a point which constantly changes its direction of motion.

84. An **Angle** is the opening between two lines which meet each other. The point in which the lines meet is called the **Vertex**, and the lines are called **Sides**.

85. A **Right Angle** (a square corner) is formed by one line standing on another so as not to incline on either side.

86. An **Acute Angle** (a sharp corner) is an angle which is less than a right angle.

87. An **Obtuse Angle** (a blunt corner) is an angle greater than a right angle.

88. A **Triangle** is the figure formed by three lines and the determined points, or by three points and the determined lines.

89. A **Scalene Triangle** is one which has no two of its sides equal.

90. An **Isosceles Triangle** is one which has two of its sides equal.

91. An **Equilateral Triangle** is one whose sides are all equal.

92. A **Right Triangle** is one which has a right angle. The side opposite the right angle is called the **Hypothenuse.**

I. TRIANGLES.

PROBLEM 299.

Prove that the square described on the hypothenuse of a right-angled triangle is equivalent to the sum of the other two squares described on the other two sides.

Solution.

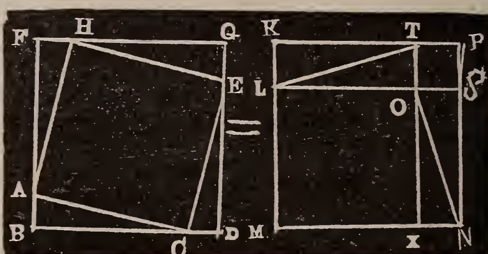

FIG. 3.

Let ABC be a right triangle. Produce BC to D, making CD = AB, and complete the square BDGF. Now make FH and GE = AB and complete ACEH.

The triangles ABC, CDE, EGH and HFA are equal, and ACEH is the square on AC, or on the hypothenuse. On MN = BD construct the square MNPK. Arrange the four triangles of the first square as drawn in the second, OP is the square on AB, and OM on BC. From each of the two squares take the four equal triangles, and there remains the square ACEH, equivalent to the sum of the squares OP and OM. Q. E. D.

Note.—The author gives this demonstration in class work, for it seems the simplest.

The following is Garfield's demonstration of the right triangle:

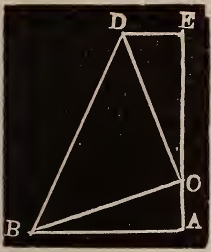

FIG. 4.

Let ABC be the given right-angled triangle. Produce AC to E, making CE = AB. At E draw ED perpendicular to CE and make it equal to AC. Draw DC and BD; then CD = CB and $\triangle$ BCD = $\frac{1}{2}\overline{BC}^2$. The area of ABDE is equal to $\frac{1}{2}$AE(AB+ED) or $\frac{1}{2}(AB + AC)^2$; it is also equal to $\frac{1}{2}(BC \times CD)$ + twice the $\triangle$ ABC or $\frac{1}{2}\overline{BC}^2 + (AB \times AC)$.

Therefore, $\frac{1}{2}(AB + AC)^2 = \frac{1}{2}\overline{BC}^2 + (AB \times AC)$; whence,

$(AB + AC)^2 = \frac{1}{2}\overline{BC}^2 + 2(AB \times AC)$, or $\overline{AB}^2 + \overline{AC}^2 = \overline{BC}^2$.

Q. E. D.

PROBLEM 300.

Given the right-angled triangle ABC, the base AC = 40 and the altitude BC = 9: what is the hypothenuse?

Solution.

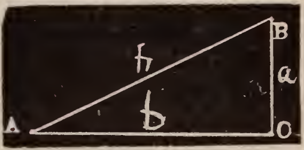

FIG. 5.

(1) AC = 40, the base.
(2) BC = 9, the altitude.
(3) $40^2 = 1600$, the square of the base, AC.
(4) $9^2 = 81$, the square of the altitude, BC.
(5) $1600 + 81 = 1681$, $\overline{AC}^2 + \overline{BC}^2$.
(6) $\sqrt{1681} = 41$, AB. $\therefore 41 =$ the hypothenuse.

PROBLEM 301.

The hypothenuse of a right-angled triangle is 97, and the altitude 65: what is the base?

Solution.

(1) AB = 97, the hypothenuse.
(2) BC = 65, the altitude.
(3) $97^2 = 9409$, the square of the hypothenuse AB.
(4) $65^2 = 4225$, the square of the altitude BC.
(5) $9409 - 4225 = 5184$, the difference of the squares of the hypothenuse and altitude.
(6) $\sqrt{5184} = 72 =$ AC.

$\therefore 72 =$ the base.

PROBLEM 302.

What is the area of a right triangle whose base is 24 feet and altitude 7 feet?

Solution.

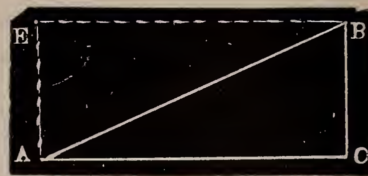

FIG. 6.

(1) We can see at once that ACB = AEB.

(2) AC = 24 ft., the base.

(3) BC = 7 ft., the altitude.

(4) $24 \times 7 = 168$ sq. ft., the product of the base and altitude.

(5) $\frac{1}{2}$ of 168 sq. ft. = 84 sq. ft., half the product of the base and altitude.

∴ 84 sq. ft. = the area of ABC.

PROBLEM 303.

What length of rope will be required to draw water from a well, it being 38 feet to the water, the sweep to be supported by an upright post 20 feet high, and standing 20 feet from the well, and the foot of the sweep to strike the ground 20 feet from the foot of the upright post?

Solution.

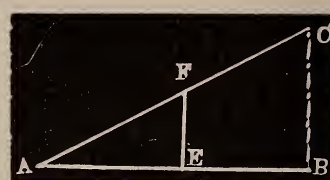

FIG. 7.

(1) AE = 20 ft.; FE, the post = 20 ft.; BE = 20 ft.; and CB will be the required length of the rope.

(2) By the similar triangles ABC and AEF, we have AE : EF :: AB : CB, or 20 ft. : 20 ft. :: 40 ft. : (CB = 40 ft.)

NOTE.—This solution was prepared by the author for the *American Mathematical Monthly*.

PROBLEM 304.

The hypothenuse of a right-angled triangle is 97 ft., and the perpendicular is 65 ft.: required the base, without squaring either of the given numbers.

Solution.

$[\sqrt{(97 + 65)(97 - 65)}] = 72$ ft., the base.

PROBLEM 305.

How long a ladder will be required to reach a window 40 feet from the ground, if the distance of the foot of the ladder from the wall is one-fifth of the length of the ladder?

Solution.

(1) From Fig. 5, AB, or $5x$, represents the length of the ladder, and AC, or x, the distance the foot of the ladder is from the wall.

(2) CB = 40 ft.

(3) Then we have $25x^2 + x^2 = 40^2$.
(4) Whence, $5x = 40.824829$ ft., the required length.

Note.—This solution was prepared by the author for the *School Visitor*.

PROBLEM 306.

A right-angled triangle, altitude 7 ft., and hypothenuse 25 ft., has the same area as a rectangle whose sides are as 7 to 3: find dimensions of the rectangle.

Solution.

(1) The base of the triangle is $\sqrt{(25^2 - 7^2)} = 24$ ft.
(2) Its area is $7 \times 24 \div 2 = 84$ sq. ft.
(3) Assuming a rectangle 7 ft. long and 3 ft. wide which is similar to the given rectangle, we find its area to be 21 sq. ft.
(4) The areas of similar figures are to each other as the squares of their homologous sides; then 21 sq. ft. : 84 sq. ft :: 7^2 : (the required length = 14 ft.)
(5) 21 sq. ft. : 84 sq. ft. :: 3^2 : (the required width = 6 ft.)

∴ The dimensions of the rectangle are 14 ft. and 6 ft.

PROBLEM 307.

Two trees, the first of which is 60, and the second 100 feet high, are 300 feet apart: what is the length of a string which, from a point between the trees, will just reach the top of each?

Solution.

(1) AD = 60 ft., the height of the tree AD, and BC = 100 ft., the height of the tree BC.
(2) Connect the tops of the trees by the line DC, and from the middle point T of DC let fall TF perpendicular to AB. Draw TE perpendicular to DC.

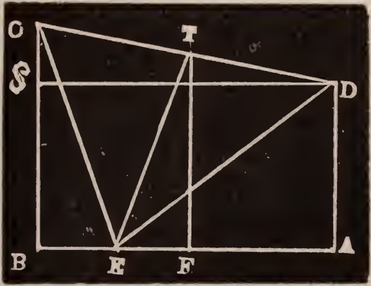

FIG. 8.

(3) CS = 100 — 60, or 40 ft.
(4) $TF = \frac{1}{2}(BC + AD) = 80$ ft.
(5) By similar triangles, DS : SC :: TF : FE, or 300 ft. : 40 ft. :: 80 ft. : (FE = $10\frac{2}{3}$ ft.)
(6) EB = FB — FE = 150 — $10\frac{2}{3}$ = $139\frac{1}{3}$ ft.
(7) $EC = \sqrt{(\overline{EB}^2 + \overline{BC}^2)} = 171.50+$ ft.

∴ CE or DE = 171.50 ft.

PROBLEM 308.

A ladder 61 ft. long being placed 7 ft. from the center of a street will just reach the top of a tower on one side and 6 ft. higher on the other side: find width of street.

Solution.

(1) From Fig. 8, AD $= x - 3$ and BC $= x + 3$.
(2) AB $= 2y$, the width of street.
(3) Then, from the two right triangles AED and EBC, $(y+7)^2 + (y-3)^2 = 61^2$, and $(y-7)^2 + (x+3)^2 = 61^2$.
(4) Expanding and subtracting one from the other, $28y = 12x$.
(5) $\therefore x = \frac{7}{3}y$.
(6) Subtracting in either equation $\frac{58}{9}y^2 = 3663$.
(7) $y = 23.841$ ft., $2y = 47.682$ ft., and AD, or $x - 3 = 52.62$ ft., BC, or $x + 3 = 58.62$ ft.

PROBLEM 309.

There are two pillars, one 110 ft. high and the other 120 ft. A ladder placed between the pillars will reach the top of either without moving it at the base, and it is further found that the ladder will reach from the top of one pillar to the top of the other: determine the length of ladder and distance between the poles.

Solution.

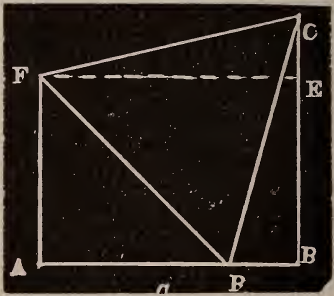

FIG. 9.

(1) Let AB = width of street.
(2) AF = 110 ft., BC = 120 ft., and CE = 10 ft.
(3) Let x = FP = PC = CF, the length of the ladder.
(4) FE = AB = $\sqrt{(x^2 - 10^2)}$, AP = $\sqrt{(x^2 - 110^2)}$, and PB = $\sqrt{(x^2 - 120^2)}$.
(5) $\sqrt{(x^2 - 10^2)} = \sqrt{(x^2 - 110^2)} + \sqrt{(x^2 - 120^2)}$.
(6) Squaring and reducing, $x^2 = \frac{53320}{3}$.
(7) $x = 133.1665$ ft.; BP $= \sqrt{(x^2 - 120^2)} = 57.735$ ft. PA $= \sqrt{(x^2 - 110^2)} = 57.735$ ft.

$\therefore x = 133.1665$ ft., PA $= 75.0555$ ft., and PB $= 57.735$ ft.

PROBLEM 310.

There are two buildings, the heights of which are 40 and 29 feet, and a ladder is placed against the higher building in such a manner that a line 46 feet long, reaching from the top of the lower to the middle of the ladder, is the least that will reach it from there to any part of it: required, the length of ladder, and the distance its foot is from each building. (*Whitney.*)

Solution.

(1) AF = 29 ft., height of the lower building.

(2) BD = 40 ft., height of the higher building.

(3) BE = x, the distance between the foot of the ladder and the higher building, y = the distance AE, and $2z$ = ED, the length of the ladder; FS = 46 ft.

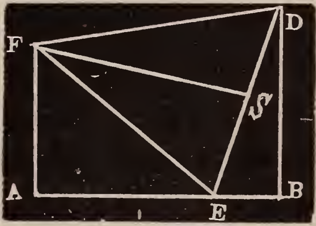

FIG 10.

(4) Draw FD and FE; then EDF is an isosceles triangle.

(5) Then we have $x^2 + 1600 = 4z^2$ . . . . (1).

(6) $(x+y)^2 + (40-29)^2 = y^2 + 29^2$ . . . (2).

(7) $y^2 + 29^2 = z^2 + 46^2$ . . . . (3).

(8) Reducing (2), $x^2 + 2xy = 720$ . . . . . (4).

(9) From (3), $z^2 = y^2 - 1275$ . . . . (5).

(10) Substituting the value of z in (1), $2y = \sqrt{(x^2 - 6700)}$.

(11) Substituting this value of $2y$ in (4), transposing, squaring and reducing, $407x = 25920$; also $x = 7.978$ ft., $y =$ 41.12 ft., and $2z = 40.788$ ft.

PROBLEM 311.

A tree 125 ft. high stands on the bank of a river 105 ft. wide; where must the tree break off so that it may remain connected at the point of breaking and its top just reach the opposite shore?

First Solution.

(1) Let AB = the height or sum of the altitude and hypothenuse.

(2) AG = the hypothenuse, and GB = the altitude.

(3) Complete the square ABFL; its area will be 125^2 ft., or 15625 sq. ft.

(4) Let BC = GD; draw GK parallel to BF and NC parallel to AB.

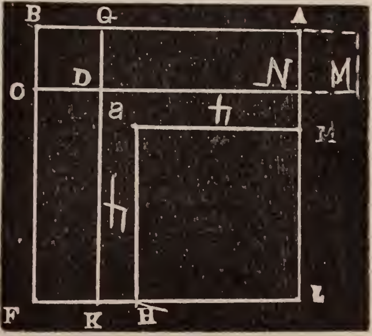

FIG. 11.

(5) GC = the square of the altitude, and DL = the square of the hypothenuse.

(6) Take LH equal to the base of the triangle formed by the broken tree, and complete the square SL.

(7) Then, SL is the square of the base, which we take from the square of DL; the difference is the gnomon.

(8) hh = the square of the altitude GC.

(9) But if from ABFL, or 15625 sq. ft., we take SL, or 11025 sq. ft., the remainder is GC + DF and AD + AM (AM = hh) = 4600 sq. ft.

(10) Then we find $\frac{1}{2}$ of 4600 sq. ft., or 2300 sq. ft. = GC + CK or AD + AM.

(11) But BC + CF = 125 ft.

(12) Then 2300 sq. ft. ÷ 125 = 18$\frac{2}{5}$ ft., the altitude, or BC.

∴ The tree must break off 18$\frac{2}{5}$ ft. from the ground.

NOTE.—This solution was prepared by the author for the *School Visitor.*

Second Solution.

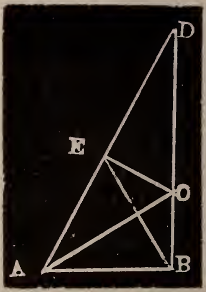

FIG. 12.

(1) Let AB = b, the width of the stream, and draw BD perpendicular to it = the height of the tree.

(2) Join AD, and bisect it in E by the perpendicular EC; join AC.

(3) It is evident that AC = CD and AC + CB = BD = a, or the sum of the hypothenuse and perpendicular.

(4) If x = the perpendicular, $a - x$ will be the hypothenuse AC.

(5) $(a - x)^2 = b^2 + x^2$, or $a^2 + 2ax + x^2 = b^2 - x^2$.

(6) Whence $x = \dfrac{a^2 - b^2}{2a}$. Now substitute the values of a^2, b^2 and a, and $x = 18\frac{2}{5}$ ft.

Rule.—*From the square of the height subtract the square of the base, and divide the difference by twice the height.*

PROBLEM 312.

Find the area of a circle inscribed in a right-angled triangle whose legs measure 15 and 20 ft., respectively.

Solution.

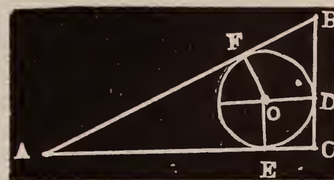

FIG. 13.

(1) Let AC = 20 ft., the base of the triangle ABC, and BC = 15 ft., the altitude.

(2) AB, the hypothenuse = $\sqrt{(\overline{AC}^2 + \overline{BC}^2)}$, or 25 ft.

(3) FDE = the inscribed circle; OD, OF, OE = the radii of the inscribed circle.

(4) $\frac{1}{2}$(AC × BC) = 150 sq. ft., the area of the △ACB.

(5) The radius of an inscribed circle is found by dividing twice the area of the triangle by its perimeter, or (150 × 2) ÷ 60 = 5 ft., the radius OE or OD.

(6) Hence, $5^2 \times \pi = 78.53$ sq. ft.

∴ 78.53 sq. ft. is the required area.

PROBLEM 313.

A ladder stands against the side of a house; if it is pulled out at the bottom 16 ft., it will not reach the top by 4 ft., and it is long enough to reach the top: find the length of the ladder.

Solution.

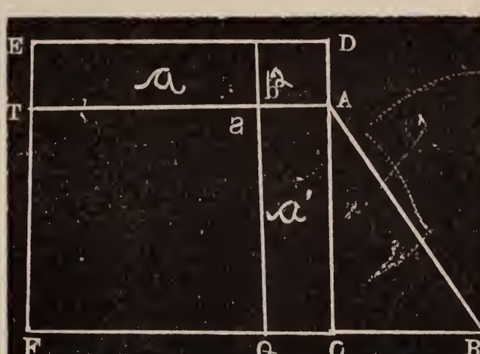

FIG. 14.

(1) Let DC = the height of the house, and AB = the ladder drawn out 16 ft. on CB.

(2) DC = AB.

(3) Construct the square DF upon DC = the square on AB.

(4) The square SF = the square on AC, the perpendicular of the △ CBA.

(5) Let b = AD, or 4 ft. = the distance the ladder slipped down when it was pulled out 16 ft.

(6) aa' = the area of the rectangles ES and SC, and b^2 = the area of the square DS.

(7) ∴ $\overline{FC}^2 - \overline{FG}^2 = aa'b^2$, or $16^2 = 256$ sq. ft.

(8) $aa'b^2 - b^2 = 240$ sq. ft., the area of the two rectangles ES and SC.

(9) ∴ The area of SC $= \frac{1}{2}$ of 240 sq. ft. $= 120$ sq. ft.

(10) SA = AD, or 4 ft.; then AC $= \frac{1}{4}$ of 120 sq. ft., or 30 ft.

(11) AC + AD = 34 ft. = DC, or AB.

∴ 34 ft. is the required length of the ladder.

PROBLEM 314.

Two lines are drawn from the acute angles of a right triangle to the middle points of the opposite sides, and measure respectively, $\sqrt{73}$ and $\sqrt{52}$: find the base and perpendicular.

Solution.

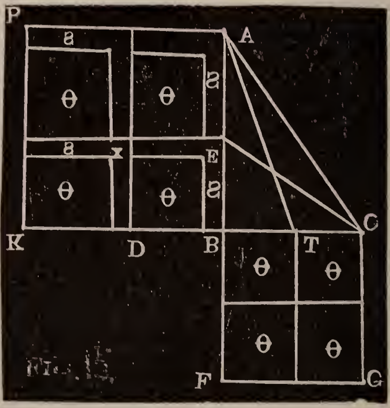

FIG. 15.

(1) Let ABC = the triangle.

(2) EC $= \sqrt{52}$ and AT $= \sqrt{73}$.

(3) Let θ represent the square on the half of BC and ED half the square on AB.

(4) Also the squares AX, PX and XK are each equal to ED.

(5) Then, if we subtract θ from either of the squares ED or XK we have the gnomon S.

(6) As EBC is a right triangle, $5\theta + S = 52$ (1).

(7) As ABT is also a right triangle, $5\theta + 4S = 73$ (2).

(8) Subtract (1) from (2) and solve for S, we find $S = 7$.

(9) Substitute the value of S in (1) we find that $\theta = 9$.

(10) ∴ BT $= \sqrt{9} = 3$, or BC $= 6$. EB $= \sqrt{\theta + S} = 4$, or AB $= 8$. AC $= \sqrt{BC^2 + AB^2} = 10$.

∴ ABC is a 6, 8 and 10 triangle.

PROBLEM 315.

There are two ladders standing in the middle of a street; one is 80 ft., and the other 90 ft., both lean against a building and the 90 ft. ladder reaches the top 20 ft. higher than the 80 ft. ladder: find height of the building.

Solution.

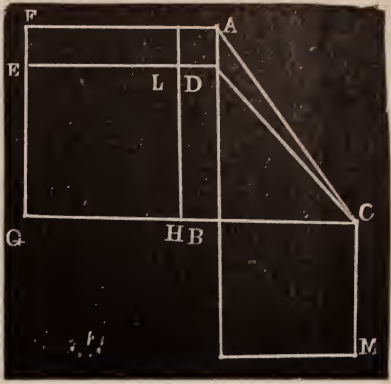

FIG. 16.

(1) Let AC and DC be the position of the ladders.

(2) $\overline{BD}^2$ = the square LG, and $\overline{BC}^2$ = the square BM.

(3) Subtract the square BM from the square AG = the two rectangles FL and LD plus the square AL = the square BM, = $\overline{AC}^2 - \overline{DC}^2$, or 1700 sq. ft.

(4) $\overline{AD}^2$ = 400 sq. ft., the square AL.

(5) 1700 sq. ft. — 400 sq. ft. = 1300 sq. ft., area of the two rectangles FL and LD.

(6) LB or FL = 1300 sq. ft $\div$ 2, or 650 sq. ft.; as LD or AD = 20 ft., then DB = 650 sq. ft. $\div$ 20, or 3.25 ft.

(7) AB = AD + DB, or 20 ft. + 32.5 ft. = 52.5 ft.

$\therefore$ 52.5 ft. = the height of the building.

PROBLEM 316.

A ladder 65 ft. long is standing against a perpendicular wall; if the base were drawn out 52 ft., what distance would the top slide down the wall?

Solution.

(1) From Fig. 14, DC = AB = 65 ft.

(2) DA = distance the top slipped down the wall, and CB = 52 ft.

(3) $\overline{CB}^2 = aa'b^2$.

(4) $\overline{DC}^2 - \overline{CB}^2$, or $aa'b^2 = \overline{SF}^2$ = 1521 sq. ft.

(5) Then, SG or AC = $\sqrt{1521}$ sq. ft. = 39 ft.

(6) DA = DC — AD = 26 ft.

$\therefore$ 26 ft. is the required distance.

NOTE.—Such problems as the above and also Fig. 11 have bothered many an applicant at county examinations. But you see they are easily solved.

PROBLEM 317.

Inscribe the greatest possible rectangle in the right triangle whose sides are 100, 80, and 60 ft.

Solution.

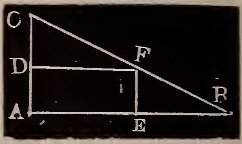

FIG. 17.

(1) Let $x =$ the length of the rectangle, and b the base, or 80 ft.

(2) CA $= a$, AB $= b$, and FE $= y$; then EB $= b - x$, $xy =$ area of the rectangle $=$ A maximum.

(3) Differentiating, $xdy + ydx = 0$ (1).

(4) The triangles FEB and ABC are similar; hence $a : b :: y : b - x$; or $ab - ax = by$ (2)

(5) Differentiating, $-adx = bdy$, whence $dy = (-adx) \div b$.

(6) Substituting this value in (1), $-axdx \div b + ydx = 0$.

(7) Clearing, $-axdx + bydx = 0$, and $-ax + by = 0$.

(8) $by = ax$ . . . (3).

(9) Then from (2) $by = ab - ax$.

(10) $\therefore ab - ax = ax$, $ab = 2ax$, $b = 2x$, or $x = b \div 2$ and $y = a \div 2$.

(11) Now as $b = 80$ ft., and $a = 60$ ft., by the above reasoning, $x = 40$ ft., and $y = 30$ ft.

$\therefore$ 40 ft. is the length, and 30 ft. the width of the maximum rectangle.

NOTE.—The sides of a maximum rectangle inscribed in a right triangle are respectively halves of the given legs.

PROBLEM 318.

A pole 495 ft. high, standing on the margin of a pond which is 408 ft. wide, was broken by the wind, the broken piece remaining attached to the stump, while its extremity rested on a post in the pond, the post being 45 ft. above the surface of the lake; and while in that position the broken piece pointed exactly to the opposite edge of the pond: required the height of the stump.

Solution.

(1) From Fig. 17, AB $= 408$ ft., the width of the pond; FE $= 45$ ft., the height of the post.

(2) $x =$ BE, $y =$ CD. AE $= 408 - x$, CF, or the broken part, is $450 - y$; then $x : 45 :: 408 - x : y$; then $xy + 45x = 18360$ . . . (1).

(3) Also, $(408 - x)^2 + y^2 = (450 - y)^2$, whence, $x^2 - 816x + 900y = 36036$ . . . (2).

(4) Also, $45 \div x = (408 - x) \div y$; then $x^2 - 408x = 45y$ (3).

(5) From these three equations we find $x = 108$ ft., and $y = 125$ ft., and AC $+ 125$ ft. $+ 45$ ft. $= 170$ ft.

$\therefore$ 170 ft. is the required height of stump.

PROBLEM 319.

The hypothenuse of a right-angled triangle is 35, the side of the inscribed square 12: what are the sides of the triangle? (*The Voice.*)

Solution.

(1) From Fig. 17, CB = 35, DF and FE = 12 ft. each, EB = x, AB = $x + 12$ and CD = y.

(2) Then CA = $y + 12$.

(3) Whence $(x + 12)^2 + (y + 12)^2 = 35^2$, or $x^2 + y^2 + 24(x + y) = 937$ . . . (1).

(4) From the similar triangles CDF and FEB, CD : DF :: FE : EB, or $y : 12 :: 12 : x$, from which $xy = 144$ (2).

(5) Multiply (2) by 2, or by doubling (2) and adding (1), we then have $y^2 + 2xy + x^2 + 24(x + y) = 1225$, or $(x + y)^2 + 24(x + y) = 1225$.

(6) Completing the square, $x + y = 25$ . . . (3).

(7) $x = \frac{144}{y}$ from (2), and substitute its value in (3), whence, $y^2 - 25y = -144$.

(8) $y - 12\frac{1}{2} = \pm 3\frac{1}{2}$, or $y = 16$ or 9, and $x = 9$ or 16.

∴ The base is 21 ft., or 28 ft., and the perpendicular is 28 ft., or 21 ft.

PROBLEM 320.

The perimeter of a right triangle is 24; the hypothenuse is 2 longer than the altitude, and the altitude 2 longer than the base: what are its sides?

Solution.

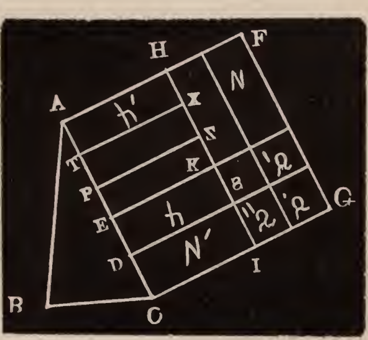

FIG. 18.

(1) Let ABC be the triangle, and AG the square of the hypothenuse, AS the square of the altitude, and AK the square of the base.

(2) The rectangles nn and AX, etc., are equal.

(3) The two rectangles nn' and the three squares $aa'a''$ = the square on the base.

(4) The square of the base $- nn' = aa'a'' = h$, or any of the rectangles, or 12; then the length of the rectangles = the length of the base.

(5) Also the area of one of the rectangles h' is 12, and one side is four less than the other side.

(6) Then by Fig. 19, let WZ be a straight line equal to the difference of the sides of the rectangle h'.

(7) Upon WZ as a diameter describe a circle, and at the extremity of the diameter draw the tangent WF equal to the side of a square having the same area as the rectangle h'.

(8) Through the point F and the center O, draw the secant FQ; then will FY, FQ be the adjacent sides of the rectangle.

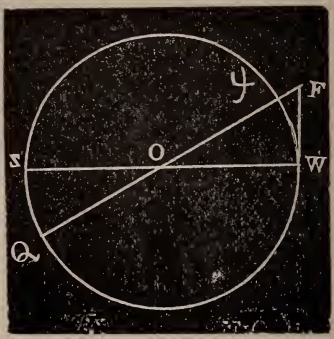

FIG. 19.

(9) $WF = \sqrt{12} = WZ = 4$, and $WO = 2$.

(10) $FO = \sqrt{(FW^2 + WO^2)} = 4$.

(11) $FQ = FO + OQ = 6$, the base of the right triangle.

(12) $AF = BC$, or 6; FD and DC each $= 2$.

(13) $\therefore AC = AF + FD + BC = 10$.

(14) $AB = AF + FD$, or 8.

$\therefore$ ABC is a 6, 8 and 10 triangle.

PROBLEM 321.

A tree is standing one hundred feet from the bank of a run; a squirrel at the height of 60 ft. runs down and runs to the water's edge; at the same time a bird flies up and then down to the squirrel at the water's edge, and the bird flew as far as the squirrel ran: find the height that the bird flew.

Solution.

(1) Let E represent the point the bird is situated when it flies upward and the squirrel runs downward.

(2) Then x represents the height the bird flew, or CE.

(3) $BC = y$, the hypothenuse of the right triangle ABC, whose base AB is 100 ft., and the perpendicular $AC = x + 60$ ft.

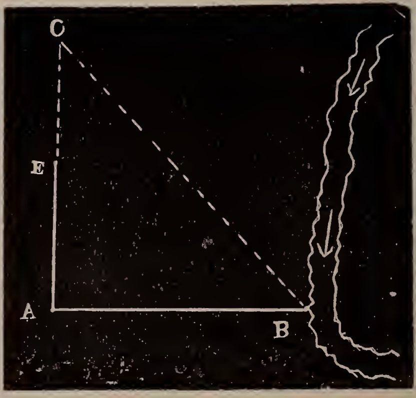

FIG. 20.

(4) Then $(x + 60)^2 +$

$100^2 = y^2$ . . . (1).

(5) $x + y = 160$ . . . (2).

(6) Find the value of y in (2), square, and substitute its value in (1), and solve for x.

(7) Hence, $x = 27\frac{3}{11}$ ft.

$\therefore 27\frac{3}{11}$ ft. = height that the bird flew.

PROBLEM 322.

Two poles, perpendicular to the same plane, are 30 ft. and 50 ft. in height: at what height from the plane will lines drawn from the top of each to the base of the other, cross?

Solution.

(1) Let AC = 50 = a; DB = 30 = b; BC = c, FE = x; EB = y; CE = $c—y$.

(2) Then by similar triangles, $a : c :: x : y$; therefore, $ay = cx$ . . . (1).

(3) Also, $b : c :: x : c—x$; therefore, $bc—by=cx$ . . (2).

(4) Now from (1) and (2), $ay+by=bc$, and $y=bc\div(a+b)$.

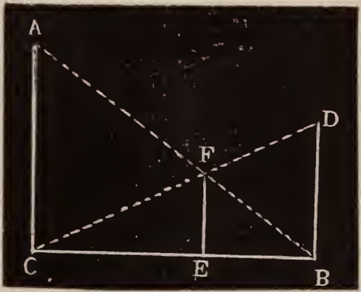

FIG. 21.

(5) By substituting the value of y in the first proportion, we have $a : c :: x : bc\div(a+b)$.

(6) $cx=abc\div(a+b)$, and $x=ab\div(a+b)$, or $\frac{ab}{a+b}$.

(7) Hence, $(50\times30)\div(50+30)=18\frac{3}{4}$ ft.

$\therefore 18\frac{3}{4}$ ft. is the required height, or FE.

Rule.—*Divide the product of their heights by their sum.*

NOTE.—It would be well to bear in mind that the height at which the lines cross is not affected by the distance the poles are apart.

PROBLEM 323.

Two trees are a certain distance apart; from the top of a to the foot of b is 60 ft.; from the top of b to the foot of a is 40 ft.; from the point where these two lines intersect to the plane is 15 ft.: what is the height of each tree and the distance apart? (*American Mathematical Monthly.*)

Solution.

(1) From Fig. 21, let DB $=x-y$; AC $=x+y$; FE $=c=15$; CD $=b=40$; AB $=a=60$; BF $=z$; and CE $=w$.

(2) By similar triangles, $w:c::w+z:x-y$, or $w(x-y)=c(w+z)$ . . . (1).

(3) $z:c::w+z:x+y$, or $(w+z)c=z(x+y)$ . . . (2).

(4) $\therefore w(x-y)=z(x+y)$ . . . (3).

(5) Put $x+y=s$ and $x-y=t$.

(6) Then $wt=zs$, or $w=\frac{zs}{t}$ . . . (4).

(7) Substituting this value of w in the proportion of (2), $\frac{2s}{t}:c::\frac{zt+zs}{t}:t$. Dividing antecedents by $\frac{z}{t}$ we have, $s:c::t+s:t$, or $st=c(t+s)$.

(8) Whence, $c=\frac{st}{s+t}=\frac{(x+y)(x-y)}{x+y+x-y}=\frac{x^2-y^2}{2x}$ . . . (5).

(9) From (5), $x^2-2cx=y^2$, whence $x=c+\sqrt{(y^2+c^2)}$.

(10) From figure, $\overline{AB}^2-\overline{AC}^2=\overline{CB}^2$, and $\overline{CD}^2-\overline{DB}^2=\overline{CB}^2$.

(11) $\therefore \overline{AB}^2-\overline{AC}^2=\overline{CD}^2-\overline{DB}^2$, or $a^2-(x+y)^2=b^2-(x-y)^2$.

(12) Whence, $x=\frac{a^2-b^2}{4y}$.

(13) Equating the two values of x, $\frac{a^2-b^2}{4y}=c+\sqrt{(y^2+c^2)}$.

(14) From this we have $y^4+\frac{1}{2}c(a^2-b^2)y=(a^2-b^2)^2$.

(15) Restoring numbers and solving the resulting equation, $y=14.060811$ ft. $x=\frac{a^2-b^2}{y}=\frac{500}{y}=35.620636$ ft. $x-y=21.499014$ ft. AB $=\sqrt{(\overline{AB}^2-\overline{AC}^2)}=\sqrt{[a^2-(x+y)^2]}=33.731178$ ft.

PROBLEM 324.

An iron bar 20 ft. long stands close against a vertical wall. If the lower end is drawn out and the upper slides down the wall until the bar assumes a horizontal position, what curve will a fly describe that remains upon the center of the bar, and what distance will it move?

Solution.

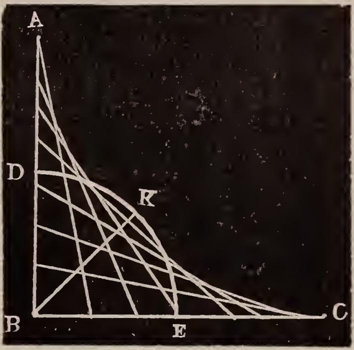

FIG. 22.

(1) Let AB = the vertical wall, BC = the horizontal position of the bar.

(2) We see from the figure the different positions of the bar as it is drawn out.

(3) The center of the hypothenuse or bar always maintains the same distance from the right angle or foot of the wall; viz., $\frac{1}{2}$ the length of the ladder.

(4) The middle point, therefore, moves through a quadrant of a circle DKF, whose radius is $10\times\frac{1}{4}\pi=15.70795$ ft.

∴ The fly moves 15.70795 ft.

PROBLEM 325.

Given the area, $a = 600$, and the hypothenuse $h = 50$; to find the other sides of the right-angled triangle by geometrical construction.

First Solution.

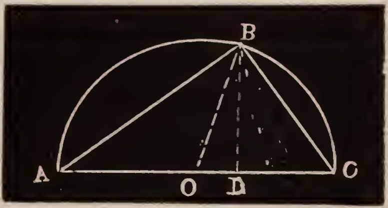

FIG. 23.

(1) Let ABC be the triangle; from B draw BD perpendicular to the hypothenuse AC, BO the radius of the circumscribing circle.

(2) Divide twice the area of the triangle by the hypothenuse = BD, or $(600\times2)\div50=24$.

(3) $OD=\sqrt{\overline{BO}^2-\overline{BD}^2}=7$; $DC=OC-OD$, or 18.

(4) $BC=\sqrt{\overline{BD}^2+\overline{DC}^2}=30$.

(5) $AB=\sqrt{\overline{AC}^2-\overline{BC}^2}=40$.

∴ ABC is a 30, 40 and 50 triangle.

Second Solution.

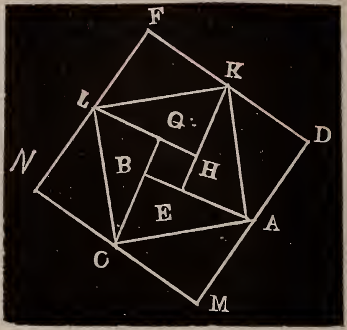

FIG. 24.

(1) Let ABC be the right triangle.

(2) Construct the square KC upon its hypothenuse, and from the corners of this square let fall the perpendiculars KE, LH and BG, this forms three new triangles, equal to ABC, and a square GE.

(3) Now, through the corners of the square KC, draw lines parallel to the perpendiculars already drawn, and we have the square DN, each of the triangles similar to ACM and equal to ABC.

(4) Therefore, □DN=□KC+4△ABC=4900.

(5) The □GE=□KC—4△ABC=100.

(6) Hence DF=AB+BC=$\sqrt{4900}$=70.

(7) HG=AB—BC=$\sqrt{100}$.

(8) Hence, AB=$\frac{1}{2}$(FD—BE)+EB=40.

(9) BC=AB—EB=30.

∴ ABC is a 30, 40 and 50 triangle.

PROBLEM 326.

The area of a right triangle is 924 sq. ft., the sum of the base and perpendicular is 89 ft.: find the sides of the triangle.

Solution.

(1) From Fig. 24, ABC=the triangle.

(2) MN=AB+BC=89, $\overline{MN}^2$=7921 sq. ft.

(3) $\overline{KL}^2$=DN—4△ABC=$\sqrt{4225}$, or 65 ft.=AC.

(4) GE=KC—4△ABC=529 sq. ft., EB=$\sqrt{529}$=23 ft.

(5) BC=$\frac{1}{2}$(FN—GB)=33 ft.

(6) AB=AE, or BC+BE, or 56 ft.

∴ ABC is a 33, 56 and 65 triangle.

PROBLEM 327.

Show that the difference between the hypothenuse and the sum of the two legs of a right-angled triangle is equal to the diameter of the inscribed circle.

Solution.

(1) Let ABC be the triangle, and O the center of the inscribed circle. BE, BF, AE, AK, KC and FC are tangents to the same circle; hence EB =FB, AE=AK and KC=FC, also AE= OE=OK.

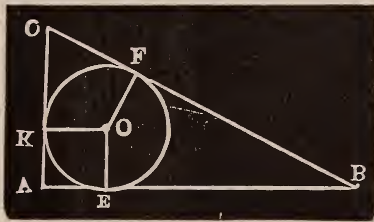

FIG. 25.

(2) EB+KC=FB+CF.

(3) EB+KC+AE+AK—(FB+CF)=AE+AK.

∴ AF+AK=FO+OE, the diameter of the circle.

NOTE.—Solved by the author for the *Teachers' Review.*

PROBLEM 328.

From a right-angled triangular piece of tin whose sides measure $a = 13$, $b = 12$ and $c = 5$ inches, cut out, first, the greatest possible circle; and after that, the greatest possible rectangle: find the length and breadth of the rectangle.

Solution.

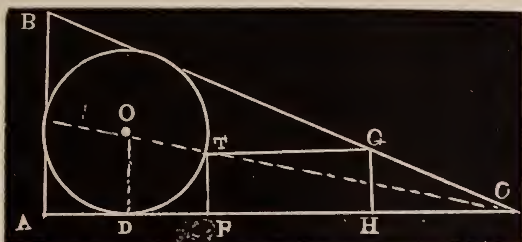

FIG. 26.

(1) Let ABC represent the tin triangle, and OD=AD, the radius of the inscribed circle.

(2) Join OC, cutting the circumference of the circle in T.

(3) From T draw TG parallel to AC, and TP perpendicular to AC. TG and TP will be the length and breadth respectively of the required maximum rectangle.

(4) The perimeter of the triangle ABC=30 in.

(5) The radius of the inscribed circle is found by dividing twice the area by its perimeter, or (12×5)÷30=2 in.

(6) DC=AC—AD=10 in.

(7) $OC=\sqrt{(\overline{DC}^2+\overline{OD}^2)}=10.1981$ in.

(8) $TC=OC-OT=8.1981$ in.

(9) Then by sfmilar triangles, OC : OD :: TC : TP, or 10.1981 : 2 :: 8.1981 : (TP=1.6077 in.).

(10) Also, AB : AC :: GH : HC, or 5 : 12 :: 1.6077 : (HC=3.8584 in.)

(11) OD : DC :: TP : PC, or 2 : 10 :: 1.6077 : (PC=8.0388 in.)

(12) ∴ $PH=PC-HC=4.1804$ in.

∴ 1.6077 in.=the breadth, and 4.1804 in.=the length of the rectangle.

PROBLEM 329.

To determine a right-angled triangle, having given the hypothenuse $h=100$, and the difference of the two lines drawn from the two acute angles to the center of the inscribed circle $= d$ or $20\sqrt{5}$.

Solution.

(1) Let ABC=the triangle, AO and OB the two lines drawn from the acute angles to the center O, x=AE, EB=100$-x$, and r=OE, the radius of the inscribed circle.

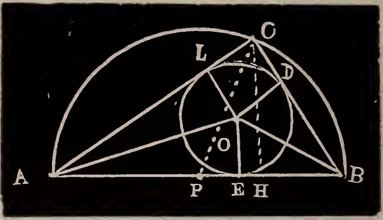

FIG. 27.

(2) $\overline{AO}^2=x^2+r^2$, and OB $=(100-x)^2+r^2$.

(3) ∴ $x^2+r^2-[(100-x)^2+r^2]=20\sqrt{5}$ squared, or 2000.

(4) Squaring and reducing, $200x=12000$. We find $x=60$.

(5) AL=AE, LO=OD=LC, and EB=BD. EB=AB—AE, or 100—AE, or 60=40.

(6) $\overline{EB}^2+\overline{OE}^2$, or $40^2+r^2=20\sqrt{5}=\overline{OD}^2$.

(7) $r=20$. AL+LC=80. BD+DC=60.

∴ ABC is a 60, 80 and 100 triangle.

NOTE.—d, or $20\sqrt{5}=\sqrt{\overline{AO}^2-\overline{OB}^2}$.

PROBLEM 330.

The perimeter of a right-angled triangle is 240 ft., and the radius of the inscribed circle is 20 ft.: find the sides of the triangle.

Solution.

(1) Fig. 27. The perimeter of the triangle ABC is 240, LC, or CD is 20, the radius of the inscribed circle.

(2) LC+CD=40, and 240—40=200, the sum of the lines AL, AE, EB and BD, which are tangents to the same circle.

(3) Hence, AL=AE, EB=BD, and AL+DB=AE+EB=100, the hypothenuse.

(4) △AOB=△AOL+△BOD, because they have the same bases and altitudes.

(5) P being the middle point of AB, CP=50.

(6) The area of the triangles AOL, BOD and AOB is $[(100\times 20)\div 2]\times 2=2000$.

(7) The area of LD is $20^2=400$.

(8) The area of △ABC is 2000+400=2400, and its altitude $CH=(2400\times 2)\div 100=48$.

(9) $PH=\sqrt{(50^2-48^2)}=14$; BH=50—14=36;
$BC=\sqrt{(48^2+36^2)}=80$.

∴ The sides of the triangle are 60, 80 and 100.

NOTE.—This solution was prepared by the author for the *School Visitor.*

PROBLEM 331.

A tree 120 ft. in height, standing on a level, is broken off so that the part broken off is 30 ft. longer than the stump. If the parts remain in contact, what is the length of the path through which the top of the tree passes in falling to the ground?

Solution.

(1) Let AC represent the tree, 120 ft., B the point where the trunk broke, and AB=(120—30)÷2=45 ft., the stump.

(2) Then, BC, the part broken off, is 75 ft.

(3) The arc CD is the path described by the top, C.

(4) By trigonometry, ∠ABD=53°7′50″; hence, CBD=126° 52′ 10″.

(5) As 360° is to the number of degrees in the arc $(126\frac{313}{360})$, so is the circumference of the circle $(150\times\pi)$ to the length of the arc, CD, or 166.07 ft.

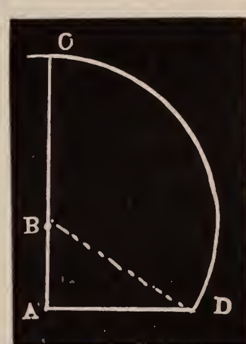

FIG. 28.

NOTE.—This solution was prepared by the author for the *School Visitor.*

PROBLEM 332.

A statue *c* ft. high stands on a column *d* feet high: how far from the foot of the column must a boy stand that the statue may subtend the greatest possible visual angle?

Solution.

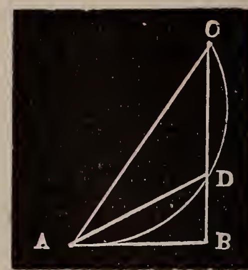

FIG. 29.

(1) Let BD$=c$, the height of the column; CD$=d$, the height of the statue; and $x=$AB, the distance the boy stands from the foot of the column.

(2) Describe a circle which shall pass through D and C, and tangent to AB, A being the point of tangency.

(3) The angle DAC being an inscribed angle, is greater than if A were moved either way on AB, for it would be exterior to the circle.

(4) Since angle A is measured by half of the arc AD, and angle C by half of the same arc, we have the similar triangles ABD, ABC.

(5) $c:x::x:c+d$. Whence, $x=\sqrt{[c(c+d)]}$.

(6) Now, A is the point at which the boy can see the statue at the best advantage. Let $c=120$ ft, and $d=60$ ft.; then $x=146.96+$ ft.

PROBLEM 333.

Find the three sides of a right-angled triangle whose perimeter is 120 ft., and whose base is $\frac{3}{4}$ of its perpendicular.

Solution.

(1) The base and perpendicular being as 3 to 4, the hypothenuse is $\sqrt{(3^2+4^2)}=5$, proportionally.

(2) This triangle whose perimeter is $3+4+5=12$, is similar to the triangle whose sides are required; hence, $120\div12$, or 10 times these, are 30, 40 and 50 ft., the required sides.

PROBLEM 334.

Required the sides of a right triangle which shall contain the greatest area under the shortest perimeter, when the square of the area shall be equal to the product of the three sides.

Solution.

(1) Let $x=$the base, $y=$the perpendicular; then, $\sqrt{(x^2+y^2)}=$ the hypothenuse.

(2) By the conditions we must have $\frac{1}{4}x^2y^2=xy\sqrt{(x^2+y^2)}$, or $xy=4\sqrt{(x^2+y^2)}$.

(3) Squaring, we have $x^2y^2=16x^2+16y^2$, which gives $x=4y\div$

$\sqrt{(y^2-16)}$.

(4) Substituting this value of x in the value for the area, we have $2y^2 \div \sqrt{(y^2-16)}$, a maximum.

(5) Differentiating, dividing by dx and equating to zero, we have $2y^2=64$, or $y=4\sqrt{2}$, and $x=4\sqrt{2}$. Whence the hypothenuse=8.

PROBLEM 335.

The diameter of the circumscribing circle of a right triangle added to the diameter of the inscribed circle, is $71\frac{3}{4}$, and the perpendicular is $\frac{3}{4}$ of the base: find the sides.

Solution.

(1) The hypothenuse is the diameter of the circumscribing circle. Hence, the sides of the triangle are some multiple of 3, 4 and 5.

(2) Let AB=3, BC=4, and AC=5.

(3) The diameter of the inscribed circle is $2(AB \times BC) \div (AB+BC+AC)=2$.

(4) ∴ The diameter of an inscribed circle is equal to 4 times the area of the triangle divided by its perimeter.

(5) The diameter of the circumscribing circle is $AB \times BC \times AC$=diameter×2 times the triangle; or diameter=(AB $BC \times AC) \div$twice the area of the triangle, or 5.

(6) Hence, the sum of the diameters=7.

(7) In the problem it is $\frac{287}{4}$. ∴ The required is $\frac{41}{4}$, and the sides are $30\frac{3}{4}$, 41 and $51\frac{1}{4}$.

NOTE.—We should remember that the continued product of the three sides of a triangle divided by twice the area of the triangle is equal to the diameter of the circumscribing circle.

PROBLEM 336.

Two men, A and B, started from the same point at the same time; A traveled southeast for 10 hr., and at the rate of 10 mi. per hr., and B traveled due south for the same time, going 6 mi. per hr; they turned and traveled directly towards each other at the same rates respectively, till they met: how far did each man travel?

Solution.

(1) Let C be the starting point, CA=the distance A travels southeast, and CB = the distance B traveled south.

(2) Then CA=100 mi., and CB=60 mi.

(3) Now draw AD perpendicular to CB produced to D.

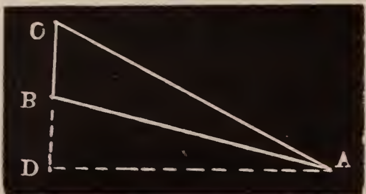

FIG. 30.

(4) As the $\angle$ D=a right angle, and $\angle$ C=45°, then CD=AD.

(5) $2\overline{AD}^2=\overline{AC}^2=100^2$, whence $AD=50\sqrt{2}$, and $BD=50\sqrt{2}-60$.

(6) In the right triangle ADB, $AB=\sqrt{(50\sqrt{2})^2+(50\sqrt{2}-60)^2}$ $=\sqrt{13600-6000\sqrt{2}}=71.517261+$ mi.

(7) A and B together travel 16 miles per hour, and the time required, until they meet in traveling AB, is $\frac{1}{16}$ of AB= 4.469228+ hr.

(8) ∴ A traveled 44.69828+ mi., and B, 26.81897+ mi. of the distance AB.

∴ The total distance traveled by A is 144.69828+ mi., and by B, 86.81897+ mi.

NOTE.—This solution was prepared for the *American Mathematical Monthly* by G. B. M. Zerr, A. M., Ph. D., President of Russell College, Lebanon, Va.

PROBLEM 337.

On a globe 20 ft. in diameter, 30 ft. above a plane, there is a boy 4 ft. tall to his eyes: how much surface on the plane is hidden from him?

Solution.

(1) 30 ft.=EB, the distance the globe is above the plane; 20 ft.=DE, the diameter of the globe; and AD=4 ft., the distance the boy's eye is above the ball.

(2) 14 ft.=AD+DL=AL.

(3) $AF=\sqrt{\overline{AL}^2-\overline{LF}^2}=4\sqrt{6}$ ft.

(4) Now, by the similar triangles ALF and ABC, AF : LF :: AB : BC, or $4\sqrt{6}$: 10 :: 54 : BC, from which $BC=540\div4\sqrt{6}=135\div\sqrt{6}$ $=135\sqrt{6}\times\frac{1}{6}=\frac{135\sqrt{6}}{6}$ ft.

(5) ∴ $\pi\overline{BC}^2=\pi\left(\frac{135\sqrt{6}}{6}\right)^2$=area of the surface hidden from the boy.

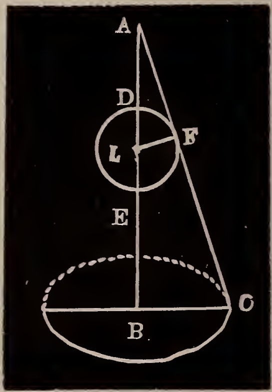

FIG. 31.

PROBLEM 338.

What is the area of the largest square that can be inscribed in a semicircle, the diameter of the circle being 20 inches?

Solution.

(1) Let AB be the diameter of the circle.

(2) Draw SB equal and perpendicular to AB, tangent to the circle; E and SO to the center of the circle.

(3) Let K be the point where SO cuts the circle.

(4) From the point K, draw KC parallel to SB; TK parallel to AB, TX parallel to KC. Hence the square is completed.

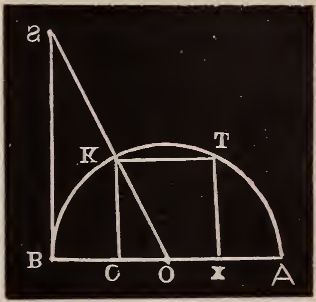

FIG. 32.

(5) Then, $\overline{OK}^2 = 4\overline{OC}^2 + \overline{OC}^2$, or $5\overline{OC}^2$.

(6) But $\overline{OK}^2 = 100$; $\therefore$ $5\overline{OC}^2 = 100$; $\overline{OC}^2 = 20$, and $OC = \sqrt{20}$, or 4.472+ in.

(7) CX = the side of the square, or $4.472 \times 2 = 8.944+$ in.

(8) $\overline{CX}^2 = 79.99+$ sq. in., area of the inscribed square.

PROBLEM 339.

I have an inch board 5 ft. long, 17 in. wide at one end and 7 in. at the other: how far from the larger end must it be cut straight across so that the two parts shall be equal? (*R. H. A., p. 407, prob. 101.*)

Solution.

(1) Let ABCD be the board.

(2) DC = 7 in., the width of the small end, and AB = 17 in., the width of the large end.

(3) CB = 5 ft., or 60 in., the length of the board. AL = 10 in.

(4) Then, by similar triangles, AL : AB :: DL : KB, or 10 : 17 :: 60 : (BK = 102 in.).

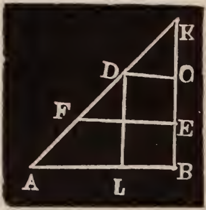

FIG. 33.

(5) The area of $\angle$DCK $= (7 \times 42) \div 2 = 147$ sq. in.

(6) The area of $\angle$ABK $= (102 \times 17) \div 2 = 867$ sq. in.

(7) Then, the area of DABC $= 867 - 147 = 720$ sq. in.

(8) Area of FECD $= 720 \div 2 = 360$ sq. in.

(9) The area of FEK $= 360 + 147 = 507$ sq. in.

(10) $\therefore$ $867 : 507 :: 102^2 : \overline{KE}^2$. EK = 78 in.

(11) 102 in. — 78 in. = 24 in.

$\therefore$ EB = 2 ft., the distance the board must be cut from the large end.

PROBLEM 340.

The sides of a triangular field are 39, 40 and 50 rd.: from what point in one of the shorter sides must a line be drawn parallel with the other shorter side, so as to divide the field into two equal parts?

Solution.

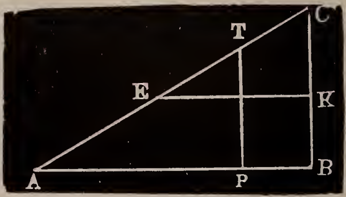

FIG. 34.

(1) Let ABC be the triangle, and draw EK parallel to AB, cutting off the half of AB or $[(40\times30)\div2]\div2=300$ sq. rd.

(2) Then we have the proportion, $600 : 300 :: 40^2 :$ (EK, or 28.284 rd.)

(3) EK=28.284 rd., length of the line that divides the triangle into equal parts; and it is parallel to AB, and AB =40 rd.

(4) Also, draw TP parallel to CB, whose side is 30 rd.

(5) ∴ $600 : 300 :: 30^2 : CB^2$, from which CB=21.213 rd.
∴ TP=21.213 rd., dividing the field into equal parts parallel to CB.

PROBLEM 341.

Two trees stand on opposite sides of a stream 40 ft. wide; the height of one tree is to the width of the stream as 8 is to 4, and the width of the stream is to the height of the other as 4 is to 5: what is the distance between their tops?

Solution.

(1) From Fig. 9, AB=40 ft., the width of the stream. CB and FA=the height of the trees.

(2) Then by the conditions of the problem, one tree is twice the width of the river.

(3) ∴ CB=80 ft., FE=40 ft., FA=50 ft., the height of the 2d tree.

(4) CE=30 ft.; FC, the distance between their tops=
$\sqrt{(\overline{FE}^2+\overline{CE}^2)}=50$ ft.

∴ The required distance is 50 ft.

PROBLEM 342.

The lengths of the lines that bisect the acute angles of a right-angled triangle being 48 and 60 respectively, to determine the three sides of the triangle.

Solution.

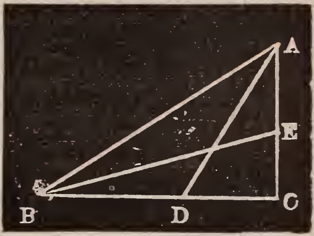

FIG. 35.

(1) Let $BC=x$, $AC=nx$, $AB=mx$, $AD=48=a$, $BE=60=b$.

(2) $\overline{AB}^2=\overline{AC}^2+\overline{BC}^2$, or $m^2x^2=n^2x^2+x^2$, or $m^2=n^2+1$.

(3) $\therefore m^2-n^2=1$ . . . (1).

(4) Area ABC = area BAD + area DAC.

(5) Area $ABC=\frac{1}{2}AB\times AC\sin A=\frac{1}{2}mnx^2\sin A=mnx^2\sin\frac{1}{2}A\cos\frac{1}{2}A$.

(6) Area $BAD=\frac{1}{2}AB\times AD\sin DAB=\frac{1}{2}amx\sin\frac{1}{2}A$.

(7) $\therefore DAC=\frac{1}{2}AC\times AD\sin DAC=\frac{1}{2}anx\sin\frac{1}{2}A$.

(8) $\therefore mnx^2\sin\frac{1}{2}A\cos\frac{1}{2}A=\frac{1}{2}amx\sin A\frac{1}{2}+\frac{1}{2}anx\sin\frac{1}{2}A$.

(9) $\therefore 2mnx\cos\frac{1}{2}A=am+an$.

(10) From triangle, $DAC=\cos\frac{1}{2}A=\dfrac{AC}{AD}=\dfrac{nx}{a}$.

(11) $\therefore am+an=\dfrac{2mn^2x}{a}$ . . . (2).

(12) Similarly from area ABC=area ABE+area CBE.

(13) We find $b+mb=\dfrac{2mx^2}{b}$ . . . (3).

(14) (2)÷(3) gives $\dfrac{(m+n)a^2}{(m+1)b^2}=n^2$ . . . (4).

(15) Eliminating n, using (1) and (4), and calling $\dfrac{a^2}{b^2}=d=.64$.

(16) $m^6+2m^5-(1+2d)m^4-(4+2d)m^3-(1-2d)m^2+2(1+d)m+d^2+1=0$.

(17) Restoring numbers, $m^6+2m^5-2.28m^4-5.28m^3+28m^2+3.28m+1.4096=0$.

(18) Reducing by Horner's method to 2 decimal places, $m=1.25$; then $a=.75$.

(19) Substituting these values of m and n in (2), $x=57.24+$ $nx=42.93+$, and $mx=71.55+$.

$\therefore$ The sides of the triangle are, $AC=42.92+$, $BC=57.24+$, and $AB=71.55+$.

PROBLEM 343.

A barn is 60 ft. long, 20 ft. high to the square; the rafters, exclusive of eave projection, are 25 ft. long, and the roof is ⅜ pitch: find cost of siding at $20 per M.

Solution.

(1) From Fig. 36, let ABC be the gable end of the barn, and AC=BC=25 ft., the length of the rafters.

(2) By the conditions of the problem, CD=3 ft., and AB, the span=8 ft.

(3) Then, AD=4 ft., that is, we suppose that ABC and ADC are similar to ABC, the given gable end.

(4) $AC=\sqrt{(\overline{AD}^2+\overline{CD}^2)}=5$, whence ADC is a 3, 4 and 5 right triangle, or the smallest integral triangle.

(5) Now, as AC is 5 in the small triangle, then we have AB, the hypothenuse of the large triangle $=5\times5=25$.

(6) Likewise, AD=20 ft., and CD=15 ft.; then AB=40 ft., the width of the span.

(7) The area of $ABC=\frac{1}{2}(AB\times CD)=300$ sq. ft., or $300\times2=600$ sq. ft., the area of both gable ends.

(8) The barn is 20 ft. high to the square, then the area of the square is 40×20, or 800 sq. ft., and $800\times2=1600$ sq. ft., area of both ends, excluding the gable ends.

(9) $60\times20\times2=2400$ sq. ft., area of both sides of the barn.

(10) $\therefore$ 2400 sq. ft.+1600 sq. ft.+600 sq. ft.=4600 sq. ft., total number of square feet in the barn, or $4\frac{3}{5}$ M.

(11) $\$20\times4\frac{3}{5}=\$92$, the cost required.

NOTE.—*A Gable* is the triangular portion of the end of a building, bounded by the sides of the roof and a line joining the eaves.

A Gable End is the triangular-topped end of a house or barn.

The Pitch is the inclination of a roof. The common pitch has a rafter three-quarters the length the span. In the above problem the pitch is $\frac{3}{8}$ of the span AB, or CD=$\frac{3}{8}$ of AB, or 15 ft. The Gothic has a full pitch, the rafters being the length of the span; the Greek has a pitch $\frac{1}{9}$ to $\frac{1}{7}$ of the span; the Roman has a pitch from $\frac{1}{6}$ to $\frac{2}{9}$ of the span; and the Elizabethan has rafters longer than the span.

II. THE EQUILATERAL TRIANGLE.

PROBLEM 344.

If the side of an equilateral triangle is $2s$, find its perpendicular.

Solution.

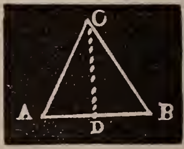

FIG. 36.

(1) In the triangle ABC, draw the perpendicular CD.

(2) $AD=s$, ADC is a right triangle.

(3) Then, $\overline{AC}^2-\overline{AD}^2=\overline{CD}^2$.

(4) Now, as $AC=2s$, we have $4s^2-s^2=\overline{CD}^2$.

$\therefore CD=s\sqrt{3}$.

Rule.—*Multiply half of the side by the* $\sqrt{3}$.

NOTE.—It would be well to remember that the $\sqrt{3}=1.73205$.

PROBLEM 345.

To find the area of an equilateral triangle having given the side.

Solution.

(1) From Fig. 36 we know that AD is s, or half the side of the triangle, and $CD=s\sqrt{3}$.

(2) Then the area is $AD\times CD$, or $s\times s\sqrt{3}=s^2\sqrt{3}$.

Rule.—*Multiply the square of half the side by* $\sqrt{3}$.

PROBLEM 346.

In an equilateral triangle having given the lengths of the three perpendiculars, drawn from a certain point within, on the three sides; to determine the perpendicular.

Solution.

(1) Let ABC represent the equilateral triangle, and DE, DF, DG, the given perpendiculars.

(2) From the points D, draw the lines DA, DB, DC to the angular points; and let fall the perpendicular CH on the base AB.

(3) Put the three given perpendiculars, $DE=a$, $DF=b$, $DG=c$, and put $s=AH$, or BH, half the side of the equilateral triangle.

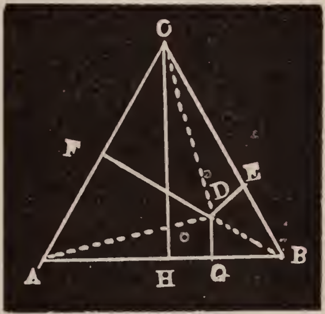

FIG. 37.

(4) Then AC or $BC=2s$, and by right angles, the perpendicular $CH=\sqrt{AC^2-AH^2}=\sqrt{4s^2-s^2}=\sqrt{3s^2}=s\sqrt{3}$.

(5) The area of a triangle is expressed by $\frac{1}{2}$ the product of the base and height, and a triangle is equal to half a rectangle of equal base and height.

(6) ∴ The whole triangle ABC $\text{is}=\frac{1}{2}AB\times CH=s\times s\sqrt{3}=s^2\sqrt{3}$.

(7) The triangle $ADB=\frac{1}{2}AB\times DG=s\times c=cs$.

(8) The triangle $BCD=\frac{1}{2}BC\times DE=s\times a=sa$.

(9) The triangle $ADC=\frac{1}{2}AC\times FD=s\times b=bs$.

(10) But the three last triangles are equal to ABC, that is $s^2\sqrt{3}=as+bs+cs$, hence dividing by s, $s\sqrt{3}=a+b+c$.

$\therefore s\sqrt{3}$=HC=FD+DG+ED.

PROBLEM 347.

Three horses are tethered each to a rope 15 rd. in length to the corners of an eqilateral triangle whose side is 30 rd: over how much space can they graze, and how much remains ungrazed?

Solution.

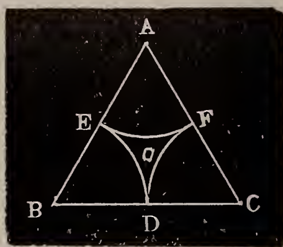

FIG. 38.

(1) Let ABC be the triangle. CFD, BDE and AFE=the three sectors grazed over by the horses. O=space ungrazed.

(2) Since the angle of an equilateral triangle is 60°, because the three angles of every triangle are equal to two right angles, each sector is $\frac{1}{6}$ of a circle. The three sectors=$\frac{1}{2}$ of a circle.

(3) Area of circle, radius 15 rd.$=15^2\times\pi=225\pi$ sq. rd.

(4) $225\pi\div2=353.43$ sq. rd., area grazed over.

(5) Area of CBA$=\overline{BD}^2\sqrt{3}$, or $15^2\sqrt{3}=389.711$ sq. rd.

(6) 389.711 sq. rd.—353.43 sq. rd.=36.281 sq. rd., part not grazed over.

PROBLEM 348.

A goat is tethered by a cord 30 ft. long, to the corner of a shed in tthe form of an equilateral triangle, side 20 ft.: find area over which it may graze outside of the shed.

Solution.

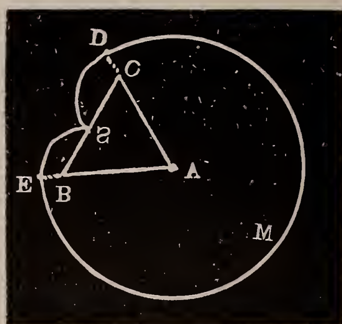

FIG. 39.

(1) Let ABC be the shed.

(2) The goat can graze with the 30 ft. cord over a circle, except a sector of 60°, or angle A.

(3) EMD=300°, or $\frac{300}{360}=\frac{5}{6}$ of a circle, which has $30^2\pi\times\frac{5}{6}$ =2356.2 sq. ft.

(4) Angle DCS=120°, and angle EBS=120°.

(5) Angle EBS+angle DCS= 240°, or $\frac{2}{3}$ of circle with a radius of 10 ft.

(6) $\therefore$ The area of BES+SCD$=10^2\pi\times\frac{2}{3}=209.44$ sq. ft.

(7) Whole area grazed over is 2356.2 sq. ft.+209.44 sq. ft.= 2565.64 sq. ft.

PROBLEM 349.

At 80 cents a rod, what would be the cost of fencing a field in the form of an equilateral triangle, its altitude being 6 rd.?

Solution.

(1) Let ABC be the triangle.

(2) AO bisects the angle A, then angle OAD=30°. Then AOD is a right-angled triangle.

(3) Now, OD+DE=CO, or the radius of the circle.

(4) CO or AO=$\frac{2}{3}$ of 6 rd.=4 rd.

(5) AB=AO$\sqrt{3}$, or 4$\sqrt{3}$=6.92820 rd.

(6) The perimeter is 6.92820×3= 20.78460 rd.

(7) 20.78460×.$80=$166.2768, cost of fencing the triangle.

FIG. 40.

NOTE.—As AOD is a right triangle and the angle OAD=30°, then AO=4 and OD=2 rd. It follows that the hypothenuse is twice the side opposite the acute angle of a right triangle which is 30°. Also if the hypothenuse be given, then the side opposite the 30° angle is half of the hypothenuse.

Prove that AO$\sqrt{3}$=AB.

A figure with six sides is called a hexagon; the side is equal to the radius of the circumscribing circle. Now, if I unite the alternate angles of the regular hexagon, as AB, BC, and CA, I have a regular triangle, called an equilateral triangle. Join AD, then ADB is a right triangle. Then OD= DB.

$\overline{AB}^2=\overline{AD}^2-\overline{DB}^2=4\overline{AO}^2-\overline{AO}^2$, or $(\overline{DB}^2)=3\overline{AO}^2$. Whence AB=AO$\sqrt{3}$; that is,the side of an equilateral triangle is equal to the radius of the circumscribing circle multipied by the square root of 3.

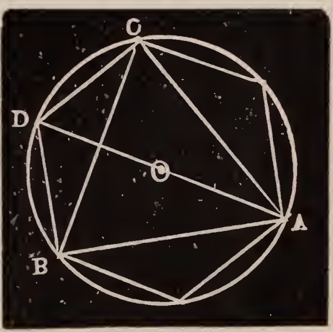

FIG. 41.

PROBLEM 350.

The area of an equilateral whose base falls on the diameter and whose vertex falls on the middle point of the arc of a semicircle, is 80 sq. ft.: what is the diameter of the semicircle?

Solution.

(1) Let SPC be the semicircle, and ABC the triangle.

(2) CD is the radius of the circle, also the perpendicular of the triangle.

(3) The area of an equilateral triangle is found by squaring half the side and multiplying by $\sqrt{3}$, then we must get the side by reversing this operation.

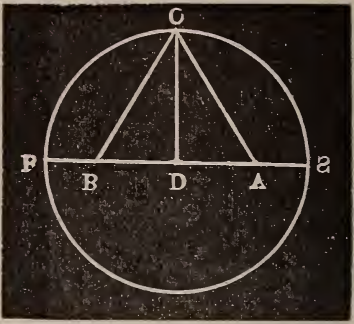

FIG. 42.

(4) $\sqrt{(80 \div 1.732)} \times 2 = 13.59$ ft.

(5) Since the perpendicular equals half the side multiplied by $\sqrt{3}$, we have $13.59 \div 2 \times \sqrt{3} = 11.77$ ft., perpendicular, or radius of the circle.

(6) ∴ The diameter is $11.77 \times 2 = 23.54$ ft.

PROBLEM 351.

John has a garden in the form of an equilateral triangle; from the corners to a spring within the garden, the distances are $a=20$, $b=28$, $c=31$ rods: find side of garden.

Solution.

(1) Let ABC be the given triangle, O the spring.

(2) Put $AO=a=20$. $BO=b=28$. $CO=c=31$.

(3) Let s=side of triangle ABC. The area$=\frac{1}{4}s^2\sqrt{3}$.

(4) Draw AFB=AOB. With B as a center and b as a radius, describe the circle FOD. With C as a center and c as a radius, describe the circle DOE. Join EF, FD and DE.

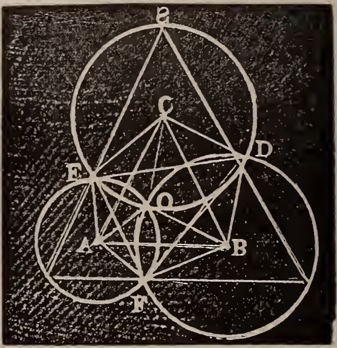

FIG. 43.

(5) Now angle EAF=2 times angle CAB=120°.

(6) ∴ EF is equal to a side of an equilateral triangle inscribed in a circle whose radius is a. ∴ $EF=a\sqrt{3}$.

(7) In the same manner find $FD=b\sqrt{3}$, and $ED=c\sqrt{3}$.

(8) Area of triangle $EAF=100\sqrt{3}$, of triangle $FBD=196\sqrt{3}$, of triangle $DCE=240\frac{1}{4}\sqrt{3}$, and of triangle EFD $=823.18129$ sq. rd.

(9) ∴ The area of the entire polygon AFBDCE=1731.9935 sq. rd.

(10) But this is double the area of triangle ABC. ∴ Area of triangle ABC=875 996785 sq. rd.

(11) Equating the two expressions for the area, we have $\frac{1}{4}s^2\sqrt{3}=875.996785$ sq. rd.

∴ $s=44\,97874$ rd., the side of the triangle ABC.

NOTE.—Solved by the author for the *Teachers' Review.*

PROBLEM 352.

A deer is tethered from one corner of an equilateral wall whose side is 100 ft., by a rope 175 ft. long: over what area can it graze?

Solution.

(1) Let AOB be the triangle.

(2) We see that it can graze over $\frac{5}{6}$ of $(175^2\times\pi)=$ 80176.06 sq. ft.

(3) Next over two sectors ATF and BET.

(4) We also have a right triangle BDT having BD=50 ft., and TB 75 ft., to find the angle DBT.

(5) By trigonometry, $\angle DBT=48\frac{683}{3600}°$, and $\angle EBT=71\frac{2917}{3600}°$, and $\angle TBE+\angle TAF=143\frac{1117}{1800}°$.

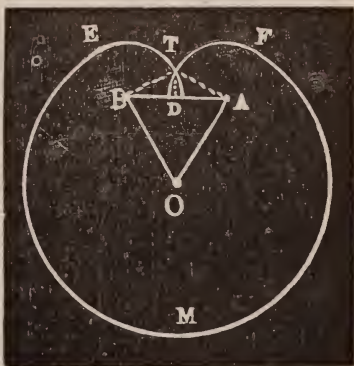

FIG. 44.

(6) From this we have $360° : 143\frac{1117}{1800}° :: 75^2\times\pi : (7049.96$ sq. ft.), the area of the two sectors.

(7) It also grazes over the triangle ABT, whose base is AB=100 ft., and altitude TD or $\sqrt{(75^2-50^2)}=25\sqrt{5}$ ft.

(8) This gives $(100\times25\sqrt{5})\div2=2795\,08$ sq. ft.

(9) Total area grazed over is 80176.06+7049.96+2795.08= 90021.10 sq. ft.

PROBLEM 353.

I have a lot in the form of an equilateral triangle whose sides are 100 ft. At each corner stands an orange tree; the height of the first tree is 10 ft., the second 20 ft., and third 30 ft. At what distance from the foot of each tree must a ladder be placed, so that without moving it at the base it may reach the top of each? Find the length of the ladder.

Solution.

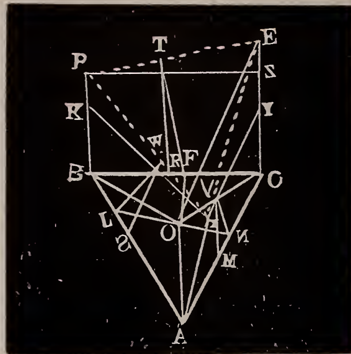

FIG. 45.

(1) Let ABC be the garden, and AX, BP and CE the trees at the corners.

(2) Join the tops of the trees by the lines XP, PE and EX. From W, T and V, the middle points of XP, PE and EX, draw WL, TF and VN perpendicular to XP, EP and EX, and at L, F and N draw perpendiculars to AB, BC and CA in the equilateral triangle ABC, meeting at O.

(3) Then O is equidistant from X, P and E; L is equidistant from X and P; and OL being perpendicular to the plane ABPX, every point of OL is equidistant from X,P.

(4) For the same reason OF and ON are equidistant from E and P, F and X.

(5) Therefore, O, their point of intersection, is equally distant from X, P and E.

(6) Draw XK, PZ and XY perpendicular to PB, EC, and also is XY perpendicular to EC.

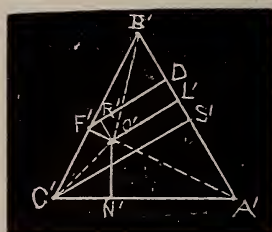

FIG. 46.

(7) Now, if the student refers to Fig. 7, it will be easy to understand Fig. 45. Let us examine Fig. 46.

(8) Let A′B′C′ be the triangle whose sides are 100 ft., L=point L′, S=S′, F=F′, etc.

(9) Draw F′D parallel to O′L′, or to C′S′, and R′O parallel to DL; then the two triangles F′DB′ and O′R′F′ are similar.

(10) We know that S, R and M are the middle points of AB, BC and CA, and that the triangles XBP and WSL, PEZ and VMN, XYE, VMN are similar.

(11) The figure AXPB is a trapezoid, and $WS=\frac{1}{2}(AX+PB)$ $=15$ ft., $TR=\frac{1}{2}(EC+PB)=25$ ft., and $VM=\frac{1}{2}(EC+XA)$ $=20$ ft.

(12) By similar triangles, XK : PK :: WS : SL, from which $SL=1\frac{1}{2}$ ft.

(13) PZ : EZ :: TR : FR, or $FR=2\frac{1}{2}$ ft.

(14) YX : EY :: VM : NM, or MN=4 ft.

(15) From this we find LB=48½ ft, AL=51½ ft., CF=47½ ft., FB=52½ ft., CN=46 ft., and NA=54 ft.

(16) $\angle S'C'B'=30°$, also $\angle DF'B'$, as F'B'=FB, or 52½ ft.

(17) From the note under Fig. 40, DB'=½ of F'B, or 26¼ ft.

(18) $FD=\sqrt{(\overline{F'B'}^2+\overline{B'D}^2)}=54.466$ ft.

(19) R'O', or DL'=L'B'—B'D=22¼ ft.

(20) By similar triangles, F'D : DB':: R'O' : F'R', or F'R'= 12.99 ft.

(21) R'D, or O'L'=DF—F'R=32.47+ ft.

(22) $B'O'=\sqrt{(\overline{O'L'}^2+\overline{DB}^2)}=58.36+$ ft.

$A'O'=\sqrt{(\overline{O'L'}^2+\overline{L'A'}^2)}=60.88+$ ft.

(23) By similar triangles, DB' : F'B':: F'R' : F'O', or F'O'= 25.98 ft.

(24) $C'O'=\sqrt{(\overline{C'F'}^2+\overline{O'F'}^2)}=54.15$ ft.

$OE=\sqrt{(\overline{OC}^2+\overline{CE}^2)}=61.7+$ ft., length of the ladder.

∴ AO=60.88+ ft., OB=58.36+ ft., CO=54.15+ ft., and 61.7+ ft.=the length of the ladder.

PROBLEM 354.

On the three sides of any plane triangle construct equilaterals; join the centers of these triangles; prove that the resulting triangle is equilateral.

Solution.

(1) Let ABC be any triangle, having the equilateral triangles ABK, BDC and ACE described on AB, BC and CA.

(2) Let G, H, F be the centers of these triangles.

(3) Now, we are to prove that GHF is an equilateral. Circumscribe the three triangles and GHF by circles.

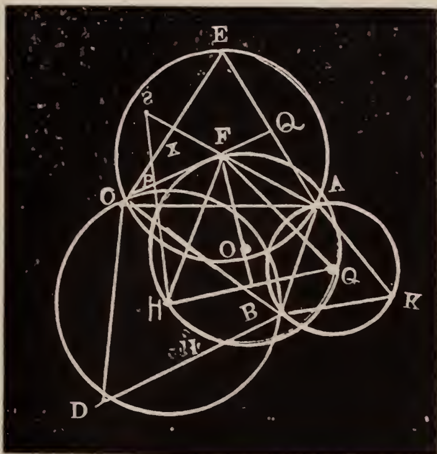

FIG. 47.

(4) Draw SPH perpendicular, FAC rather at right angles with AC, to intersect AF if produced, in S.

(5) AS passes through F; then it bisects $\angle FAC$, which makes $\angle XAC=30°$.

(6) In like manner angle $QCA=30°$, from this it is easy to see that SFP is an equilateral, and angle $FPH=120°$.

(7) ∴ FGHP is a quadrilateral and the sum of their opposite angles$=180°$, as angle $FPH=120°$. Angle $FGH=180°-120°=60°$. In the same manner it may be proven that HFG and FHG are each equal to 60°; therefore FGH is equilateral.

NOTE.—This solution was prepared by the author for the *American Mathematical Monthly*.

PROBLEM 355.

A wheel 8 ft. in diameter is run by means of a belt from a wheel 1 ft. in radius; the distance between their centers is 12 ft., and the belt is crossed between them, causing them to revolve in opposite directions: what is the length of the belt used?

PROBLEM 356.

A ladder 20 ft. long leans against a perpendicular wall at an angle of 30°: how far is its middle point from the bottom of the wall?

Solution.

(1) If the ladder makes an angle of 30° with the wall, it must make an angle of 60° with the base.

(2) Suppose on the opposite side of the wall a similar ladder be placed, and meet the wall at an angle of 30°, the angle at the base would be 60°.

(3) Then, with the two ladders and the base line, we have an equilateral triangle, each side 20 ft. and each angle 60°.

(4) Now, joining the middle points in the sides of any equilateral triangle divides the triangle into four similar triangles.

(5) The middle points of the base line and of the ladder are the middle points of the equilateral triangle.

(6) Hence, the line that joins these points is $20\div2$, or 10 ft.

PROBLEM 357.

An equilateral triangle and a square have equal areas: find the ratios of their perimeters.

Solution.

(1) As the area of an equilateral triangle equals $\frac{1}{4}$ of the square of a side multiplied by $\sqrt{3}$, or 1.73205, we find the side by reversing the operations in this rule.

(2) When the area is 1, the side is $\sqrt{(1\div 1.73205)}\times 4=\frac{3}{4}$, nearly, and the perimeter is $3\times\frac{3}{4}=2\frac{1}{4}$.

(3) The side of the square whose area is 1, and the perimeter 4.

(4) The required ratio, then, is $2\frac{1}{4}$ to 4, nearly, or 9 to 16.

PROBLEM 358.

The area of one equilateral triangle is $\frac{9}{16}$ of another; if the radius of the inscribed circle of the larger triangle is 8 inches, what will be the circumference of the circle inscribed in the smaller?

Solution.

(1) Ratios of any homologous lines, such as sides, altitudes, radii of inscribed or circumscribed circles, is $\sqrt{\frac{9}{16}}=\frac{3}{4}$.

(2) Then the radius of the required circle is $\frac{3}{4}$ of 8, or 6 in., and its circumference is $6\times 2\pi$ or 12π.

PROBLEM 359.

A park of three equal sides has a drive along the boundary occupying .19 of the whole park; if the nearest distance from the drive to the center is 9 rd., what is the area of the inscribed circle?

Solution.

(1) The area of the park inside of the drive is .81 of the whole park.

(2) The 9 rd. must be $\sqrt{.81}$, or .9 of the nearest distance from the center of park to its side.

(3) 9 rd.$\div$.9=10 rd., said distance, which is the radius of the inscribed circle.

(4) Area of circle is $10^2\pi$, or 100π.

FORMULAS USED IN SOLUTIONS.

1. Area=one-half the side squared and multiplied by $\sqrt{3}=$ 1.73205.
2. Side$=\sqrt{(\text{area}\div\sqrt{3})}\times 2$.
3. Altitude=one-half the side multiplied by $\sqrt{3}$.
4. Radius of inscribed circle=one-third of the altitude.
5. Radius of circumscribed circle=two-thirds of the altitude.
6. All equilateral triangles are similar.
7. Each angle is $\frac{1}{6}$ of a circle of 60°.
8. Side=the radius of the circumscribed circle multiplied by $\sqrt{3}$.

NOTE.—The student should commit the above rules; it will often be the saving of much labor.

III. SCALENE TRIANGLES.

PROBLEM 360.

What is the area of a triangle whose base is 20 ft., and altitude 12 ft.?

Solution.

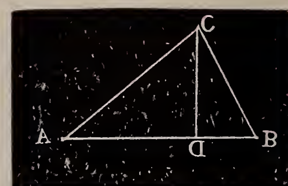

FIG. 48

(1) 20 ft.=base.
(2) 12 ft.=altitude.
(3) $(20\times12)\frac{1}{2}=120$ sq. ft.

$\therefore$ 120 sq. ft.=area of the triangle ABC.

Rule.—*Multiply the base by the altitude and take half the product.*

PROBLEM 361.

Find the area of a triangle whose sides are 13, 14 and 15 ft. respectively.

Solution.

FIG. 49.

(1) 13 ft.+14 ft.+15 ft.=42 ft., sum of the sides.

(2) $\frac{1}{2}$ of 42=21 ft., half the sum of the sides.

(3) 21 ft.—13 ft.=8 ft., 21 ft.—14 ft.=7 ft., 21 ft.—15 ft.=6 ft.

(4) $21\times8\times7\times6=7056$, product of the half sum and the three remainders.

(5) $\sqrt{7056}=84$ sq. ft., area of ABC.

Rule.—*Add the three sides together and take half the sum; from the half sum subtract each side separately; multiply the half sum and the three remainders together and extract the square root of the product.*

NOTE.—The demonstration of this rule can be found in any good geometry.

Suppose we were to find the greatest inscribed square in the above problem.

(1) ABC represents the given triangle, GDEF the inscribed square.

(2) AB$=b$, CH$=h$, the side of the inscribed square by x; then will CK$=h-x$.

(3) But there may be three different squares inscribed in any scalene triangle not right-angled. Let AB=15, CB=14 and AC=13, or c, b, a. Also, let $\frac{1}{2}(a+b+c)=s$.

(4) By formula, CH, or $h=\frac{2}{c}\sqrt{s(s-c)(s-b(s-a)}=11\frac{1}{5}$.

(5) Then the perpendiculars upon CB=12, and AC=$12\frac{12}{13}$.

(6) By similar triangles, AB : GF :: CH : CK; that is, $c : x :: h : h-x$.

(7) $x=\frac{ch}{c+h}$, or $6\frac{54}{131}$ in. described upon AB; also $6\frac{6}{13}$ in. on CB, and $6\frac{162}{337}$ in. on AC.

(8) ∴ $6\frac{54}{131}$ in., $6\frac{6}{13}$ in., and $6\frac{162}{337}$ in. are the sides of the inscribed squares, and it is proven to be true that in any acute-angled triangle the inscribed square described on the least side of the triangle will be the greatest.

(9) Hence, $6\frac{162}{337}$ in. is the side of the square described on the least side of the triangle, which will be the greatest.

NOTE.—This problem was prepared by the author for the *Teachers' Review*.

PROBLEM 362.

The area of a triangle is 84 sq. ft.; two of the sides are 13 and 15: find the other side.

Solution.

(1) From Fig. 48, AB=15 ft., CB=13 ft.

(2) CD=84×2÷15=11.2 ft.

(3) CDB is a right triangle, with its hypothenuse 13, and altitude 11.2 ft.

(4) BD$=\sqrt{(13)^2-(11.2)^2}=6.6$ ft.

(5) AD=15—6.6=8.4 ft., the base of the right triangle ADC.

(6) CA$=\sqrt{(11.2)^2-(8.4)^2}=14$ ft., the third side.

PROBLEM 363.

In a triangle whose sides are 13, 14, 15, required the length of each line from the angles to a common point, which shall divide the triangle into three equal parts.

Solution.

(1) Let ABC be the triangle.

(2) We know that the area of the triangle is 84; then each part is 28.

(3) The common point is at the intersection of lines drawn parallel to the sides and one-third the distance from the side opposite angle.

(4) AC=13, CB=15, and AB=14.

(5) EO=(28×2)÷14=4.

(6) FO=(28×2)÷13=$\frac{56}{13}$.

(7) OD=(28×2)÷15=$\frac{56}{15}$.

(8) AH=$\frac{14}{3}$. AX, or OH =$\frac{13}{3}$; ∴ HE= $\sqrt{(\overline{OH}^2-\overline{OE}^2)}=\frac{5}{3}$.

(9) AE=$\frac{14}{3}+\frac{5}{3}=\frac{19}{3}$; AO= $\sqrt{(\overline{AE}^2+\overline{OE}^2)}=$ $\frac{1}{3}\sqrt{505}=7.49073+$.

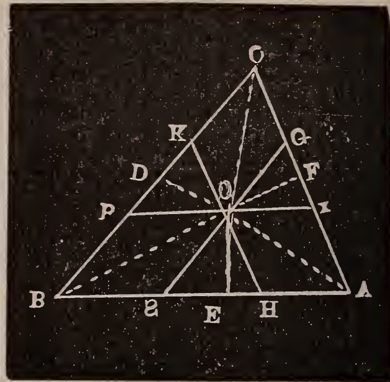

FIG. 50.

(10) BC=$\sqrt{(\overline{BE}^2+\overline{OE}^2)}=$ $\frac{1}{3}\sqrt{673}=8.6474+$.

(11) OG=$\frac{1}{3}$ of 15=5. FG=$\sqrt{\overline{OG}^2-\overline{OF}^2}=\frac{33}{13}$.

(12) CF=$\frac{33}{13}+\frac{18}{3}=\frac{268}{39}$.

(13) CO=$\sqrt{[(\frac{268}{39})^2+(\frac{56}{13})^2]}=\frac{1}{3}\sqrt{592}=8.11035+$.

PROBLEM 364.

In a triangular garden whose sides are a=300 ft., b=250 ft., and c=200 ft., is a spring equally distant from each corner: how far is it from each corner?

Solution.

(1) Let BAC be the garden.

(2) CA=300 ft., BC=250 ft., AB=200 ft., and O represents the position of the spring.

(3) BO, OA and OC are radii of the circumscribing circle and the required distance.

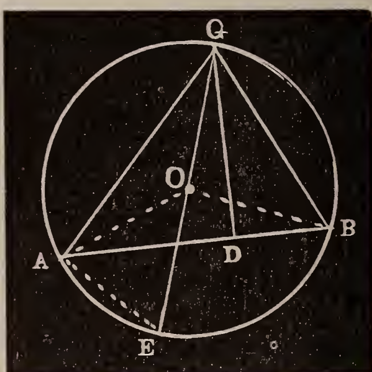

FIG. 51.

(4) Let $\frac{a+b+c}{2}=s$; then, by formula, CD= $\frac{2}{c}\sqrt{s(s-a)(s-b)(s-c)}=$ 248+.

(5) Now as the triangles CBD, CAE are similar, we have CB : CE :: CD : CA.

(6) Substituting the values of CB, CD, CA in the above proportion, we find CE, the diameter of the circle $=302+$ ft.

$\therefore$ BO, AO and CO $=151+$.

NOTE.—This problem was taken from the *Teachers' Review.* The solution there is different from the one above.

PROBLEM 365.

A mathematical teacher amuses six of his best students by laying off a triangular space on the level school yard, measuring exactly 231 ft., 250 ft. and 289 ft. He takes his position in the center, three boys occupy positions on the vertices and three girls on the sides equidistant from the teacher. The girls figure the distance they are from the teacher, and the boys find out how far each is from the teacher and the two nearest girls. (*Hancock Co. test*)

Solution.

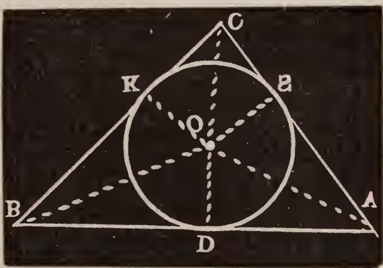

FIG. 52.

(1) Let ABC be the triangle, and O the position of the teacher, A, B, C the positions of the boys, and S, D, K that of the girls.

(2) Let AB $=c$, CB $=a$ and CA $=b$, and let S $=\frac{1}{2}(a+b+c)$.

(3) The area of ABC $=\sqrt{s(s-a)(s-b)(s-c)}$ $=27720$ sq. ft. The perimeter of the triangle $=a+b+c=770$ ft.

(4) Dividing twice the area of the triangle by the perimeter gives the radius of the inscribed circle, or the distance the girls are from the teacher.

(5) $\therefore (2770\times2)\div770=72$ ft., the radius of the inscribed circle.

(6) CA $=231$ ft., AB $=250$ ft. and CB $=289$ ft.

(7) Let $x=$ CS and CK. Then AC$-x=$ SA and CB$-x=$ KB.

(8) Now, AS, or AC$-x=$ AD, and KC, or CB$-x=$ DB.

(9) We have 231 ft.$-x+$289 ft.$-x=$ 250 ft., or $x=135$ ft.

(10) AS $=$ 231 ft.$-$135 ft. $=$ 96 ft., and 289 ft.$-$135 ft. $=$ 154 ft.

(11) CO $=\sqrt{72^2+135^2}=153$ ft. AO $=\sqrt{72^2+96^2}=120$ ft., and OB $=\sqrt{72^2+170^2}=170$ ft.

NOTE.—This problem was first published by J. S. Royer, the publisher of the *School Visitor*, and was solved by the author and others.

SCALENE TRIANGLES WHOSE AREAS ARE INTEGRAL.

4	13	15	13	40	45	20	37	51	39	41	50
13	14	15	15	34	35	25	39	56	39	85	92
7	15	20	15	37	44	25	52	63	40	51	77
11	13	20	17	39	44	25	51	52	41	51	58
10	17	21	25	29	36	25	74	77	41	84	85
12	17	25	25	39	40	26	51	55	48	85	91
13	20	21	29	35	48	29	52	69	50	69	73
17	25	26	39	41	50	34	65	93	51	52	53
17	25	28	13	68	75	35	53	66	52	73	75
13	37	40	15	41	52	36	61	65	43	61	68
			17	55	60	37	91	66			

IV. CIRCLE.

PROBLEM 366.

Prove that AB is 3 times ET, or KT is $\frac{1}{3}$ of AO.

Proof.

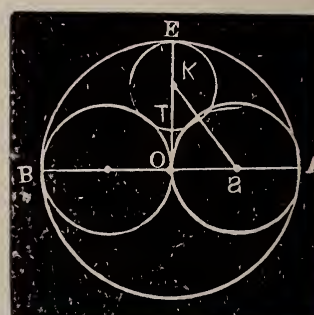

FIG. 53.

(1) Let KT$=r$, SO$=$R, and ET $=x$.
(2) SOK is a right-angled triangle.
(3) Then SK$=$R$+r$, hypothenuse.
(4) OK$=2$R$-r$, the perpendicular.
(5) SO$=$R, the base of the right triangle.
(6) $R^2+(2R-r)^2=(R+r)^2$.
(7) $r=\frac{2}{3}R$, and $R=\frac{3}{2}r$.
(8) Now as $x=2r$, then $x=\frac{4}{3}R$.
Suspose SO, or R$=3$, then $x=\frac{4}{3}$ of 3, or 4.
(9) Then as SO can be applied on AB 4 times, AB$=12$.
(10) Then ET : AB :: 1 : 3.
(11) Also, KT$=2$, and AO$=6$; then KT : AO :: 1 : 3.

PROBLEM 367.

From a circular piece of pasteboard whose diameter measures 60 in., John cut the largest circles that could be measured from it; Mary cut the two largest circles from what remained: what is the area of one of John's and Mary's pieces?

Solution.

(1) From Fig. 53, let AB be the diameter of the given circle,

AO the diameter of one of John's and KT the diameter of one of Mary's pieces.

(2) Then AO=30 in., or SO=15 in.

(3) The area of $AO=15^2\pi=225\pi$ sq. in.

(4) $EO=\frac{1}{3}$ of 60, or 20 in. Then, one of Mary's pieces$=10^2\pi$, or 100π.

PROBLEM 368.

A circular field is inclosed by 12-foot boards: how many acres in the field, if there are as many acres in the field as there are boards in the fence surrounding it, the fence being 4 boards high?

Solution.

FIG. 54.

(1) Let AB be the diameter of the circle.

(2) AO=R, the radius.

(3) $2R\pi$=circumference.

(4) $\frac{8R\pi}{12}$=the number of boards.

(5) One acre contains 43560 sq. ft.

(6) $\therefore \frac{R^2\pi}{43560}$=the number of acres in the field.

(7) Since the number of boards in the circumference equals the number of acres in the field, $\frac{8R\pi}{12}=\frac{R^2\pi}{43560}$.

(8) Clearing of fractions, $12R=348480$, and $R=29040$.

(9) $\frac{8R\pi}{12}=19360\pi$, by substituting the value of R.

$\therefore 19360\pi$=no. of acres.

PROBLEM 369.

A gardener is desirous of laying out a flower garden containing 10 sq. rd. in the form of a crescent, bounded by a quadrant and semi-circle: what is the diameter of the semi-circle?

Solution.

(1) The center of the large circle, O, will be on the circumference of the smaller circle, ADC.

(2) ACDF is a quadrant. The right triangle ADC=10 sq. rd., or AD=the side of a square containing 20 sq. rd.

(3) The lune or crescent CFAM will contain 10 sq. rd., for the quadrant ACDF is equal to the semicircle CAMC.

(4) Now, taking away the common segment X, we have left the triangle ACD= the lune CAMC.

(5) As the triangle is equal to 10 sq. rd., then the crescent contains 10 sq. rd.

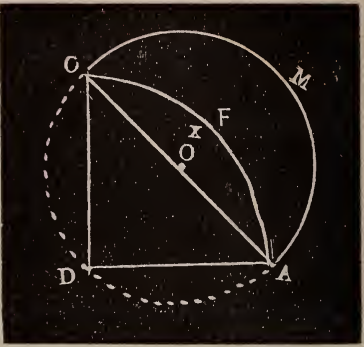

FIG. 55.

(1) Let ABC be a right triangle, and describe a semicircle on each side.

(2) The area of ABC equals the area of the two lunes Z and X taken together.

(3) The area of the semicircle described on AB=the sum of the semicircles described upon AC and CB.

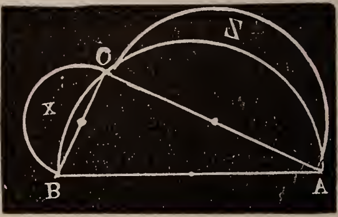

FIG. 56.

(4) Now from each take the two segments cut off by AC and CB, and the triangle ABC remains of the larger semicircle, and two lunes Z and X of the two smaller semicircles. Q. E. D.

PROBLEM 370.

May and Ruth were cutting out tickets for a recital, from a piece of cardboard, and the part remaining was in the form of a segment of a circle, the height and base measuring exactly 2 in. and 12 in. The girls, being mathematically inclined, set to work to find the diameter of the circle from which the segment was cut: what was the diameter?

Solution.

(1) Let ABC be the segment, and CD=2, AD=12 in.

(2) Join CD, and produce CD to E, join EB. Then the two

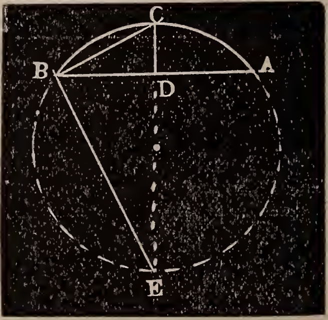

FIG. 57.

triangles CDB and BDE are similar.

(3) We have CD : DB :: DB : DE, or 2 : 6 :: 6 : DE, or DE= 18 in.

(4) DE+DC=EC, or 20 in.

∴ The diameter was 20 in.

PROBLEM 371.

Three equal circles touch each other externally, and thus enclose one acre of ground: what is the diameter in rods of each of these circles?

Solution.

(1) Draw three equal circles to touch each other externally and join the three centers ABC, thus forming an equilateral triangle.

(2) Let R represent the radius of the three equal circles; then it is obvious that each side of the triangle is equal to 2R.

FIG. 58.

(3) This triangle encloses the given area, and three equal sectors. As each sector is a third of two right angles, the three sectors are equal to a semicircle.

(4) The area of a semicircle whose radius is $R=\frac{\pi R^2}{2}$.

(5) The area of the whole triangle is $\frac{\pi R^2}{2}+160$.

(6) But the area of the equilateral triangle is $R^2\sqrt{3}$. (Fig. 38.)

(7) $\therefore R^2\sqrt{3}=\frac{\pi R^2}{2}+160$, or $R^2(2\sqrt{3}-\pi)=320$.

(8) $R^2=\frac{320}{2\sqrt{3}-3.1416}=\frac{320}{0.3225}=992.248$.

(9) $\therefore R=31.48+$ rd.

PROBLEM 372.

Find the length of the longest and shortest chords that can be drawn through a point 6 in. from the center with a radius of 12 inches.

Solution.

(1) The longest possible chord through any point is the diameter drawn through it, and the shortest is one perpendicular to the diameter.

(2) Therefore, AB is the longest, and KP the shortest chord.

(3) FB=FO+OB=18 in. AF=6 in.

(4) The products of the segments AF and FB=the product of KF and FP.

(5) $6\times18=\sqrt{108}=10.39+$ in., or $\frac{1}{2}$ of KP.

(6) $2\times10.39=20.78$ in., the shortest chord.

FIG. 59.

PROBLEM 373.

Two parallel chords in a circle, on opposite sides of the diameter, are each 20 ft. long, and the perpendicular distance between them is 8 ft.: what is the diameter of the circle?

Solution.

(1) Let ABCD be the circle, and DC, AB =20 ft., EF=8 ft., FO=4 ft., and DE=10 ft.

(2) From D draw DO, then OED is a right triangle.

(3) $DO=\sqrt{(10^2+4^2)}=10.77$, the radius of the circle.

(4) $\therefore$ 10.77 ft.$\times2=21.54$ ft., the diameter.

FIG. 60.

PROBLEM 374.

Required the area of the segment, of which the chord is 50 ft., and the height 3 ft.

Solution.

(1) 50 ft.=the base of the segment, 3 ft.=the altitude.

(2) 27 cu. ft.=cube of the height of the segment.

(3) 100 ft.=twice the base. $27\div100=.27$ sq. ft., the quotient of the cube of the height by twice the base.

(4) $3\times50=150$ sq. ft.=product of the height and base.

(5) $\frac{2}{3}$ of 150=100 sq. ft.=$\frac{2}{3}$ of height and base.

(6) 100 sq. ft.+.27 sq. ft.=100.27 sq. ft.

$\therefore$ 100.27 sq. ft.=area of the segment.

Rule.—*Divide the cube of the height by twice the base, and increase the quotient by two-thirds of the height and base.*

PROBLEM 375.

E. F. Wieser cut off a segment of a circular cardboard, which had just $\frac{1}{3}$ of the circumference, and contained 130 sq. in.: what was the diameter of the cardboard?

Solution.

FIG. 61.

(1) CDB$=\frac{1}{3}$ of the circumference.

(2) BCD=the segment, angle COB$=120°$, and angle COD$=60°$.

(3) AO, or CO$=r$, the radius of the circle.

(4) CF$=\frac{1}{2}r\sqrt{3}$, from Fig. 36.

(5) The area of the sector COB$=\frac{1}{3}r^2\pi$, and the area of the triangle COB=CF× FO, or $\frac{1}{4}r^2\sqrt{3}$.

(6) ∴ The area of the segment = CBD$=\frac{1}{3}r^2\pi-\frac{1}{4}r^2\sqrt{3}=130$ sq. in.

(7) We find $r=14.5+$ in.

∴ 29+ in. is the diameter of the cardboard.

PROBLEM 376.

From a point without a circle two tangents are drawn, forming with each other an angle of 60°, and the length of each tangent is 20 in.: what is the diameter?

Solution.

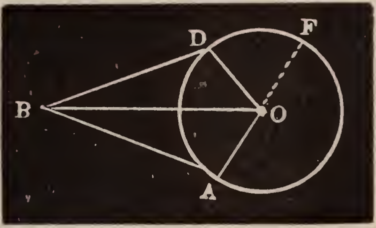

FIG. 62.

(1) Let DB and AB, the tangents=20 ft., and the angle ABD$=60°$.

(2) The angle A is a right triangle because it is formed by a tangent and radius.

(3) OB bisects the angle DBA; hence, angle ABO$=30°$, and angle BOA$=60°$.

(4) ∴ The triangle BAO=half of an equilateral triangle whose sides are equal to OB.

(5) OA$=\frac{1}{2}$OB. AB$=$OA$\sqrt{3}$.

(6) ∴ AO$\sqrt{3}=20$, or OA$=\frac{20}{\sqrt{3}}=11.548+$ in.

(7) $11.548\times 2=23.096$ in., the diameter.

PROBLEM 377.

A horse is tethered from one corner of a square 20-acre field: how long must the rope be so that it can graze over 20 acres outside the field?

Solution.

(1) Let AOCB be the field, and O the point at which the horse is tied.

(2) Let R=OH, the radius or the length of the rope by which the horse is tied.

(3) DOF=$\frac{1}{4}$ of a circle, or a quadrant, but the horse grazes over $\frac{3}{4}$ of a circle, or FED.

(4) As $\frac{3}{4}$ of a circle=20 A., then there must be $6\frac{2}{3}$ A. in the quadrant DOF.

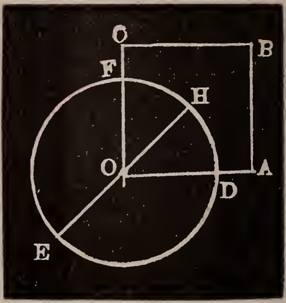

FIG. 63.

(5) $26\frac{2}{3}$ A.=area of the circle.

(6) $R^2\pi$=area of the circle.

(7) ∴ $R^2\pi=26\frac{2}{3}$ A., or $4266\frac{2}{3}$ sq. rd.

(8) R=36.85 rd., the length of the rope.

PROBLEM 378.

A wheel 8 ft. in diameter is run by means of a belt from a wheel 1 ft. in radius; the distance between their centers is 12 ft., and the belt is crossed between them, causing them to revolve in opposite directions: what is the length of the belt used?

Solution.

(1) OM and O′M′ are parallel; OMP and O′M′P are similar triangles.

(2) Hence, we have the proportion, O′M′ : OM :: O′P : OP.

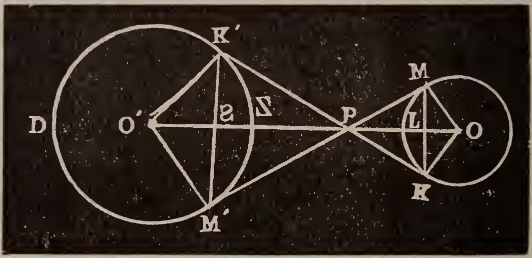

FIG. 64.

(3) Now, PO′=x, OP=12—x.

(4) 4 : 1 :: x : 12—x, whence x=9.6 ft.

(5) OP=12 ft.—9.6 ft.=2.4 ft.

(6) $PK'=\sqrt{\overline{PO'}^2-\overline{K'O'}^2}$=8.72 ft.

(7) $KP=\sqrt{\overline{OP}^2-\overline{OK}^2}$=2.18 ft.

(8) KK′=KP+PK′=10.9 ft.

(9) Hence, MM′+KK′=21.8 ft.=length of the belt not on the wheels.

(10) $K'S=(PK'\times K'O')\div PO'=3.63$ ft.
(11) $K'M'=3.63\times 2=7.26$ ft.
(12) $SO'=\sqrt{\overline{K'O'}^2-\overline{K'S}^2}=1.68$ ft.
(13) Area of segment $K'ZM'=(\overline{ZS}^3\div$by twice $K'M')+\frac{2}{3}(ZS\times K'M')=12.0886$ sq. ft.
(14) Area of triangle $M'O'K'=\frac{1}{2}(K'M'\times SO')=6.0984$ sq. ft.
(15) 12.0886 sq. ft.+6.0984 sq. ft.=18.186 sq. ft., area of sector $O'K'ZM'$.
(16) $4^2\pi$=area of large wheel, 8π=circumference of the large wheel.
(17) The arc $K'ZM'=\pi 4^2 : 18.186 :: 8\pi :$ (9 05 ft.)
(18) 8π—9 05=16 08 ft=$K'DM'$, part of circumference covered by the belt.
(19) Area of the sector $MLKO=4^2 : 1^2 :: 18.186 :$ (1.13 sq.ft.)
(20) 2π=circumference of small wheel.
(21) The arc MLK=3.1416 : 1.13 :: 6.2832 : (2.26 ft.)
(22) 6.28 ft., circumference of small wheel—2.26, arc MK= 4.02 ft., part of small wheel covered by belt.
(23) 4.02 ft.+16.08 ft.+21.18 ft.=41.18 ft., required length.

(*Eaton.*)

If the belt is not crossed, what is its length?

Solution.

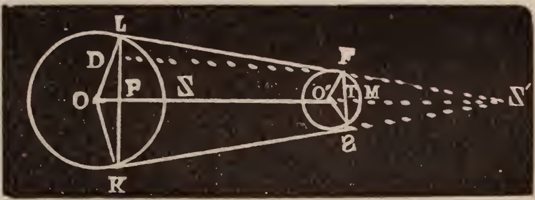

FIG. 65.

(1) Let F, L, S and K be the points where the belt touches the wheels.
(2) $LZ'O$ and LFD are similar triangles.
(3) LD : LO :: FD : Z'O; hence 3 : 4 :: 12 : (16=OZ'.)
(4) $Z'O'=Z'O-O'O=4$ ft $Z'F=\sqrt{\overline{Z'O'}^2-\overline{FO'}^2}=3.87$ ft.
(5) $FL=\sqrt{\overline{FD}^2-\overline{LD}^2}=\sqrt{12^2-3^2}=11.618$ ft.
(6) 11.618 ft.×2=23.236 ft., length of belt not touching the wheels.
(7) Area of the triangle $OLZ'=(Z'L\times LO)\frac{1}{2}=30.98$ sq. ft.
(8) $LP=(30.98\times 2)\div 16=3.872$ ft., LK=7.744 ft.
(9) $PO=\sqrt{\overline{LO}^2-\overline{LP}^2}=1.003$ ft.
(10) PZ=ZO—PO=2.997 ft.
(11) Area of the segment $LZK=(\overline{ZP}^3\div$by twice $LK)+$

$\frac{2}{3}$(ZP×LK)=16.488 sq. ft.

(12) Area of the triangle LKO=$\frac{1}{2}$(LK×PO)=3.883 sq. ft.

(13) Hence, the area of the sector OLKZ=16.488 sq. ft.+3.883 sq. ft.=20.371 sq. ft.

(14) $4^2\pi$: 20.371 :: 8π : (10.85 ft.)=arc LZK, or part that the belt does not touch of the large wheel.

(15) 8π—10.85 ft.=14.28 ft., part of large wheel covered by the belt.

(16) $4^2 : 1^2$ :: 20.371 : 1.273 sq. ft., area of sector FMS, or part of small wheel covered by belt.

(17) $.5^2\pi$: 1π :: 1 273 : .509 ft., length of belt on small wheel.

(18) .509 ft.+23.236 ft.+14.28 ft.=38.025 ft., required length.

PROBLEM 379.

A circular lot 15 rd. in diameter is to have three circular grass beds just touching each other and the larger boundary: what must be the distance between their centers, and how much ground is left in the triangular space about the center? (*Ray's Higher Arith.*)

Solution

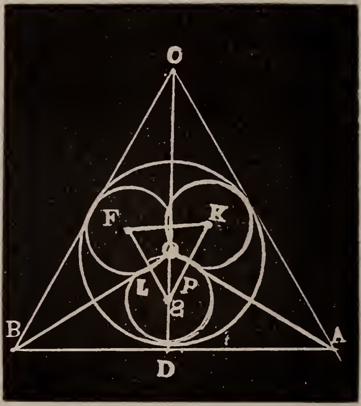

FIG. 66.

(1) OD=7$\frac{1}{2}$ rd.=R, the radius of the lot.

(2) AO=15 rd.

(3) AB=AO$\sqrt{3}$=25.98075 rd. CD=22$\frac{1}{2}$ rd.

(4) Area of ABC=$\frac{1}{2}$(AB×CD)=292.28343+ sq. rd.

(5) The area of the three triangles AOB, COB and COA=$\frac{1}{3}$ of 292.28343 sq. rd.=97.42731 sq. rd.

(6) Let SD, or SL=r, the radius of the small circle.

(7) r=(AOB×2)÷ AB+OB+AO, or twice the area of AOB divided by the perimeter=3.4807.

(8) ∴ SF=6.9614 rd. The area of sector PSL=$\frac{1}{6}$ of the small circle, because the angle PSL=60°.

(9) $\frac{1}{2}r^2\pi$=3 times $\frac{1}{6}r^2\pi$=area of the three sectors in the triangle KSF.

(10) $r^2\sqrt{3}-\frac{1}{2}r^2\pi=r^2(\sqrt{3}-\frac{1}{2}\pi)$=1.9537 sq.rd., the area of the space enclosed.

∴ SF=6.9614 rd., and 1.9537 sq. rd.=area of the curvilinear space.

PROBLEM 380.

If 2 ft. be the radius of a circle, how far from the center must a chord be drawn parallell to divide the semicircle into two equal parts?

Solution.

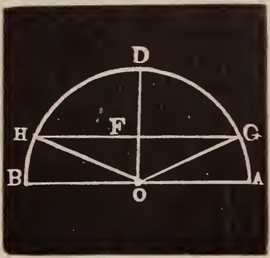

FIG. 67.

(1) Let ADB be the semicircle, AB the diameter, GH the line that divides the semicircle into two equal parts.

(2) On OD, perpendicular to AB, let $OF=x$, the distance required, and θ=angle HOB. r=OB, or radius.

(3) Then $x=r\sin\theta$, $FH=r\cos\theta$.

(4) Area of sector $HOB=\frac{1}{2}r^2\theta$.

(5) Area of sector $DOH=\frac{1}{2}r^2(\frac{1}{2}\pi-\theta)$.

(6) Area of triangle $FOH=\frac{1}{2}r^2\sin\theta\cos\theta$.

(7) We then have $r^2\theta+r^2\sin\theta\cos\theta=r^2(\frac{1}{2}\pi-\theta)-r^2\sin\theta\cos\theta$.

(8) Transposing and reducing we have $\sin2\theta+2\theta=\frac{\pi}{2}$.

(9) We will find the value of θ by double position, and it is found as follows:

(10) Let $\theta=20°=\frac{\pi}{9}$ for first trial.

(11) $\therefore \sin40°+\frac{2\pi}{9}=\frac{\pi}{2}$, or $\sin40°=\frac{5\pi}{18}=.87267$, but $\sin40°=.64279$.

(12) $\therefore .64279=.87267$.

(13) $.87267-.64279=.22988$ . . . (1).

(14) Let $\theta=22°=\frac{11\pi}{90}$.

(15) $\therefore \text{Sin}44+\frac{11\pi}{45}=\frac{\pi}{2}$, or $\sin44°=\frac{23\pi}{90}=.80285$.

(16) $\therefore .69466=.80285$.

(17) $.80285-.69466=.10819$ . . . (2).

(18) $(1)-(2)=.12169$, $22°-20°=2°$.

(19) $\therefore \phi : 2° :: .22988 : .12169$.

(20) $\therefore \phi=3.77812°=3°\ 46'\ 47''$.

(21) $\text{Sin}47°\ 33'\ 34''+47°\ 33'\ 34''=90°$.

(22) $\text{Sin}47°33'34''=42°26'26''=\frac{76393\pi}{324000}=.74073$.

(23) $\therefore .73798=.74073$.

(24) .74073—.73798=.00275 . . . (3).
(25) 23° 46′ 47″—22°=1° 46′ 47″=.6407″.
(26) (2)—(3)=.10544.
(27) ϕ : .6407″ :: .10819 : .10544.
(28) ∴ ϕ=.6574″=1°49′34″.
(29) ∴θ=23°49′34″, and sin47°39′8″=42°20′52″=$\frac{38113\pi}{162000}$=.73911.
(30) ∴ .73907=.73911.
(31) .73911—.73907=.00004.
(32) 23° 49′ 34″—23° 46′ 47″=0° 2′ 47″=167″.
(33) .00275—.00004=.00271.
(34) ∴ ϕ_2 : 167″ :: .00275 : 00271.
(35) ∴ ϕ_2=169.5″=2′ 49.5″.
(36) θ=23° 46′ 47″+2′ 49.5″.
(37) θ=23° 49′ 36.5″.
(38) Sinθ=.40397.
(39) ∴ x=.40397r or .80794 ft.

∴ .80794 ft., is the required distance.

PROBLEM 381.

What is the length of a chord cutting off $\frac{1}{3}$ part of a circle, whose diameter is 50 ft.?

Solution.

(1) From Fig. 67, the segment GDH is twice the segment ABHG, or $2r^2\theta+2r^2\sin\theta\cos\theta=r^2(\frac{1}{2}\pi-\theta)-r^2\sin\theta\cos\theta$.
(2) Transposing and reducing, we have $\sin 2\theta+2\theta=\frac{\pi}{3}$.
(3) Find the value of θ by double position as follows:
(4) Let $\theta=14°=\frac{7\pi}{90}$ for first trial.
(5) ∴ $\text{Sin}28°+\frac{7\pi}{45}=\frac{\pi}{3}$, or $\sin 28°=\frac{8\pi}{45}$=.55851.
(6) But sin28°=.46947.
(7) ∴ .46947=.55851.
(8) .55851—.46947=.08904 . . . (1).
(9) Let $15°=\frac{\pi}{12}$, and $30°+\frac{\pi}{6}=\frac{\pi}{3}$, or $30°=\frac{\pi}{6}$=.5235.
(10) ∴ .5000=.52359, and .52359—.5000=.02359 . . . (2).
(11) (1)—(2)=.06545, 15°—14°=1°21′.
(12) ϕ : 1° :: .08904 : .06545.
(13) ∴ ϕ=1.3604°=1° 21′ 38″.

(14) $14°+1° 21' 38''=15° 21' 38''$.

(15) $\therefore \theta=15° 21' 38''$.

(16) Let angle DOB$=\lambda=90°$.

(17) $\therefore \lambda-\theta=84° 38' 13''$=angle DOH.

(18) SinDOH=.96426+.

(19) Multiply the diameter of the circle by .96426+, and the result will be the length of the chord.

(20) $\therefore .96426\times 50=48.21300$ ft.=GH.

PROBLEM 382.

A circular field 80 rd. in diameter, is divided into three equal parts by two parallel fences: find length of fences, and the distance each is from the center.

Solution.

FIG. 68.

(1) Let AB and PF be the two fences, and KO, OE the distance from the center.

(2) OB$=r$, angle BOT$=\theta$.

(3) This is the same as the preceding equation for θ, i. e., $\sin 2\theta+2\theta=\frac{\pi}{3}$, or $\theta=15° 21' 38''$.

(4) Angle KOT$=\lambda=90°$.

(5) $\therefore \lambda-\theta=74° 38' 13''$=KOB$=\psi$.

(6) $\sin\psi=.96426+$.

(7) $\therefore$ AB=DΓ the diameter$\times .96426+=77.1408$ rd.

(8) KO$=r\sin\theta=10.595$ rd.

$\therefore$ 77.1408 rd. is the length of the fence, and 10.595 rd. is the distance from the center.

PROBLEM 383.

A circle containing one acre is cut by another whose center is on the circumference of the given circle, and the area common to both is ½ acre: find the radius of the cutting circle.

Solution.

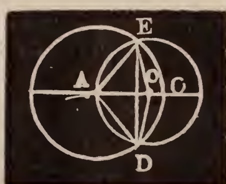

FIG. 69.

(1) Designate by O the center of the circle, and by A the center of the cutting circle.

(2) EA=R, the radius.

(3) With A as a center and radius R, draw the arc ECD, cutting the circumference at E and D.

(4) Join OD and AE.

(5) Put $r=\text{OA}=4\left(\frac{10}{\pi}\right)=7.136$ rd., radius of the 1st circle.

(6) Put angle EAO=angle OEA=θ.

(7) Then angle EOA=$\pi-2\theta$.

(8) Sector EOA=$\frac{\pi r^2(\pi-2\theta)}{2\pi}$, sector EAC=$\frac{\pi R^2\theta}{2\pi}$, triangle EAO=$\frac{1}{2}Rr\sin\theta$.

(9) Then we have $\frac{\pi r^2(\pi-2\theta)}{2\pi}+\frac{\pi R^2\theta}{2\pi}-\frac{1}{2}Rr\sin\theta=\frac{\pi r^2}{4}$.

(10) But $R=2r\cos\theta$.

(11) Substituting the value of R, we have $\pi r^2(\pi-2\theta)+4\pi\theta r^2\cos^2\theta-\pi r^2\sin2\theta=\frac{1}{2}\pi^2r^2$, or $2\theta\cos2\theta-\sin2\theta=\frac{1}{2}\pi$, or $\sin2\theta-2\theta\cos\theta=\frac{1}{2}\pi$ . . . (1).

(12) Let $\phi=2\theta$, and the equation becomes, $\sin\phi-\phi\cos\phi=\frac{1}{2}\pi$ . . . (2).

(13) It is obvious that ϕ cannot be less than $\frac{1}{2}\pi$. Let us assume $\phi=\frac{1}{2}\pi$, and substitute this in (2).

(14) Then $\sin\phi=1$. $\frac{1}{2}\pi\cos\phi=0$. Now, 1—0=1, which is less than $\frac{1}{2}\pi$.

(15) Assume $\phi=135°=\frac{3}{4}\pi$, and $\sin\phi=.70711$

$$-\tfrac{3}{4}\cos\phi=1\ 66609$$

$$\overline{2.37320}$$, which is greater than $\frac{1}{2}\pi$.

(16) Then,

$$\begin{array}{lll} 2.37320 & 135° & 1.57080=\tfrac{1}{2}\pi. \\ 1. & 90° & 1. \\ \hline 1.37320 : & 45° :: & .57080 : 18.7° \end{array}$$

the correction to be added to the 1st assumed value of $\phi=\frac{1}{2}\pi$, giving 108.7°.

(17) Let us now assume 108° and 110° as the values of ϕ.

(18) For $\phi=108°=\frac{3}{5}\pi$, we have $\sin\phi=$ .95106

$$-\tfrac{3}{5}\pi\cos\phi=\ .58249$$

$$\overline{1.53355}$$, less than $\frac{1}{2}\pi$.

(19) For $\phi=110°=\frac{11}{18}\pi$, we have $\sin\phi=$ .93969

$$-\tfrac{11}{18}\pi\cos\phi=\ .65663$$

$$\overline{1.59632}$$, greater than $\frac{1}{2}\pi$.

(20) Then, as before,

$$\begin{array}{lll} 1.59632 & 110° & 1.57080 \\ 1.53355 & 108° & 1.53355 \\ \hline .06277 : & 2° :: & .03725 : 1°\ 11' \end{array}$$

which added to 108°=109° 11'.

(21) Now, take $\phi=109°\ 11'$ and $109°\ 12'$.

(22) For $\phi=109°\ 11'=\frac{6551}{10800}\pi$, we have $\sin\phi=\ .94447$

$-\frac{6551}{10800}\pi\cos\phi=\ .62616$

1.57063, less than $\frac{1}{2}\pi$.

(23) For $\phi=109°\ 12'=\frac{91}{150}\pi$, we have $\sin\phi=\ .94438$

$-\frac{91}{150}\pi\cos\phi=\ .62679$

1.57117, greater than $\frac{1}{2}\pi$.

(24) As before

$$\begin{array}{r} 1.57117 \\ 1.57063 \\ \hline .00054 \end{array} : \begin{array}{r} 109°\ 12' \\ 109°\ 11' \\ \hline 1' \end{array} :: \begin{array}{r} 1.57080 \\ 1.57063 \\ \hline .00017 \end{array} : 18.8''$$

which added to $109°\ 11'$ gives $\phi=109°\ 11'\ 18.8''$.

(25) Dividing by 2, $\theta=54°\ 35'\ 39.4''$.

(26) $\therefore\ R=2r\cos\theta=8.26926$ rd.

PROBLEM 384.

A horse is tied to a stake in the circumference of a 10-acre field: how long must the line be to allow him to graze over just one acre inside the field?

Solution.

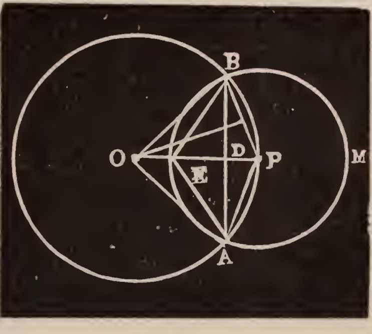

FIG. 70.

(1) Let P be the point to which the horse is tied, B and A the points in the circumference of the field to which the horse can graze, and O the center of the field.

(2) Let $BO=r=\sqrt{1600\div\pi}=$ the radius of the field, and angle BPO = angle OBP$=\theta$.

(3) Draw $OD=r\sin\theta$ perpendicular to $BP=2PD=2r\cos\theta$, the length of the line.

(4) Now BOP is isosceles, angle PBO=angle BPO, angle $BOP=(\pi-2\theta)$, and BP=R.

(5) Sector $BPE=\dfrac{\pi R^2(\pi-2\theta)}{2\pi}$, sector $BOP=\dfrac{\pi r^2\theta}{2\pi}$, $\triangle OAP=\frac{1}{2}Rr\sin\theta$.

(6) Then we have $\dfrac{\pi R^2(\pi-2\theta)}{2\pi}+\dfrac{\pi r^2}{2\pi}-\frac{1}{2}Rr\sin\theta=\dfrac{\pi R^2}{20}$.

(7) But $r=2R\cos\theta$.

(8) Substituting the value of r, we have $\pi R^2(\pi-2\theta)+4\pi\theta R^2\cos^2\theta-\pi R^2\sin 2\theta=\frac{1}{10}\pi R^2$, or $\sin 2\theta-2\theta\cos 2\theta=\frac{9}{10}\pi$.

(9) Solving this equation by the method of double position, we find $\rho=76^\circ\,21'\,45''$.

(10) $BP=2r\cos\rho=10.6418$ rd., the length of the line.

PROBLEM 385.

Find the area of the largest five-pointed star that can be cut from a circular piece of pasteboard 20 inches in diameter.

Solution.

FIG. 71.

(1) Draw the circle, and dividing the circumference into five equal parts, join the alternate points of division, as in the figure.

(2) Then ABCDE will be the star inscribed in the circle whose center is O.

(3) The angle $ECA=36^\circ$, being measured by half the arc EA.

(4) Since OC bisects the angle, then angle $FCO=18^\circ$, and and angle $FOC=36^\circ$.

(5) Hence, angle $CFO=180^\circ-(36^\circ+18^\circ)=126^\circ$.

(6) The sine of the angle opposite the given side is to the sine of the angle opposite the required side, as the given side is to the required side.

(7) From this we have $\sin 126^\circ : \sin 18^\circ :: 10 : (FO=3.832.)$

(8) Now, it is obvious that the triangle FOC is $\frac{1}{10}$ of the star.

(9) Multiplying half the product of the two sides by the sine of the included angle, gives the area of the triangle FOC, or $\triangle FOC=\frac{1}{2}(10\times 3.832)\times\sin 36^\circ=11.2619$ sq. in.

(10) Area of the star is $10\times 11.2619=112.619$ sq. in.

NOTE.—This figure was supposed to possess mysterious properties, and was called "Health." It was used as a badge by the secret society founded by Pythagoras about 550 B. C., for the pursuit of mathematics.

PROBLEM 386.

A deer is tied to a stake in the circumference of a 10-acre field: how long must the line be to allow it to graze over just one acre outside the field?

Solution.

(1) From Fig. 70, let the larger circle be the field, P the point and PB the tethering line outside the field.

(2) Let $OB=r$, and λ=angle BPO.

(3) Then $BP=2r\cos\lambda$, arc $BMA=2\pi-2\lambda$, and its length $=2r\cos\lambda(2\pi-2\lambda)$.

(4) $BOP=\pi-2\lambda$; then the area of $BOP=\frac{1}{2}r^2\sin(\pi-2\lambda)=\frac{1}{2}r^2\sin2\lambda$.

(5) Area of $BOAP=r^2\sin2\lambda$; arc $BP=\pi-2\lambda$.

(6) Area of the sector $BOP=\frac{1}{2}r^2(\pi-2\lambda)$ and sector $O-BPA=r^2(\pi-2\lambda)$.

(7) $\therefore$ The area of the portion of the circle outside is sector $P-BMA+2\triangle BOP-$ sector $O-BPA=4r^2\cos^2\lambda(\pi-\lambda)+r^2\sin2\lambda-r^2(\pi-2\lambda)$, or $\cos2\lambda(2\pi-2\lambda)+\sin2\lambda=-\frac{9}{10}\pi$.

(8) Then, by position, we obtain the value of λ.

(9) Let $\lambda=70°$.

(10) $\therefore 2(\pi-\frac{5}{12}\pi)\cos150°+\sin150°=-\frac{9}{10}\pi$.

(11) $-.86603\times\frac{7}{6}\pi+\frac{1}{2}=-\frac{9}{10}\pi$.

(12) $\therefore$ .5=.34677.

(13) Let $\lambda=77°$.

(14) $\therefore -1.028615\pi+.43837=-\frac{9}{10}\pi$.

(15) $\therefore$ .43837=.40406.

(16) $\left.\begin{array}{l}.5-.34677=.15323\\ .43837-.40406\end{array}\right\}$ .15323—.0341=.11892.

(17) $\therefore 2° : x° :: .11892 : .15323$.

(18) $\therefore x°=2.2407°=2°\ 14'\ 26\frac{1}{2}''$.

(19) $\therefore \lambda=77°\ 14'\ 26\frac{1}{2}''$.

(20) $\therefore -1.030385\pi+.43080=.9\pi$.

(21) $\therefore$ .43080=.40962.

(22) .43080—.40962=.02118.

(23) .03431—.02118=.01313.

(24) $.2407 : x :: .01313 : .03431$.

(25) $\therefore x=.6898$.

(26) $\therefore 77.61797°=77°\ 37'\ 44\ 3''$.

(27) $\therefore -1.033037\pi+.41854=-.9\pi$.

(28) $\therefore$ .41854=.41795.

(29) .41954—.41795=.00059.

(30) .02118—.00059=.02059.

(31) 77.62898—77 2407=.38828.

(32) $.38828 : x :: .02059 : .02118$.

(33) $\therefore x=.39944$.

(34) $\lambda=77.64014°=77°\ 38'\ 24.5''$.

(35) $PB=2r\cos\lambda=9.66+$ rd.

PROBLEM 387.

Three men own a grindstone 2 ft. 8 in. in diameter: how many inches must each grind off to get an equal share, allowing 6 in. waste for the aperture? (*R. H. A., p. 406, prob. 84.*)

Solution.

(1) Let AT be separated into three equal parts, AH, HD and DT.

(2) Let perpendiculars at the points of division be drawn to intersect the circumference constructed on AB as a diameter, and these points F and E be joined with B; they will be radii that will determine rings of equal area.

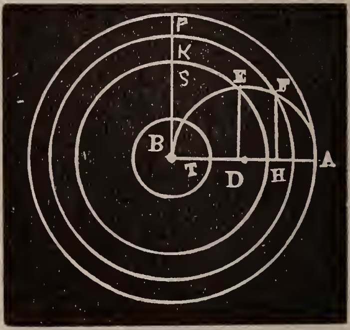

FIG. 72.

(3) BT is the radius of the aperture, or 3 in., which subtracted from 16 in. leaves AT=13 in.

(4) The area of the stone$=16^2\pi=256\pi$.

(5) The area of the aperture is $3^2\pi=9\pi$.

(6) The area to be ground off is $256\pi-9\pi=247\pi$.

(7) Each man's share will be $\frac{1}{3}$ of $247\pi=82\frac{1}{3}\pi$.

(8) $256\pi-82\frac{1}{3}\pi=173\frac{2}{3}\pi$, area of the stone after the first grinds.

(9) $173\frac{2}{3}\pi-82\frac{1}{3}\pi=91\frac{1}{3}\pi$, area of the stone after the second grinds.

(10) Dividing the area of a circle by π and extracting the square root give the radius.

(11) $AB=\sqrt{(256\pi\div\pi)}=16$ in.

(12) $HB=\sqrt{(73\frac{1}{3}\pi\div\pi)}=13.178$ in.

(13) $SB=\sqrt{(173\frac{1}{3}\pi\div\pi)}=9.557$ in.

(14) ∴ The first man grinds off $16-13.178=2.822$ in.; the second $13.178-9.557=3.621$ in., and the third $9.557-3=6.557$ in.

PROBLEM 388.

Find the radius of the largest circle that can be drawn in a quadrant of a circle, radius 20 in.

Solution.

(1) Bisect the given arc TB in S.

(2) Let fall the perpendicular SF, join O with S, and produce it, making SP=SF.

(3) Join P and F; draw SL parallel to PF, O'L to OT.

(4) Then with the center O' and radius $O'S = r$, describe the circle SLE and it will be the inscribed circle.

(5) Since the triangles OSL and OPF are similar, and OF = OS, or $20 \div \sqrt{2} = 14.142$ in.; then by similar triangles, we have OP : OS :: OF : OL, or 34.142 : 20 :: 14.142 : OL = 8.284 in. $\therefore r = 8.284$ in.

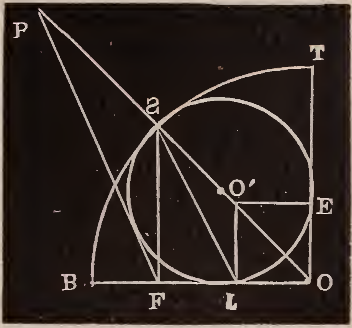

FIG. 73.

PROBLEM 389.

The radius of a circle is 6 in.; through a point 10 in. from the center, tangents are drawn: find the lengths of these tangents, and also the chord joining the points of contact.

Solution.

(1) Let O be the center of the given circle, and P the given point.

(2) On OP as a diameter describe a circumference at B and C.

(3) Draw BO and CO.

(4) Angles OBP and OCP are right angles, being inscribed in a semicircle.

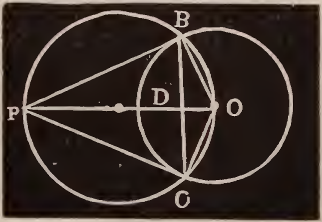

FIG. 74.

(5) OP is 10 in.; OB, 6 in.

(6) BP or CP, $\sqrt{(10^2 - 6^2)} = 8$ in.

(7) Area of OBP is $(8 \times 6) \div 2 = 24$ sq. in., and its altitude BD $= (24 \times 2) \div 10 = 48$ in.

(8) Hence, the chord is 96 in., and one of the tangents 8 in.

PROBLEM 390.

A horse is tethered to one corner of a barn 40 ft. square, by a rope 100 ft. long: over how much ground can he graze?

Solution.

(1) Let ADCO be the barn, and OF the rope 100 ft. long.

(2) The area of $EnF = \frac{3}{4}$ of $100^2\pi = 23562$ sq. ft.

(3) CE=AF=AS=CP=60 ft.

(4) Area of the two quadrants CEP and $AFS = \frac{1}{2}$ of $60^2\pi = 5654.88$ sq. ft.

(5) The area included in DKGL has been grazed twice, hence its area must be taken from the area already found.

(6) DP=DS=20 ft.

(7) HG=OD=$40\sqrt{2}$, or 56.56.

(8) AH=$\frac{1}{2}$OD=HD=28.28 ft

(9) AG=CG=60 ft.

(10) HG $= \sqrt{(\overline{AG}^2 - \overline{AH}^2)} =$ 52.91 ft.

(11) DG=52.91–28.28=24.63ft.

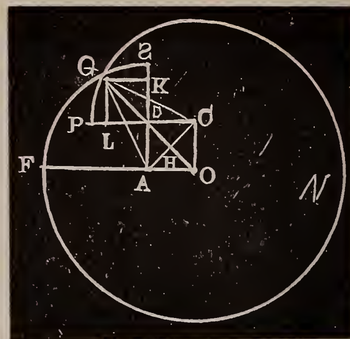

FIG. 75.

(12) $\sqrt{(24.63^2 \div 2)}$=DL=17.41 ft., side of square KL, and its area is 17.41^2=303.108 sq. ft.

(13) The segments KSG and PLG are equal.

(14) KS=LP=20 ft.—17.41 ft.=2.59 ft.

(15) Area of segments is $(\overline{KS}^3 \div KG \times 4) + \frac{2}{3}$ of $(KS \times KG \times 2)$ =60.372 sq. ft.

(16) Area grazed twice is 303.108+60.372=363.48 sq. ft.

(17) ∴ The horse grazes over 23592+5654 88—363.48=28853.4 sq. ft.

PROBLEM 391.

A stake to which a horse is tethered, is 120 ft. from the corner of a barn 60 ft. long, 40 ft. wide, and in line with the long side of the barn: if the rope is 120 ft. long, over how much ground can the horse graze?

Solution.

(1) Let FCEA represent the barn.

(2) O represents the point at which the horse is tethered.

(3) OC=20ft., CE=40ft.

(4) OE $= \sqrt{(40^2 + 20^2)}$ =44.721+ ft.

(5) 44.721 : 20 :: sin90° : sin26° 34′ = angle CEO.

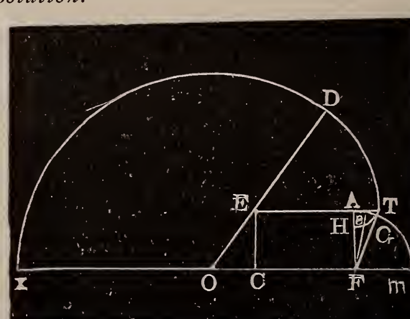

FIG. 76.

(6) The large angle FXDO=270°+26° 34′=$296\frac{17}{30}$°.

(7) 360° : $296\frac{17}{30}$° :: $120^2\pi$: 37267.753 sq. ft., area of FODX.

(8) $(40 \times 20) \div 2$=400 sq. ft., area of triangle COE.

(9) Angle DET=90°—26° 34′=63° 26′.

(10) DE=120 ft.—44.72 ft.=75.28 ft.

(11) 360° : $63\frac{13}{30}$° :: 75.28²π : 3137.07 sq. ft., area of TED.

(12) MF=120 ft.—(60 ft.+20 ft.)=40 ft.

(13) π(40²÷4)=1256.64 sq. ft., area of MFA.

(14) AT=120 ft.—(60 ft.+44.75 ft.)=15.28 ft.

(15) π(15.28² ft.÷4)=182.374 sq. ft., area of ATH.

(16) The area of AGH is common to the two sectors FMA and ATH, hence its area must be subtracted from the two.

(17) Area of GAH=area of sector AGH+(area of sector FGA—area of triangle FGA).

(18) In the triangle FGA, FA=40 rd., and AS=7.64 rd.

(19) 40 : 7.64 :: sin90° : sin11°.

(20) Angle AFS=11°. Angle FAS=90°—11°=79°.

(21) 360° : 79° :: 15.28²π : 160.96 sq. ft., area of sector AGH.

(22) 360° : 22° :: 40²π : 307.61 sq. ft., area of sector FGA.

(23) Then the area of the triangle FGA=299.95 sq. ft.

(24) 160.96+(307.61—299.95)=168.62 sq. ft., common part of both sectors.

(25) 37267+400+3137.07+1256.64+182.374—168.62=42075.22 sq. ft., part grazed over by the horse.

PROBLEM 392.

The distance over a mountain is 2 miles; the horizontal distance, through, is 1 mile: how many more pickets would be required to build a fence over than through, the pickets to be set perpendicular to the horizon in each case, supposing 1200 pickets are required to build the fence through the mountain?

Solution.

(1) It will take exactly the same number. The accompanying cut will illustrate plainly.

(2) AB=1 mi.; ACB=2 mi.; the perpendicular lines represent the pickets.

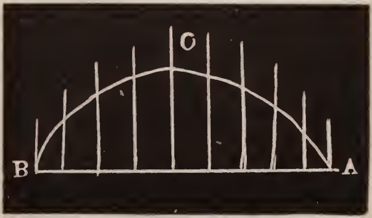

FIG. 77.

V. ELLIPSE.

93. An **Ellipse** is a plane curve of such a form that if from any point in it two straight lines be drawn to two given fixed points, the sum of these straight lines will always be the same.

94. These two fixed points are called the **foci.**

Practically, a tolerably accurate ellipse may be drawn on paper by sticking two pins in it to represent the foci, putting over these a bit of thread knotted together at the ends, inserting a pencil in the loop, and pulling the sheet tight as the figure is described.

The importance of the ellipse arises from the fact that the planets move in elliptical orbits, the sun being in one of the foci—a fact which Kepler was the first to discover. When we calmly reason upon the immeasurable distances, and the awful rapidity of motion, with the mass of matter moving in this beautiful curve, we are constrained to acknowledge that all our boasted knowledge is as nothing in the wondrous dispensation of Him who "telleth the number of the stars and calleth them all by their names."

95. In Fig. 78, AA′ is the major axis, or the diameter which passes through the foci. BB′ is the minor axis, or the diameter which is perpendicular to the major axis.

To Find the Area.—*Multiply half the sum of the two diameters by* π *and the result is the area.*

PROBLEM 393.

If the axes of an ellipse are 60 and 80 ft., what are the areas of the two segments into which it is divided by a line perpendicular to the major axis, at the distance of 10 ft. from the center?

Solution.

(1) Let A′DAD′ represent a circle, EAS the segment of the ellipse A′BAB′, and DAD′ the segment of the circle.

(2) CG=10 ft., $\overline{DG}^2$=A′G×GA, or 27,000 ft.

(3) DG=$\sqrt{1500}$, or 38.729 ft.

(4) DD′=77.458 ft.

(5) The area of DAD′=$(h^3 \div 2b)$ $+\frac{2}{3}$ of hb=1723.46 sq. ft.

(6) Area of the segment of circle : the area of the segment

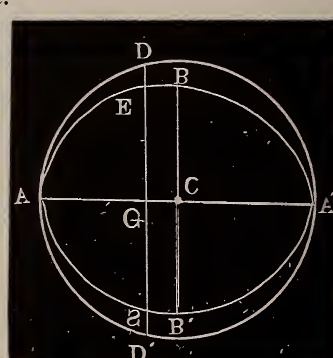

FIG. 78.

of the ellipse :: CB : CA′.

(7) ∴ 1723.46 : the segment of the ellipse :: 40 : 30.

(8) From which we find the area of the segment of the ellipse to be 1292.56 sq. ft.

(9) The area of the ellipse is $(40\times30)\pi=3769.92$ sq. ft. (by the above rule). $3769.92-1292.56=2477.36$ sq. ft., area of EAS.

NOTE.—This problem was prepared by the author for the *Teachers' Review*. We wish to prepare an extensive work on the ellipse in the future.

VI. CATENARIAN CURVE.

96. A curve formed by a chain or rope of uniform density, hanging freely from any two points not in the same vertical line.

The catenary was first observed by Galileo, who proposed it as the proper figure for an arch of equilibrium. He imagined it to be the same as the parabola. Its properties were first investigated by John Bernovilli, Huygens and Leibnitz. It is now universally adopted in suspension bridges. Each wire assumes its own catenary curve.

PROBLEM 394.

What length of rope is needed that if the ends are tied to two poles of the same height 100 ft. apart, the rope will sag 25 ft., midway between the poles?

Solution.

(1) Let DE and PF be the poles; EF=100 ft.

(2) Let D and P be the fixed points to which the ends of the rope are attached; the rope will rest in a vertical plane.

(3) The curve DCP is called the common catenary.

(4) Let C be the lowest point of the catenary.

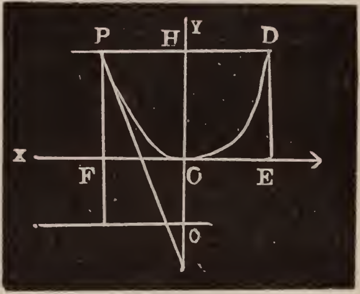

FIG. 79.

(5) Take this as the origin of coordinates, and let the horizontal line through C be taken for the axis of x, and the vertical line through C for the axis of y.

(6) Let (x, y) be any point, P, in the curve.

(7) Denote the length of the arc CP, by S.

(8) Let c be the length of the rope whose weight is equal to the tension at C.

(9) Now, if we move the origin to the point, O, at a distance equal to c below C, by putting $y-c$ for y we have $y=\frac{c}{2}\left(e^{\frac{x}{c}}+e^{-\frac{x}{c}}\right)$, which is the equation of the catenary.

(10) When $x=o$, $y=c$.

(11) x varies from o to 50, and y from c to $c+25$ at the same time.

(12) 2S=the length of the rope.

(13) To find c we have (e=2.7182818, the base of the Napierian Logarithms),—$e^z=1+z+\frac{z^2}{1.2}+\frac{z^3}{1.2.3}+\frac{z^4}{1.2.3.4}$, etc.

(14) $e^{-z}=1-z+\frac{z^2}{1.2}-\frac{z^3}{1.2.3}$, etc.

(15) $e^z+e^{-z}=2+z^2+\frac{2z^4}{1.2.3.4}$.

(16) Now put $\frac{x}{c}$ for z, and $y=c\left(1+\frac{x^2}{2c^2}+\frac{x^4}{24c^4}\right)$, etc.

(17) Then when $x=50$, $y=c+25$.

(18) Then $c+25=c\left(1+\frac{(50)^2}{2c^2}+\frac{(50)^4}{24c^4}, \text{etc.}\right)$ and $\frac{(50)^4}{24c^3}+\frac{(50)^2}{2c}=25$. Whence $c=53.71607$.

(19) $dS=\sqrt{dx^2+dy^2}=ydy\div(y^2-c^2)$.

(20) The limits of y of being c and $c+25$, we have, $S=\int_c^{c+25}\frac{ydy}{\sqrt{y^2-c^2}}=\sqrt{(c+25)^2-c^2}=\sqrt{50c+625}=57.5356$.

(21) 2S=115.0712 ft., or the length of the rope.

VII. CYCLOID.

97. A **Cycloid** is the curve which is produced when a circle rolls forward on a straight line.

A familiar example of it is the bicycle wheel moving along a smooth road. If a mark be made at any point on the circumference of a wheel, it will describe a series of cycloids.

The curved figure thus produced is not, as the etymology suggests, "of the form of a circle"; were it so, then the point of the circumference commencing its revolutions at a given spot on the road, would, when that revolution was completed, return to that spot again. It does not so return; but when having completed its revolution, it afresh touches the road, it is at an advanced point in it compared with the spot at which it before came into contact with it.

98. Now, if the circle OP rolls on the straight line AX, the point P will describe the arc of a cycloid APHX.

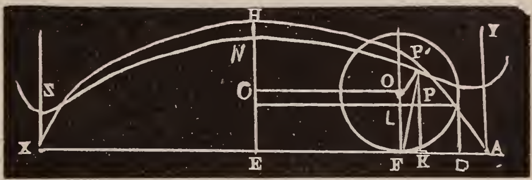

FIG. 80.

To find its equation, let A be the origin, O the center of the circle, r=OF, P=any point in the curve, and we have x= AD, y=PD=LF.

But AF=arc PF=$r\text{vers}^{-1}\frac{y}{r}$. DF=PL=$\sqrt{BL\,.\,LF}$.

$\therefore x=r\text{vers}^{-1}\frac{y}{r}-\text{DF}=r\text{vers}^{-1}\frac{y}{r}-\sqrt{2ry-y^2}$, which is the equation.

$\frac{dx}{dy}=\frac{y}{\sqrt{2ry-y^2}}$. $\therefore S=\sqrt{2r}\int_o^{2r}(2r-y)^{-\frac{1}{2}}dy=$

$\left(-2(2r)^{\frac{1}{2}}(2r-y)^{\frac{1}{2}}\right)_o^{2r}=4r$, which is $\frac{1}{2}$ the cycloidal arc.

The length of the branch AHX is eight times the radius, and the area of AHX is three times the area of the generating circle.

PROBLEM 395.

Suppose a fly sits on the rim of a carriage wheel 5 ft. in diameter: what distance will the fly move while the wheel makes one revolution on a plane surface?

Solution.

(1) Let A, Fig. 80, be the initial position of the fly.
(2) APH and X are subsequent positions as the wheel revolves.
(3) The fly describes the cycloidal curve APHX, whose base is the circumference of the wheel, or 5π, 15.708 ft.=AX.
(4) The branch AHX=5×4, or $2\frac{1}{2}\times8=20$ ft.
(5) The area of AHX=$5^2\pi\times3$=58.905 sq. ft.

PROBLEM 396.

Suppose a fly lights on the spoke of a carriage wheel 5 ft. in diameter, 6 in. up from the ground: what distance will the fly move while the wheel makes one revolution on a level plane?

Solution.

(1) From Fig. 80, let P be the generating point at a distance from the center$=$P′O$=b$, A the origin, $r=$OF, $\theta=$P′OF, $x=$AK, $y=$P′K.

(2) Then $x=r\theta - b\sin\theta$, and $y=r-b\cos\theta$, are the equations of the locus, in which $b<r$.

(3) $dy=b\sin\theta d\theta$.

(4) $dx=(r-\cos\theta)d\theta$, $\sqrt{(dy)^2+(dx)^2}=\sqrt{(r^2+b^2-2rb\cos\theta d\theta}$.

(5) Let S=the number of feet the fly moved while the wheel makes one revolution.

(6) Then, since $r=2\frac{1}{2}$ and $b=2$, we have

$$\int_0^\pi \sqrt{(r^2+b^2-2rb\cos\theta d\theta)}=\int_0^\pi \sqrt{(41-40\cos\theta d\theta)}.$$

(7) Put $\theta=2\phi$; then $S=18\int_0^{\frac{\pi}{2}}\sqrt{1-\frac{80}{81}\cos^2\phi d\phi}=$

$$9\pi\left[1-\frac{20}{81}-3\left(\frac{1}{1\cdot2}\right)^2\left(\frac{20}{81}\right)^2-5\left(\frac{1\cdot3}{1\cdot2\cdot3}\right)^2\left(\frac{20}{81}\right)^3-7\left(\frac{1\cdot3\cdot5}{1\cdot2\cdot3\cdot4}\right)^3\left(\frac{20}{81}\right)^4\right]$$
$=18.84$ ft.

NOTE.—PNZ is a prolate cycloid, or the path described by the point P′; also the curve of the cycloid's origin would fall to the left of YA. I think the student will understand this; see *Wood's Co-ordinate Geometry*.

VIII. CURVE OF PURSUIT.

99. If a point moves along any path, and another point is made to move towards it according to any law, the path of the latter is called a **Curve of Pursuit**.

PROBLEM 397.

A fox 40 rd. due south of a hound is running due east, 25 ft. per second; the hound gives chase at the rate of 30 ft. per second: how far will each run before the fox is caught?

Solution.

(1) It is evident that the hound is all the time running directly after the fox.

(2) The curved line BC represents the path of the hound.

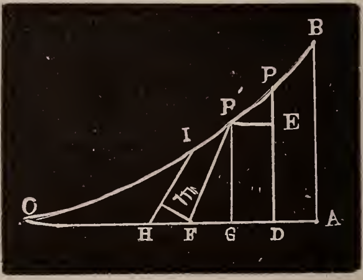

FIG. 81.

(3) Let P and F be their position at any instant, and P′ and H their positions at the next instant.

(4) Put $AD=x$, $BP=s$, $AF=y$, and $PG=w$.

(5) Then $EP'=dx$, $PP'=ds$, and $PP'-HM=dw$, (A).

(6) Now, let the rate of the hound be m times that of the fox.

(7) Then $mdy=ds$. Put $AB=b$, $AC=a$.

(8) Then from the triangles PP′E and HFM, we get $dy : ds :: HM : dx$.

(9) $HM=\frac{dydx}{ds}$.

(10) By placing for ds its value mdy, $HM=\frac{dydx}{mdy}=\frac{dx}{m}$.

(11) From (A) we get, by substituting, $ds-\frac{dx}{m}=dw$, or $mdy=\frac{dx}{m}+dw$, or $m^2dy=dx+mdw$ . . . (1).

(12) Integrating (1), $m^2y=x+mw+C$ . . . (2).

(13) When $x=o$, $y=o$, and $w=b$, $C=-mb$ and (2) becomes $m^2y=x+mw-mb$.

(14) Now, m is a constant; if we can find its value for any time in the race, we can take it as the value of m.

(15) Put $x=a$ and $y=a$, which is the case at the close of the race, and we get $m^2a=a-mb$ . . . (3).

(16) From which $a=\frac{mb}{m^2-1}$.

(17) In the given problem $b=40$ rd., and $m=\frac{6}{5}$.

(18) Substituting these, we get $a=109\frac{1}{11}$ rd., the fox's distance.

(19) $ma=130\frac{10}{11}$ rd., the hound's distance.

(*Prof. G. E. Kelly.*)

PROBLEM 398.

A bird sits on the top of a pole 50 ft. high: how high above the pole must a boy, standing 100 ft. from the pole, aim his arrow, to hit the bird, the velocity of the arrow at starting being 100 ft. per second?

Solution.

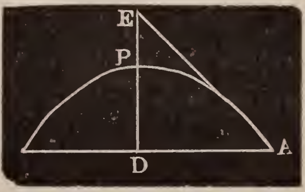

FIG. 82.

(1) Let A represent the point where the arrow leaves the gun, D the foot of the tree, and E the point directly over the bird, where the arrow is aimed.

(2) Let $a=50$, the height of the tree, $b=AD=100$, $v=100$, $i=$initial

velocity of arrow; t=time required for the arrow to make its flight to P, and x=EP, the distance required.

(3) Then AE=vt, the hypothenuse, and as x is the distance a body would fall in t seconds, we have $t^2=2x\div g$.

(4) Also $AE^2=v^2t^2=2v^2x\div g=(x+a)^2+b^2=x^2+2ax+a^2+b^2$

(5) Hence, $x^2+\left(2a-\frac{2v^2}{g}\right)x=-(a^2+b^2)$.

(6) $x=\frac{v^2}{g}-a\pm\left[\left(\frac{v^2}{g}-a\right)^2-(a^2+b^2)\right]$.

(7) Substituting, $x=\frac{10000}{3216}-50\pm[(\frac{10000}{3216}-50)^2-2500+10000]$ $=260.94527\pm235.78048=25.1648$, or 496.7257 ft.

NOTE.—Solved by the author for the *Teachers' Review*.

IX. PARALLELOGRAMS AND SOLIDS.

100. A **Parallelogram** is a quadrilateral whose opposite sides are parallel.

101. A **Rectangle** is a parallelogram whose angles are all right angles.

102. A **Rhombus** is a quadrilateral whose sides are equal.

103. A **Square** is a rectangle whose sides are all equal.

PROBLEM 399.

The diagonal of a square is equal to the side multiplied by $\sqrt{2}$.

Solution.

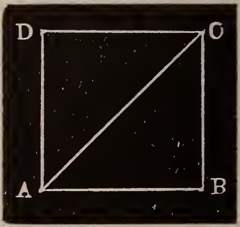

FIG. 3.

(1) Let ABCD be the square, and AC the diagonal.

(2) $\overline{AC}^2=\overline{AB}^2+\overline{CB}^2=2\overline{AB}^2$, or $2\overline{CB}^2$.*

(3) $AC=\sqrt{\overline{AB}^2+\overline{CB}^2}=\sqrt{2\overline{AB}^2}=AB\sqrt{2}$

$\therefore AC=AB\sqrt{2}$.

PROBLEM 400.

What is the side of a square whose diagonal is 20 ft.?

Solution.

(1) 20 ft.=diagonal.

(2) 20 ft.$\div\sqrt{2}$=14.84 ft.=side of the square.

Rule.—*Divide the diagonal by the* $\sqrt{2}$.

*This shows that the square of the diagonal of a square is equal to twice the square of one side, or twice the square itself.

PROBLEM 401.

The area of a square is 800 sq. ft.: what is the diagonal?

Solution.

(1) 800 sq. ft. = area of the square.

(2) $800 \times 2 = 1600$ sq. ft., double the area.

(3) $\sqrt{1600} = 40$ ft., the diagonal.

Rule.—*Double the area and extract the square root.*

PROBLEM 402.

What is the side of a square garden whose area is 3600 sq. ft.?

Solution.

(1) 3600 sq. ft. = area of the garden.

(2) $\sqrt{3600} = 60$ ft., side of the garden.

$\therefore$ 60 ft. = the side of the garden.

PROBLEM 403.

What is the area of a square whose diagonal is 10 ft.?

Solution.

(1) 10 ft. = the diagonal = AC.

(2) $10 \times 10 = 100$ sq. ft.

(3) 100 sq. ft. $\div 2 = 50$ sq. ft., the area of the square.

Rule.—*Divide the square of the diagonal by* 2, *the quotient will be the area.*

PROBLEM 404.

The square of half a line is equal to $\frac{1}{4}$ the square of the whole line.

Solution.

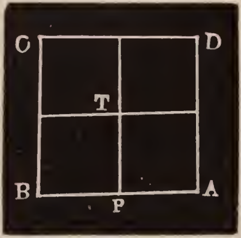

FIG. 84.

(1) Let $s = AB$.

(2) $AP = \frac{1}{2}s = \frac{s}{2}$.

(3) $\overline{AB}^2 = s^2$, $\overline{AP}^2 = \frac{s^2}{4}$.

$\therefore \overline{AP}^2 = \frac{1}{4}$ of s^2.

PROBLEM 405.

If the square ABCD has 40 acres in it, find the distance from C to a point $\frac{1}{3}$ the distance from B to A, as F.

Solution.

(1) $\overline{BD}^2=40$ acres, or 6400 sq. rd.
(2) $AB=\sqrt{6400}=80$ rd.
(3) $FB=\frac{1}{5}$ of 80 rd. $=16$ rd.
(4) Now, as FCB is a right-angled triangle, $FC=\sqrt{\overline{CB}^2+\overline{FB}^2}=\sqrt{80^2+16^2}=81.5+$ rd.

$\therefore$ CF=81.5+ rd.

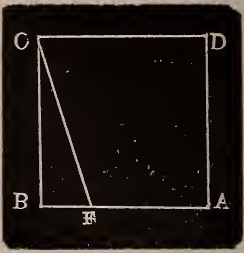

FIG. 85.

PROBLEM 406.

From a point in the side and 8 ch. from the corner of a square field containing 40 A., a line is run, cutting off $19\frac{1}{2}$ A.: how long is the line?
(*R. H. A.*)

Solution.

(1) Let ABCD be the square field, and P the point 8 ch. from D.
(2) Draw PE parallel to DC, PF cutting off $19\frac{1}{2}$ A. above the line, or PFCD.
(3) DC, or DA=20 ch.; then, the area of the rectangle DPEC$=20\times8=160$ sq. ch., or 16 A.
(4) The triangle EPF contains $19\frac{1}{2}-16=3\frac{1}{2}$ A.

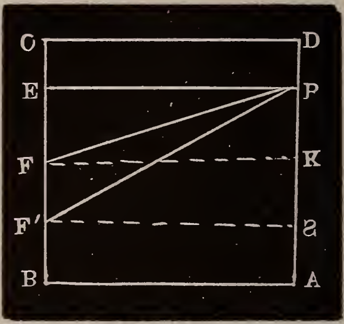

FIG. 86.

(5) EF, the side of the triangle$=3\frac{1}{2}\times2=7$ A., or 70 sq. ch.
(6) $70\div20=3\frac{1}{2}$ ch.
(7) $PF=\sqrt{20^2+3\frac{1}{2}{}^2}=20.3+$ ch.
(8) $\therefore$ PF=20.3 ch., when the $19\frac{1}{2}$ A. is taken above the line PF.
(9) Now, we will take it below the line PE.
(10) It is evident that the line PF will fall below its present position, as PF′ shown in the figure.
(11) Now, the quadrilateral PABF′ contains $19\frac{1}{2}$ A.
(12) DPEC+PABF$=35\frac{1}{2}$ A.
(13) PEF′=40 A.$-35\frac{1}{2}$ A.$=4\frac{1}{2}$ A.
(14) Then, the right triangle PEF′$=4\frac{1}{2}$ A.
(15) $4\frac{1}{2}\times2=9$ A., or 90 sq. ch., area of the rectangle PEF′S.
(16) EE$=90\div20=4\frac{1}{2}$ ch.; that is, the area of a rectangle divided by its length gives its width, or vice versa.

(17) $PF'=\sqrt{20^2+4\frac{1}{2}^{\,2}}=20.5$ ch.

$\therefore PF'=20.5$ ch.

PROBLEM 407.

In a square field which contains 10 A., a line is drawn from the southwest corner to the middle of the north side; from the southeast corner to the middle of the north side; and from the southwest corner to the northeast corner: find the area of the triangle enclosed by the three lines.

Solution.

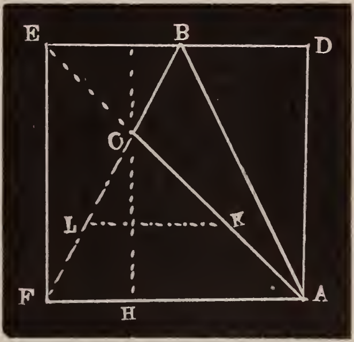

FIG. 87.

(1) Let ADEF be the field.

(2) The side DE is easily found to be 40 rd., or $\sqrt{(10\times160)}$.

(3) The area of the triangle BEF is $20\times40\div2$, or 400 sq. rd., which is also the area of the triangle ABE, for it has the same base BE and the same altitude EF.

(4) Subtract the area of the triangle BEC from this, and the result is the area required.

(5) Since BE is half AF, the altitude of the triangle BCE is half the altitude of the similar triangle ACF, for

$$BE : AF :: BC : CF :: EC : AC :: GC : CH.$$

(6) $\therefore$ The altitude of the triangle is equal to $\frac{1}{3}$ of 40, or $13\frac{1}{3}$ rd., and its area is $20\times13\frac{1}{3}\div2$, or $133\frac{1}{3}$ sq. rd.

(7) Hence, $400-133\frac{1}{3}=266\frac{2}{3}$ sq. rd., the area of the triangle ABC.

PROBLEM 408.

A man takes for himself a central square of 40 acres, within a square of 160 acres, and gives to his four sons rectangular farms equal in every respect and making up the remainder of the farm. The father and sons are to have dwellings centrally located in their respective farms: how far will each son be from his nearest brother, and from his father?

Solution.

(1) Let ABCD be the farm of 160 A., and LFXM the father's farm.

(2) Draw EL, MP, XN, FY, and ARXY, RBPM, PCEL and EDYF will be the sons' farms.

(3) $AB=BC=160$ rd.
(4) $LF=FX=80$ rd.
(5) $YF=EL=MP=40$ rd.
(6) $YD=AR=120$ rd.
(7) $OK=ON+NK=40$ rd.$+$ 20 rd.$=60$ rd.
(8) $S'K=\frac{1}{2}AR-NX=60$ rd.$-$ 40 rd.$=20$ rd.
(9) Then, $S'O=\sqrt{(60^2+20^2)}$ $=63.24$ rd.$=$the distance each son's house is from the father's.

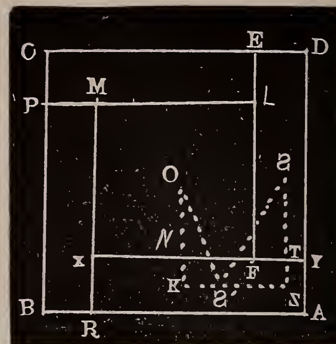

FIG. 88.

(10) $ST=60$ rd., and $TZ=20$ rd.
(11) $\therefore$ $SZ=60$ rd.$+20$ rd.$=80$ rd.
(12) $ZS'=40$ rd.
(13) Then, $SS'=\sqrt{(40^2+80^2)}=89.44+$ rd.$=$the distance one son is from the other.

PROBLEM 409.

John has a square wheat field containing 10 A., and hires Mart to cut it; he starts to cut at the southeast corner of the field: how far is the machine from the southeast corner of the square after cutting 5 A., the machine being a 6 ft. cut?

Solution.

(1) Let ABCD be the wheat field, and EFGH the part that remains after cutting an integral number of rounds.
(2) $KLMN=5$ A., the part that remains providing the machine cuts the first 5 A. in an integral number of rounds.
(3) $AD=AB=40$ rd.$=660$ ft.
(4) $NK=KL=28.28$ rd.$=$ 466.62 ft.

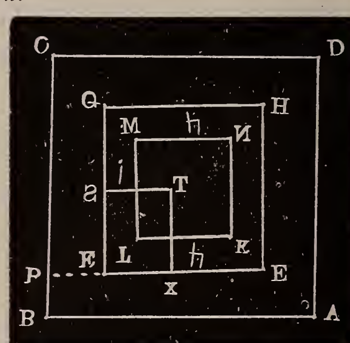

FIG. 89.

(5) $FP=40$ rd.-28.28 rd.$=11.72$ rd.$\div 2=5.86$ rd.$=96.69$ ft.
(6) $96.69\div 6$, the width of the machine$=16$, or the integral number of rounds, and .49 rd. remaining.
(7) .49 rd.$=$*is*, 1.38 ft., twice the width.
(8) $EF=KL+$twice *is*$=468$ ft.
(9) Area of $EFGH=468^2=219024$ sq. ft., and area of KLMN

=5 A., or 217800 sq. ft.

(10) The area of the part that is not cut, to have a remainder of 5 A., is *hh*, or 219024—217800=1224 sq. ft.

(11) Now this 1224 sq. ft. is to be cut by taking a full swath, at the southeast corner of EFGH.

(12) XFST represents the swath containing 1224 sq. ft.

(13) Now, we have the breadth of this rectangle (6 ft.); we find its length by dividing by its breadth, or XT=SF= 1224÷6=204 ft.

∴ The machine is 204 ft. from the southeast corner of FGHE.

NOTE.—The latter part of the problem has reference to the square EFGH.

PROBLEM 410.

Through a man's farm of 1000 acres, lying in the form of a square, runs a railroad in a straight line on the diagonal: what does the right of way cost at $200 an acre, the strip being 100 ft. wide?

Solution.

(1) Let ABCD be the farm, and FGCHEA be the strip occupied by the railroad.

(2) The diagonal AC= $\sqrt{(1000\times160\times2)}$= 565.68551 rd., or 9333.81091 ft.

(3) EAF and HGC are isosceles triangles.

(4) HC=CG=AF=EA=50ft.

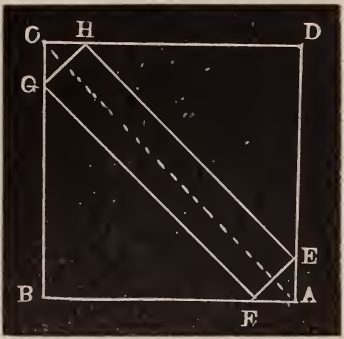

FIG. 90.

(5) If we take a rectangle 100 ft wide and 9333.81091 ft. long, we have the area of the railroad+4 small equal isosceles right triangles.

(6) The area of all is $50^2\times2$=5000 sq. ft.

(7) [(9333.81091×100)—5000]÷43560, the number of square feet in 1 A.=21.3123 A.

(8) 21.3123×200=$4262.46, the cost of right of way.

PROBLEM 411.

By cutting off a strip 3 in. wide from around a square board, I found that ⅘ remained: find the area of the board.

Solution.

(1) Let ABCD be the board.
(2) Let MNFE be $\frac{4}{9}$ of the whole area.
(3) $MN=\sqrt{\frac{4}{9}}=\frac{2}{3}$ of the side AB.
(4) $AB=\frac{3}{3}$, then $\frac{3}{3}-\frac{2}{3}=\frac{1}{3}$, twice NT.
(5) $NT=\frac{1}{2}$ of $\frac{1}{3}=\frac{1}{6}$.
(6) $NT=3$ in.
(7) $\frac{1}{6}=3$. $\frac{6}{6}$, or $AB=6\times3=18$ in.
(8) $\overline{AB}^2=18^2$ in. $=324$ sq. in., area of ABCD.

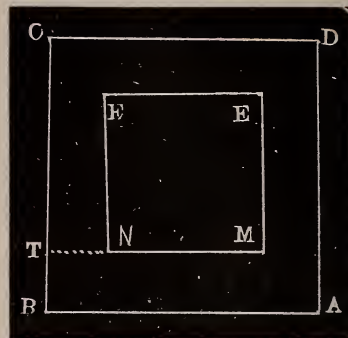

FIG. 91.

PROBLEM 412.

Required the length of a piece of carpet that is a yard wide, with square ends, that can be placed diagonally in a room 40 ft. long and 30 ft. wide, the corners of the carpet just touching the walls.

Solution.

(1) Let ABCD be the room, and GFEH the strip of carpet.
(2) Let $x=EC=AG$, and $y=CF=AH$.
(3) Then, by similar triangles, $40-x : 30-y :: y : x$.
(4) Or, $40x-x^2=30y-y^2$. (1).
(5) We also have $x^2+y^2=3^2$, or $x=\sqrt{(9-y^2)}$ . . (2).

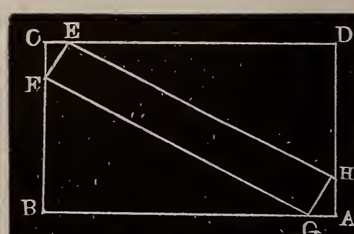

FIG. 92.

(6) Substituting in (1) and reducing, $y^4-30y^3+616y^2+135y-3579.75=0$. . . . (3).
(7) $y=2.4337$, $x=1.7541$.
(8) Hence, the length is $\sqrt{(40-x)^2+(30-y)^2}=47.144+$ ft.

NOTE.—This solution was prepared by the author for the *American Mathematical Monthly.*

PROBLEM 413.

What is the area of a path 1 yard wide, passing diagonally across a lot 20 rd. wide and 30 rd. long, the diagonal of the lot passing through the middle of the path?

Solution.

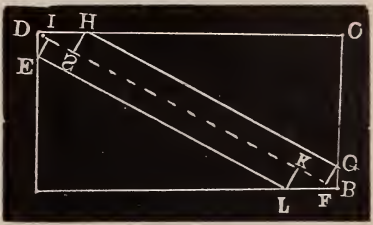

FIG. 93.

(1) Let ABCD represent the lot; HGLE, the path.

(2) EI, HS, GF and LK are each perpendicular to DB, and are each 18 in.

(3) By similar triangles, DC : CB :: DS : HS, and DS=2.25 ft.

(4) DC : CB :: GF : FB, from which FB=1 ft.

(5) The total area of the four triangles is 4.875 sq. ft.

(6) $\overline{DC}^2+\overline{BC}^2=\overline{DB}^2$, from this we find DB=593.23+ ft.

(7) 593 23 ft.—3 25 ft. (DS+1)=589.98+ ft., SF or IK.

(8) 589.98×3 ft.=1769.94+ sq. ft., area of the two rectangles.

(9) 1769.94 sq. ft.+4.875 sq. ft.=1774.815 sq. ft., total area.

PROBLEM 414.

Find the diagonal of a rectangular field containing 13½ A., whose length is to its breadth as 12 to 5.

Solution.

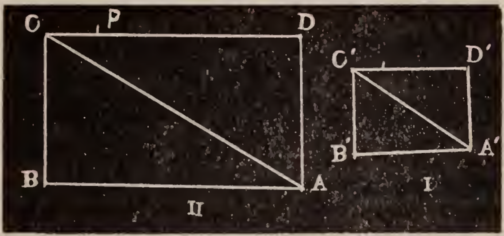

FIG. 94.

(1) Let ABCD be the field.

(2) Its area is 13½ × 160 = 2160 sq. rd.

(3) Assume A'B'C'D' as a similar field whose sides are D'A'=5, A'B'=12.

(4) Then, similar figures are to each other as the squares of their like dimensions.

(5) Area of A'B'C'D'=5×12=60 sq. rd.

(6) We have A'B'C'D' : ABCD : $\overline{A'D'}^2$: $\overline{AD}^2$, or 60 : 2160 :: 5^2 : $\overline{AD}^2$.

(7) From this we find AD=30 rd.

(8) A'B'C'D' : ABCD :: $\overline{D'C'}^2$: $\overline{DC}^2$, or 60 : 2160 :: 12^2 : $\overline{DC}^2$.

(9) From this DC=72 rd.

(10) $AC=\sqrt{(\overline{AB}^2+\overline{BC}^2)}$=78 rd.

∴ The diagonal AC=78 rd.

PROBLEM 415.

The diagonal of a rectangle is 109 and its perimeter 302; required, the sides.

Solution.

(1) The diagonal of a rectangle divides it into two equal right-angled triangles.

(2) Let x represent the perpendicular, y the base and h the hypothenuse of one of the triangles.

(3) Then, we have $x+y=151$ . . . (1).

(4) $x^2+y^2=109^2$ . . . (2).

(5) Subtracting (2) from the square of (1), we have $2xy=10920$ . . . (3).

(6) Subtracting (3) from (2), we have $x^2-2xy+y^2=961$.

(7) Extracting the square root, we have $x-y=31$ . . . (4).

(8) From (1) and (4), we find $x=91$ and $y=60$.

(9) Hence 91 and 60 are the sides of the rectangle.

PROBLEM 416.

A corn field is 25 rd. longer than wide and contains 900 sq. rd.: what are its dimensions?

Solution.

(1) Let x represent the width, and $x+25$ the length.

(2) $x(x+25)$, or $x^2+25x=900$.

(3) Completing the square and extracting the square root, we find $x=20$ rd., and $x+25=45$ rd.

PROBLEM 417.

A rectangular flower garden 60 yd. long and 40 yd. wide, has a walk 6 ft. wide around it, and paths of the same width through it, joining the middle points of the opposite sides: find in square yards the area of one of the four flower beds enclosed by paths.

Solution.

(1) Let ABCD be the garden.

(2) The area is $60\times40=2400$ sq.yd.

(3) Let S, S′, S″, S‴, be the flower beds.

(4) The length of the side walk $=60$ yd.$\times2+36$ yd.$\times2=192$ yd.

(5) The length of the inside walks is 56 yd.$+$34 yd.$=$ 90 yd.

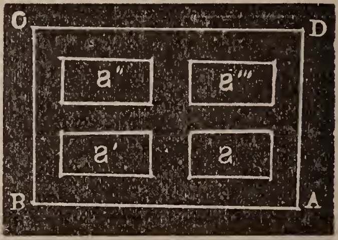

FIG. 95.

(6) 192 yd$+$90 yd.$=$282 yd., the length of the walks.

(7) Now, the area of the walks is $282 \times 2 = 564$ sq. yd.
(8) The area of the flower beds is $2400 - 564 = 1836$ sq. yd.
(9) The area of one of the beds is 1836 sq. yd. $\div 4 = 459$ sq. yd.

PROBLEM 418.

At three corners of a square garden, each side 100 ft., stand towers 20 ft., 60 ft., and 80 ft. high respectively: where must a ladder be placed in this garden, that it may reach the top of each tower without moving its base?

Solution.

(1) We must place the foot of the ladder in the line perpendicular to the side between the 20 ft. and 60 ft. towers, at a point from which the ladder will reach both towers, and, also, in the perpendicular to the second side at the point where the ladder would reach the tops of the 60 ft. and 80 ft. towers.
(2) Now, to find the first point, we have $20^2 + x^2 = 60^2 + y^2$.
(3) $x^2 + y = 100$; then $x = 66$, $y = 34$.
(4) To find the second point, we have $60^2 + u^2 = 80^2 + z^2$.
(5) $u + z = 100$; then, $u = 64$, and $z = 36$.
(6) $\sqrt{(80^2 + z^2 + y^2)} = \sqrt{(60^2 + u^2 + y^2)} = \sqrt{(20^2 + u^2 + x^2)} = \sqrt{8852} = 94.08506+$ ft., the length of the ladder.
(7) From the first tower, the foot of the ladder is $\sqrt{8852 - 20^2} = 91.934759+$ ft.
(8) From the second it is $\sqrt{8852 + 60^2} = 72.470821+$ ft.
(9) From the third it is $\sqrt{8852 - 80^2} = 49.5176736+$ ft.
(10) The height of a tower on the fourth corner, that the ladder will reach is $\sqrt{8852 - (x^2 + z^2)} = 40\sqrt{2}$, or $56.5668+$ ft.

PROBLEM 419.

A rectangular field is 64 rd. long and 50 rd. wide: if from the middle of either side we set out a right line to its opposite, where must it intersect in order to cut the field in the ratio of 1 to 2?

Solution.

(1) Let ABCD be the field, and P the point of the side AB.
(2) Draw the line PE perpendicular from P to the opposite side.
(3) Draw PF which shall divide the field as required.

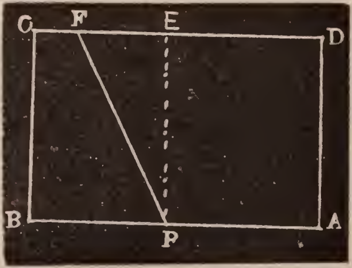

FIG. 96.

(4) The area of the field is 64×50, or 3200 sq. rd.
(5) APED=$\frac{1}{2}$ of the field, and PBCF=$\frac{1}{3}$ of the field.
(6) The triangle EPF=$\frac{1}{2}$—$\frac{1}{3}$=$\frac{1}{6}$ of the field, or $\frac{1}{6}$ of 3200 sq. rd.=533$\frac{1}{3}$ sq. rd.
(7) Since DA=50 rd., EF=(533$\frac{1}{3}$×2)÷50=21$\frac{1}{3}$ rd.
(8) EC=32 rd., half the side.
(9) FC=CE=FE=32—21$\frac{1}{3}$=10$\frac{2}{3}$ rd., the distance from the corner of the field.

PROBLEM 420.

There is a square field such that the number of rods around it is equal to the number of acres within it: how many acres does it contain?

Solution.

(1) Let ABCD be the square field.
(2) O is the center of the field.
(3) Draw OF perpendicular to AB.
(4) FE=1 rd.
(5) Join OE. Then EFO is a right triangle.
(6) Now, as EF=1 rd., the triangle EFO=1 A., or 160 sq. rd.
(7) OF=(160×2)÷1=320 sq. rd.
(8) Then CB=640 rd., the side of the square.
(9) 640 rd.×4=2560 rd., the perimeter.

∴ There are 2560 A. in the field.

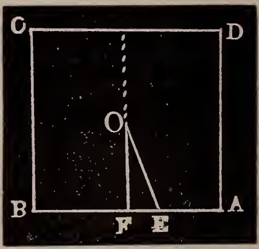

FIG. 97.

PROBLEM 421.

If 16 rails fence a rod, how many acres in that square field of such extent that every rail will fence an acre?

Solution

(1) Let the above figure represent the square.
(2) EF=1 rd. and the triangle EFO=16 A., or 2560 sq. rd.
(3) OF=(2560×2)÷1=5120 rd.
(4) CB=5120 rd.×2=10240 rd.
(5) The perimeter of the field=10240 rd.×4=40960 rd.
(6) Now as 16 rails fence a rod, and every rail will fence a rod; there will be as many acres in the field ABCD as the product of 40960×16=655360 A.

NOTE.—The above problem has bothered many an applicant at the county examination. Now, my friend, do not miss it again.

PROBLEM 422.

You have a square farm of 40 acres: find the side of that farm without using square root.

Solution.

(1) In the diagram, let ABCD=1 sq. mi. =640 A.

(2) Then EFGD=$\frac{1}{16}$ sq. mi.=40 A., the required farm.

(3) But ED=$\frac{1}{4}$ mi.=80 rd.

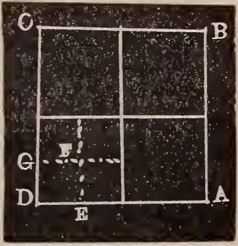

FIG. 98.

PROBLEM 423.

At the northwest corner of a rectangular field two men start to walk at the same rate, one east on the short boundary line and the other on the diagonal: where will they meet if the one turns at the south-east corner to meet the other, the field being 96 rd. long and 28 rd. wide?

Solution.

(1) From Fig. 94, let ABCD be the rectangle, AC the diagonal, and P the point of meeting.

(2) $\sqrt{\overline{AD}^2+\overline{DC}^2}$=100 rd.=AC.

(3) AD+DC=124 rd.

(4) From AD to P=112 rd.

(5) PC=[(96+28)−100]=12 rd., the point of meeting north of the southeast corner.

PROBLEM 424.

Schwarts has a garden in the form of a square; from the corners to a spring within the garden, the distances are a=40, b=50, c=80 ft.: find the side of the garden.

Solution.

(1) Let ABCD be the square and F the spring.

(2) DF=50 ft., CF=40 ft. and FB=80 ft.

(3) Draw DE=50, EC=40, CT=40, and TB=80 ft.

(4) Suppose S is a spring whose distances are AS=40, DS=50, and SB=80 ft.

(5) $\sqrt{\overline{SB}^2+\overline{BT}^2}=\sqrt{80^2+80^2}$= 113.13 ft.=ST.

(6) SE=$\sqrt{\overline{SD}^2+\overline{DE}^2}$, or $\sqrt{50^2+50^2}$=70.71 ft.

(7) In the scalene triangle STE, EK is its perpendicular, and now we will find SK and KT by a well known theorem in geometry.

(8) TS : TE + ES :: TE —ES : KT—SK, or 113.13 : 150.71 :: 9.29 : 12.37.

(9) Half the difference of the segments added to half their sum gives the greater segment, and subtracted gives the lesser segment.

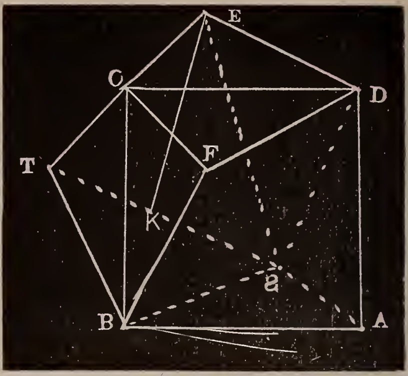

FIG. 99.

(10) Therefore, KT is 62.75 ft., and SK=45.38 ft.

(11) $KE=\sqrt{\overline{KT}^2-\overline{ET}^2}=49.6$ ft.

(12) Area of STE$=\frac{1}{2}$ST$\times$EK=2805.376 sq. ft.

(13) The area of SBT=3200 sq. ft., and area of EDS=12500 sq. ft.

(14) Therefore, the area of DSBTE=area of ABCD= 7255.375 sq. ft.

∴ AB, the side of the square garden=85+ ft.

PROBLEM 425.

A square farm contains 40 acres. It is required to lay off another park containing the same area, enclosed by an iron fence forming circular arcs only, and to find the cost of such a fence at $2 per rd.

Solution

(1) Let ABCD represent the square farm, whose sides AB, BC= $\sqrt{40\times160}=80$ rd.

(2) With a radius equal to that of the circumscribing circle draw the arcs APB and BRC.

(3) The area of the pelicoid APRCD is equal to that of the square farm, and equal to the area of the park, bounded by circular arcs only.

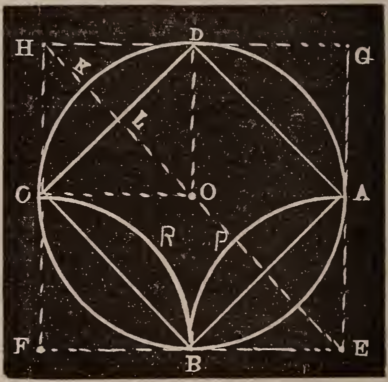

FIG. 100.

(4) The fence required is the length of the circumference of the circumscribing circle.

(5) $80 \times \pi = 251.328$ rd., the circumference of the circle, or the length of the fence.

(6) ∴ The cost is $251.328 \times \$2 = \$502.656$.

PROBLEM 426.

What is the length of each side of an octagon formed from a square whose sides are 8 ft.?

Solution.

(1) Take A, B, C, D as centers.

(2) With a radius equal to OA, describe arcs cutting the sides.

(3) Join the adjacent points, and the resulting figure NMLKFGQP is the required octagon.

(4) $OB = 4\sqrt{2}$, the diagonal of the square whose side is 4 in.

(5) The area of the octagon is the square less the area of four triangles cut off.

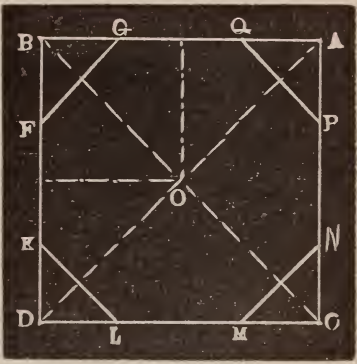

FIG. 101.

(6) Then $8 - 4\sqrt{2} = 2.3431456$ ft., side of the triangle FBG.

(7) ∴ $8 - (2.3432456 \times 2) = 3.3133088$ ft., side of the octagon.

PROBLEM 427.

The diagonal of a square circumscribed about a circle is 40 rd.: find area of inscribed square.

Solution.

(1) From Fig. 100, let EFGH be the circumscribed square, and ABCD the inscribed square.

(2) EH, the diagonal of the large square $= 40$ rd.

(3) Area of the large square is $40^2 \div 2 = 800$ sq. rd.

(4) Let $OC = r$, then $EFHG = 4r^2$, and $ABCD = 2r^2$. That is, the area of the large square is twice the area of the small square.

(5) ∴ The area of $ABCD = \frac{1}{2}$ of 800 sq. rd. $= 400$ sq. rd.

PROBLEM 428.

To make a square equal to a given rectangle ABCD.

Solution.

(1) Produce one side AB, till BE is equal to the shorter side of the rectangle ABCD, or CB.

(2) On AE as a diameter describe a circle meeting BC produced at F.

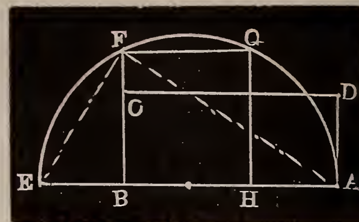

FIG. 102.

(3) Then will BF be the side of the square BFGH, equal to the given rectangle DB, as required.

(4) AB : BF :: BF : BE.

(5) Hence $\overline{BF}^2$=AB.BE, or AB.BC.

PROBLEM 429.

On a rectangular lot whose sides are 39 ft. and 49 ft., a house is built, covering 999 sq. ft.; the house is so located and of such a shape, that the lot surrounding the house is of uniform width: find the basal dimensions of the house. (*Helfrich.*)

Solution.

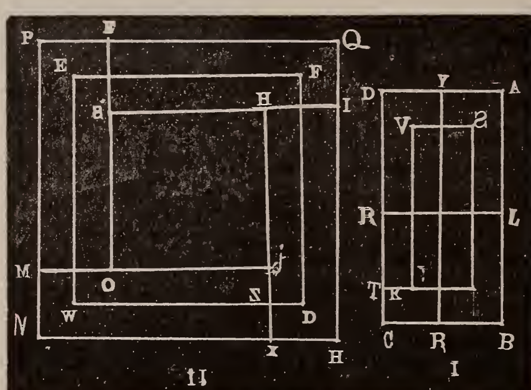

FIG. 104. FIG. 103.

(1) Let ABCD represent the rectangular lot and SK the house.

(2) Divide the lot into four equal parts as represented in Fig. 103, and arrange these four figures as shown in Fig. 104, which makes the square HN.

(3) PC=$24\frac{1}{2}$ ft., and CR=$19\frac{1}{2}$ ft.

(4) Then, the side of the square HN=$19\frac{1}{2}+24\frac{1}{2}=44$ ft.

(5) The area of HP=44^2=1936 sq. ft.

(6) The area of the lot DB $=49\times39=1911$ sq. ft.

(7) The area of the lot surrounding the house is $1911-999=912$ sq. ft.

(8) $1936-912=1024$ sq. ft., area of the square DE.

(9) DF $=\sqrt{1024}=32$ ft.

(10) $(44-32)\div2=6$ ft. $=$ ZX, the width of the lot.

(11) ZX $=$ TK. From this we can see that the width of the house is 39 ft.—12 ft.$=$27 ft., and the length 49 ft.—12 ft.$=$37 ft.

PROBLEM 430.

My lot contains 135 sq. rd., and the breadth is to the length as 3 to 5: what is the width of a road which shall extend from one corner half around the lot and occupy $\frac{1}{4}$ of the ground? (*R. H. A.*)

Solution.

(1) From Figs. 103 and 104, let AC represent the lot containing 135 sq. rd., and SK a similar lot containing 15 sq. rd., and whose sides are 3 and 5 rd.

(2) Then by similar figures, 135 sq. rd. : 15 sq. rd. :: $\overline{DA}^2 : \overline{VS}^2$, or 135 sq. rd. : 15 sq. rd. :: $\overline{DA}^2 : \overline{VS}^2$, from which DA $=9$ rd.

(3) 135 sq. rd. : 15 sq. rd. :: $\overline{AB}^2 : \overline{VK}^2$, from which AB$=$ 15 rd.

(4) Now let the rectangle IX in Fig. 104 represent AC in Fig. 103.

(5) Then construct the rectangles IS′FQ, FOMP and MJXN.

(6) They are equal to IHXH, for their lengths and breadths are equal.

(7) HN $=9$ ft. $+15$ ft. $=24$ ft.

(8) The area of HP $=24^2=576$ sq. rd.

(9) The area of the road is 135 sq. rd.

(10) The area of DE $=576-135=441$ sq. rd.

(11) FD, or FE $=\sqrt{441}=21$ rd.

(12) 24 rd.—21 rd. $=3$ rd.

(13) ZX $=\frac{1}{2}$ of $3=1\frac{1}{2}$ rd.

(14) $1\frac{1}{2}$ rd. $=24\frac{3}{4}$ ft., the width of the road.

PROBLEM 431.

How wide a strip must be plowed around a square 40-acre field to plow 75% of it?

Solution.

(1) The field contains 6400 sq. rd., and the side is 80 rd.

(2) Since 75%, or $\frac{3}{4}$ of it is plowed, the square unplowed is $\frac{1}{4}$

of the field.

(3) $\sqrt{\frac{1}{4}}$, or $\frac{1}{2}$ of it is $\frac{1}{2}$ of the side of the field, or 40 rd.

(4) Hence the width of the strip plowed is $(80-40)\div2=20$ rd.

PROBLEM 432.

The length and breadth of a ceiling are 6 and 5; if each dimension were 1 ft. longer, the area would be 304 sq. ft.: what are the dimensions? (*R. H. A.*)

Solution.

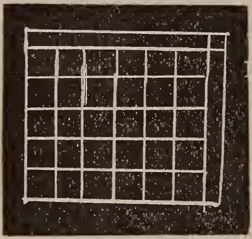

FIG. 105.

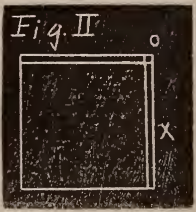

FIG. 106.

(1) 304 sq. ft.—1 sq. ft.=303 sq. ft.

(2) $6\times5=300$ squares as shown in Fig. 105.

(3) $6+5=11$ rectangles 1 ft. wide.

(4) 30 squares+11 rectangles=303 sq. ft.

(5) $303\div30=10\frac{1}{10}$ sq. ft., area of each square.

(6) 11 rectangles$\div30=\frac{11}{30}$ sq. ft., addition to each square.

(7) $\frac{11}{30}\div2=\frac{11}{60}$ ft., width of the small rectangle in Fig. 106.

(8) $(\frac{11}{60})^2=\frac{121}{3600}$ sq. ft., area of square O in Fig. 106.

(9) $10\frac{1}{10}=\frac{36360}{3600}$ sq. ft.

(10) $\frac{36360}{3600}+\frac{121}{3600}=\frac{36481}{3600}$ sq. ft.

(11) Extracting the square root of this fraction, we have $\frac{191}{60}$ ft., side of square enlarged, Fig. 106.

(12) $\frac{11}{60}$ ft.=the addition.

(13) $\frac{191}{60}-\frac{11}{60}=\frac{180}{60}=3$ ft., side of squares in Fig. 105, room 5 by 6 squares.

(14) $5\times3=15$ ft., one dimension.

(15) $6\times3=18$ ft., the other dimension.

NOTE.—Solved for the *Teachers' Review* by N. D. Moser.

PROBLEM 433.

What is the area of a trapezium the diagonal of which is 110 ft., and the perpendiculars to the diagonal 40 and 60 ft. respectively?

Solution.

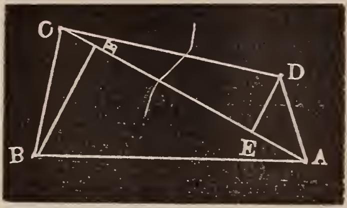

FIG. 107.

(1) Let ABCD be the trapezium, AC the diagonal.

(2) DE=40 ft., and FB=60 ft., the perpendiculars to AC

(3) 110 ft.=AC=base of the triangle ACD.

(4) $\frac{1}{2}$(AC×DE)=2200 sq. ft., area of the triangle.

(5) AC=the base of the triangle ACB.

(6) FB=altitude of ACB.

(7) ∴ $\frac{1}{2}$ of (AC× FB)=3300 sq. ft.=area of ACB.

(8) Then ACD+ACB=5500 sq. ft., area of trapezium.

PROBLEM 434.

Find the perimeter of a rhombus, area 216 sq. ft., and one diagonal 24 ft.

Solution.

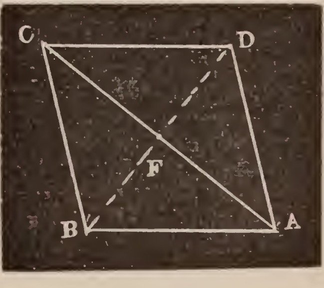

FIG. 108.

(1) Let ABCD be the rhombus.

(2) AC=24 ft., the given diagonal, and DB is the other diagonal.

(3) The triangle DBC=$\frac{1}{2}$ of 216, or 108 sq. ft.

(4) The area of the right triangle DFC=$\frac{1}{2}$ of 108=54 sq. ft.

(5) DF=(54×2) ÷ FC, or 12=9 ft.

(6) Then DB=9×2=18 ft.

(7) DC=$\sqrt{DF^2+FC^2}$=15 ft., the side of the rhombus.

(8) ∴ The perimeter is 15×4, or 60 ft.

PROBLEM 435.

The area of a rhombus is 96 sq. rd., and the diagonals are to each other as 3 to 4: find the side.

Solution.

(1) Assume a similar rhombus whose sides are A′B′, B′C′, C′D′ and D′A′, diagonals A′C′ and D′B′.

(2) F′C′=2, and D′B′=3 ft.

(3) Then the area of A′B′C′D′=D′B′× F′C′, or 6 sq. ft.

(4) We have 6 sq. ft. : 96 sq. ft :: $3^2 : \overline{DB}^2$, from which DB=12 ft.

(5) 6 sq. ft. : 96 sq. ft. :: $4^2 : \overline{AC}^2$, from which AC=16 ft.

(6) CF=8 ft., and DF=6 ft.

(7) ∴ $DC=\sqrt{\overline{DF}^2+\overline{FC}^2}=100$ ft., the side of the rhombus.

PROBLEM 436.

The side of a rhombus is 10 and the shorter diagonal is 12: what is the area?

Solution.

(1) Let ABCD be the rhombus.

(2) AB=10, DB=12 and DF=6.

(3) FCD is a right triangle and DC is the hypothenuse.

(4) $FC=\overline{DC}^2-\overline{DF}^2=8$.

(5) Area of ABCD=DB×FC, or 96.

PROBLEM 437.

How many acres in a field, two sides being parallel, 6 rd. and 14 rd., and the width 16 rd.? (*Royer.*)

Solution.

(1) Let ABCD represent the field.

(2) AB=14, DC=6 and CE=16 rd.

(3) Area of ABCD= $\frac{1}{2}$(DC+AB) × CE = 160 sq. rd., or 1 A.

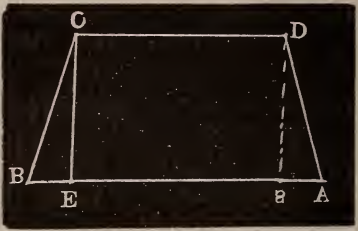

FIG. 109.

Rule.—*Multiply half the sum of the parallel sides by the altitude.*

PROBLEM 438.

Area of a trapezoid is 400, one of the parallel sides is 24, and the width 20: find the other side.

Solution.

(1) Let ABCD represent the trapezoid.

(2) AB=24, and EC=20.

(3) Then, 400÷20=20, and 20×2=40, the sum of the parallel sides.

(4) ∴ 40—24=16, or DC.

Rule.—*The sum of the parallel sides equals the area divided by the width, multiplied by* 2.

PROBLEM 439.

The parallel sides of a trapezoid are 36 and 24, and the other two sides are each 10: find the width.

Solution.

(1) Let ABCD represent the trapezoid.

(2) DC=24, AB=36, CB=AD=10.

(3) AS=EB=(36—24)÷2=6.

(4) Now, CEB is a right triangle.

(5) $EC=\sqrt{\overline{CB}^2-\overline{EB}^2}=8$.

∴ 8 is the width of the trapezoid.

PROBLEM 440.

The sides, a=6, b=4, c=5 and d=3, of a trapezium inscribed in a circle, being given, find the area of the trapezium.

Solution.

(1) Let ABCD be the trapezium.

(2) AB=6, BC=3, CD=5 and DA =4.

(3) Then by Bowser's Trigonometry, Art. 106, we have S= area and $s=\frac{1}{2}(a+b+c+d)$.

(4) $S=\sqrt{(s-a)(s-b)(s-c)(s-d)}=$ 18.97.

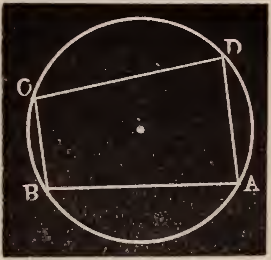

FIG. 110.

PROBLEM 441.

What is the edge of the largest cube that can be cut from a hemisphere 20 in. in diameter?

Solution.

(1) Let ALB be the hemisphere, and DE the inscribed cube.

(2) ES=x, the edge of the cube.

(3) SP=OP=$\frac{1}{2}x$, or $\frac{x}{2}$.

(4) OF=10 in., ESO is a right triangle.

(5) $\overline{OS}^2=\overline{OP}^2+\overline{SP}^2$.

(6) $\overline{OS}^2=\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^2=\frac{2x^2}{4}$

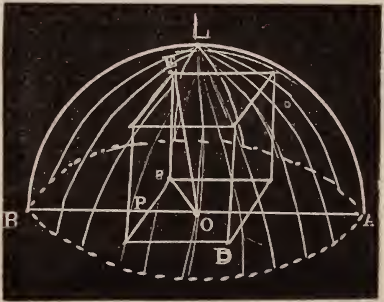

FIG. 111.

$=\frac{1}{2}x^2$.

(7) $x^2+\frac{1}{2}x^2=\overline{OE}^2$, or 10^2.

(8) $x=8.1649$ in.

∴ 8.1649 in. is the edge of the cube.

PROBLEM 442.

A tin vessel having a circular mouth 9 in. in diameter, a bottom $4\frac{1}{2}$ in. in diameter, and a depth of 10 in., is $\frac{1}{4}$ part full of water: what is the diameter of a ball which can be put in and just be covered by the water?

Solution.

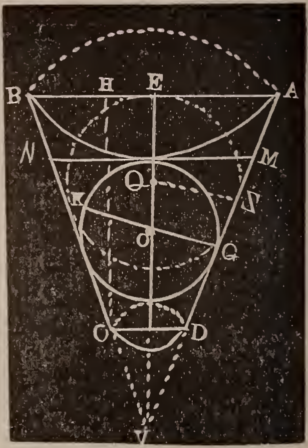

FIG. 112.

(1) Let AVB represent a middle section of the vessel.

(2) AB=9 in., DC=4.5 in., EF =10 in.

(3) The triangle BHC and BEV are similar.

(4) BH : BE :: HC : EV, or $2\frac{1}{4}$: $4\frac{1}{2}$ :: 10 : EV.

(5) From this EV=20, AV= $\sqrt{\overline{AE}^2+\overline{EV}^2}=20.5$.

(6) $9^2\times.7854\times\frac{20}{3}=424.116$cu.in., solid contents of the vessel.

(7) $(4\frac{1}{2})^2\times.7854\times\frac{10}{3}=53.0145$ cu. in., solid contents of the cone DVC.

(8) QZ=radius of the ball that will just go into the cone.

(9) $\frac{1}{2}(9\times20)=90$ sq. in., area of the triangle AVB.

(10) $9+(20.5\times2)=50$ in., the perimeter of AVB.

(11) Since the radius QZ of the inscribed circle is found by dividing the area by half the perimeter, we have QZ= $90\div25=3.6$ in.

(12) This is the radius of the inscribed sphere of the cone AVB, or $3.6\times2=7.2$ in., diameter of the ball that will just go into the mouth of the vessel.

(13) $(7.2)^3\times.5236=195.4326528$ cu. in., solid contents of the ball Q.

(14) 424.116—53.0145=371.1015 cu. in., solid contents of ADCB.

(15) 424.116—195.4326258 cu.in.=228.6733472 cu. in., solid contents of whole cone—solid contents of a ball that just goes into it.

(16) $\frac{1}{4}$ of 371.1015=92.7753 cu. in.=amount of water given in the vessel.

(17) 92.7753+53.0145=145.7898 cu. in. = amount of water given in the vessel+the solid contents of the small cone DCV.

(18) MN represents the surface of the water after the ball O is dropped in.

(19) Now by similar solids, we have 228.6733472 : 145.7898 :: $(7.2)^3$: (?) or GK, from which GK=6.196+ in., the diameter of the ball O.

NOTE.—If the ball O was 5 in. in diameter it would rest on the bottom of the vessel.

PROBLEM 443.

Find the diameter of the largest sphere that can be put in a hollow cone whose internal base is 2 ft. and altitude 3 ft.

Solution.

(1) Let MNV represent a middle section of the cone, and the inscribed sphere whose center is O.

(2) Then MN=2 ft., KV=3 ft., MV=$\sqrt{(3^2+1^2)}$=3.16227 ft.

(3) Dividing twice the area by the perimeter of MNV gives radius OG; or OG=(3×2)÷2+(3.16227×2)=.720705 ft.

(4) Hence, the diameter is 1.44141 ft.

PROBLEM 444.

A conical wine-glass whose base diameter is 6 in. and altitude 5 in., is filled with wine: if it be turned through an angle of 45°, how much wine will run out?

Solution.

(1) AB= diameter of the mouth, CF the depth and AMDN the surface of the wine.

(2) Draw CP perpendicular to AD; then the water remaining equals an oblique cone with altitude PC and base an ellipse whose axes are AD and NM.

(3) Draw FO and EG parallel to BC, and through O pass a plane perpendicular to CF.

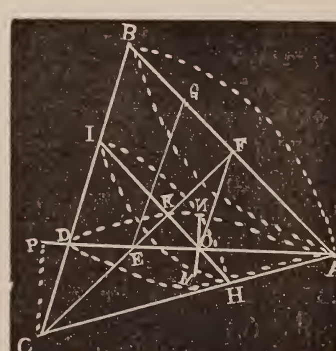

FIG. 113.

(4) The angle AEF=45°; then FE=AF=3 in., and CE=CF—FE=2 in.

(5) From AEF and CEP, AE : CE :: EF : CP=$\sqrt{2}$.

(6) From FCB and FGE, FC : FE :: FB : FG=$\frac{9}{5}$.

(7) AE=$\sqrt{(\overline{AF}^2-\overline{FE}^2)}=3\sqrt{2}$.

(8) AG=AF+FG=$4\frac{4}{5}$.

(9) From AGE and AOF, AG : AF :: AE : AO=$\frac{15}{8}\sqrt{2}$.

(10) From FBC and CIK, CF : CK :: FB : IK=$\frac{15}{8}$=HK.

(11) OH=HK—OK=$\frac{3}{4}$.

(12) From geometry, OM=$\sqrt{(OH\times IO)}=\frac{3}{2}$.

(13) AMDN=AO×MO×$\pi=\frac{45}{16}\pi\sqrt{2}$.

(14) The cone ACD is equal to $\frac{45}{16}\pi\sqrt{2}\times$ PC÷3=5.8905+ cu.in.

(15) Cone ABC—ACD=41.2335 cu. in., the amount of wine that will run out,

NOTE.—This solution was prepared for the *School Visitor* by Prof. R. H. Sunkle.

PROBLEM 445.

Required the largest cube that can be placed in a conical cup 20 in. deep and 12 in. in diameter.

Solution.

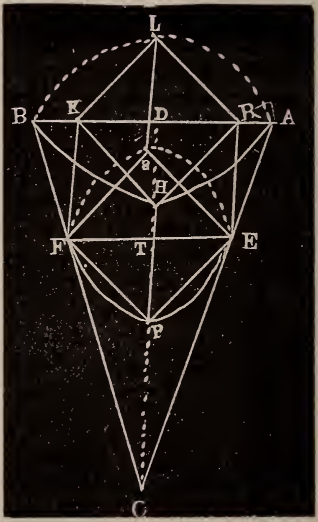

FIG. 114.

(1) Let ACB represent a section of the conical cup and cube, where EF represents the diameter of the cup on which the lower side of the cube rests and also the diagonal of the bottom of the cube.

(2) DC=20 in., DB=5 in.

(3) Let the edge of the cube=x.

(4) Then DC : DB :: TC : TF, or 20 : 6 :: 20—x : (120—6x)÷20.

(5) But the diagonal of the side of the cube is (120—6x)÷10 =$x\sqrt{2}$, whence we find x= 5.957+ in.

PROBLEM 446.

Find the maximum cylinder which can be inscribed in a conical cup 1 ft. deep and 10 in. in diameter.

Solution.

(1) Let ABC be the cone, and MN the inscribed cylinder.

(2) Let BD $=b$, AD $=a$, HD $=x$, FH $=y$, and the volume of the cylinder $=$V.

(3) We have $v=\pi y^2 x$. . . (1).

(4) From the similar triangles ADB and AHF, we have AD : BD :: AH : FH, or $a:b::a-x:y$.

(5) $\therefore y=\frac{b}{a}(a-x)$, which in (1) gives $v=\pi\frac{b^2}{a^2}(a-x)^2x$ . . (2).

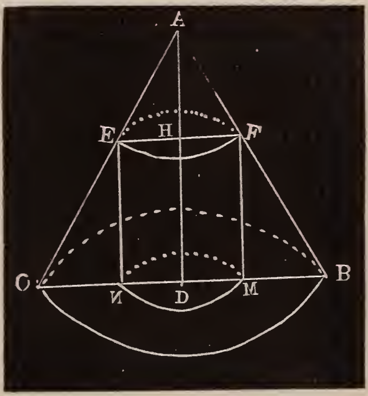

FIG. 115.

(6) Dropping constant factors, we have $u=(a-x)^2x=a^2x-2ax^2+x^3$.

(7) $\therefore \frac{du}{dx}=a^2-4ax+3x^2=0$, or $x^2-\frac{4}{3}ax=-\frac{1}{3}a^2$.

(8) $\therefore x=a$, or $\frac{1}{3}a$.

(9) $\frac{du^2}{dx^2}=-4a+6x$; $\frac{d^2u}{dx^2}=2a$, when $x=a$, $\therefore$ minimum.

(10) $\frac{d^2u}{dx^2}=-2a$, when $x=\frac{1}{3}a$, $\therefore$ maximum.

(11) Hence, the altitude of the maximum cylinder is $\frac{1}{3}$ of the cone.

(12) The second value of x in (2) gives $V=\frac{\pi b^2}{a^2}(a-\frac{1}{3}a)\frac{a}{3}=\frac{4}{27}\pi ab^2$.

(13) Volume of the cone $=\frac{1}{3}\pi ab^2$.

(14) $\therefore$ Volume of cylinder $=\frac{4}{9}$ volume of cone.

(15) $y=\frac{b}{a}(a-\frac{1}{3}a)=\frac{2}{3}b=$radius of base of cylinder.

(16) From the above the required result can easily be found.

PROBLEM 447.

From a cone, altitude 30 in. and radius of base 5 in., a 6-inch cylinder is cut as long as possible; from the top of the cone remaining another cylinder is cut, of the same length as the former and the thickest possible: how much of the volume of the original cone is cut away?

Solution.

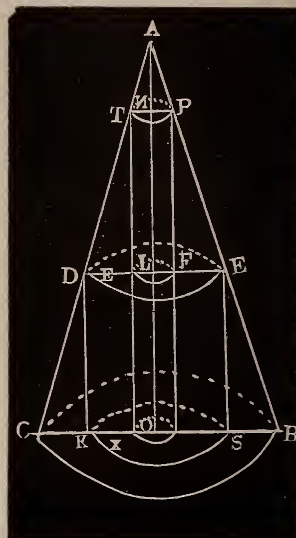

FIG. 116.

(1) Let ABC represent the cone, and SKDE and FETP the cylinders.

(2) AO=30 in., OC=5 in., LD=3 in.

(3) COA and DLA are similar triangles.

(4) We have CO : AO :: DL : AL, or 5 : 30 :: 3 : AL. From which AL=18 in.

(5) LO=30 in.—18 in.=12 in., the length of the cylinder SE, also the length of the cylinder FT.

(6) The triangles ALD and TED are similar.

(7) We have AL : LD :: TE: ED, or ED=2 in.

(8) LE=LD—ED=1 in., or radius of small cylinder.

(9) Volume of the large cylinder is $(3^2\pi\times12)=108\pi$ cu. in.

(10) Volume of the small cylinder is $(1^2\pi\times12)=12\pi$ cu. in.

(11) Volume of both cylinders$=108\pi+12\pi=120\pi$ cu. in.

(12) Volume of the cone ABC$=5^2\pi\times10=250\pi$ cu. in.

(13) Volume cut away$=250\pi-120\pi=130\pi$, or 408.4 cu. in.

PROBLEM 448.

What are the two dimensions of a conical vessel that contains 13 gallons of wine, the altitude being 12 in., and the area of the top being to the area of the bottom as 5 is to 3?

Solution.

(1) This problem is plainly solved by arithmetic.

(2) The volume of the vessel is 231×13, or 3003 cu. in., which is the product of the sum of the three basal areas and $\frac{1}{3}$ of the altitude of the frustum.

(3) Hence $3003\div4=750.75$ sq. in., sum of the areas of lower, upper and mean bases.

(4) Let these areas be as 3 : 5, and $\sqrt{15}=3.8729$.

(5) Then the lower base is $750.75\times(3\div11.8729)=189.695$ sq.in.

(6) The upper base is $750.75\times(5\div11.8729)=316.159$ sq. in.

(7) The lower diameter is $\sqrt{(189.695 \div .7854)} = 15.54$ in.

(8) The upper diameter is $\sqrt{(316.159 \div .7854)} = 20.07$ in.

PROBLEM 449.

How far must a fly walk on the shortest route from a lower to the opposite upper corner of a room 30 ft. long, 25 ft. wide and 15 ft. high? Locate the point where it leaves the floor.

Solution.

(1) Let AE be the room.

(2) AB=30 ft., the length.

(3) AG=25 ft., the width.

(4) AF=15 ft., the height.

(5) Let G be the corner at which the fly starts, and C the corner to which the fly travels.

(6) Suppose the side AFCB to be lowered as a door upon its hinges until it forms a plane with the floor.

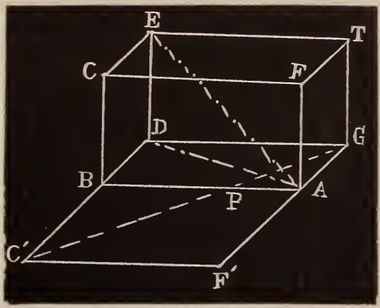

FIG. 117.

(7) The shortest path the fly can travel is on the line GC′.

(8) GF′=25+15=40 ft., F′C′=30 ft., and the triangles GAP and GF′C′ are similar.

(9) $GC' = \sqrt{(\overline{GF'}^2 + \overline{F'C'}^2)} = 50$ ft., the shortest path the fly can travel.

(10) GF′ : F′C′ :: GA : AP, or 40 : 30 :: 25 : AG, or 18¾ ft., the distance the fly leaves the floor from the corner A.

PROBLEM 450.

A cone 1 ft. high and 6 in. in diameter is perforated by a 2-inch hole, entering at the center of the base and passing through perpendicular to the base: find the part of the cone removed.

Solution.

(1) From Fig. 116, let ABC be the cone, and XP the 2-inch hole.

(2) AO=12 in., and OC=3 in.

(3) Then we have the similar triangles ANT and AOC.

(4) CO : OA :: TN : NA, or 3 : 12 :: 1 : NA, or 4 in.

(5) NO=AO—AN=8 in., the height of the cylinder.

(6) $8 \times 1^2 \times \pi = 8\pi$ = volume of the cylinder.

(7) Volume of the small cone APT is $\frac{1}{3}\times4\times1^2\times\pi=\frac{4\pi}{3}$.

(8) Total volume removed is $8\pi+\frac{4\pi}{3}$, or $\frac{28\pi}{3}$.

(9) Volume of the cone ABC is $\frac{1}{3}\times12\times3^2\times\pi=36\pi$.

(10) The total volume removed is $\frac{4\pi}{3}\div36\pi$, or $\frac{1}{27}$ part of it.

PROBLEM 451.

What is the distance from one lower corner to the opposite upper corner, of a box 12 ft. long, 6 ft. wide and 4 ft. deep?

Solution.

(1) From Fig 117, let ABF be the box.
(2) AB=12 ft., BC=4 ft., and BD=6 ft.
(3) ABD and ADE are right triangles, of which AD is the hypothenuse of the triangle ABD, and also the base of the triangle ADE.
(4) $\overline{AD}^2=\overline{AB}^2+\overline{BD}^2$.
(5) $AE=\sqrt{\overline{AD}^2+\overline{ED}^2}=14$ ft., the diagonal of the box.

PROBLEM 452.

A vine passes around a pole, which is 52 ft. long, and 3 ft. in circumference, once in every 4 ft. of its length: find the length of the vine.

Solution.

(1) One round of the vine forms the hypothenuse of a right-angled triangle, whose base is the circumference of cylinder, and perpendicular 4 ft.
(2) Hence each round takes $\sqrt{(3^2+4^2)}=5$ ft.
(3) Since there are $52\div4=13$, number of rounds, the vine must be $13\times5=65$ ft. long.

NOTE.—Each round or spire is equivalent to the hypothenuse of a right triangle. This may be shown by covering a pencil or cylinder with paper and tracing the position of the vine upon it. Then unwrap the paper from the cylinder, and the right triangles will be seen.

PROBLEM 453.

In a cubical room a line drawn from an upper corner to the opposite lower corner is 24 ft.: find the size of the room.

Solution.

(1) From Fig. 117, let BF be the room, and AE the diagonal.
(2) Let x=the edge.

(3) Then AD, the diagonal of base $=\sqrt{(\overline{AB}^2+\overline{DB}^2)}$, or $\sqrt{(x^2+x^2)}=\sqrt{2x^2}=x\sqrt{2}$.

(4) $\overline{EA}^2=\overline{AD}^2+\overline{ED}^2$, or $(x\sqrt{2})^2+x^2=3x^2$.

(5) $3x^2=576$, or $x^2=192$.

(6) $x=\sqrt{192}$, or 13.85+ ft.

PROBLEM 454.

If a cu. yd. of Klondike gold were formed into a bar $\frac{1}{2}$ in. square, without waste, what would be the length?

Solution.

(1) A cu. yd. contains 1728×27 cu. in. $=46656$ cu. in.

(2) The area of one end of bar is $(\frac{1}{2})^2$, or $\frac{1}{4}$ sq. in.

(3) The length must be $46656\div\frac{1}{4}=186624$ in.

PROBLEM 455.

A 16-inch tile will conduct as much water as how many 4-inch tiles?

Solution.

(1) Formula: Ratio of similar sides $=\sqrt{\text{(ratio of areas)}}$.

(2) The real quantity discharged is proportional to the areas of the ends of the tile, and since all circles are similar, we have from the above formula, $16^2\div4^2$, or 16 times as much.

PROBLEM 456.

If cloth for a suit of clothes for a man weighing 125 lb. costs $10, what will be the cost of enough of the same quality to make a suit for a man weighing 216 lbs., the men being similar in form and the suits similar in style?

Solution.

(1) Similar solids are to each other as the cubes, and the surfaces as the squares of their like dimensions, or their homologous lines.

(2) Therefore, we have $(\sqrt[3]{125})^2 : (\sqrt[3]{216})^2 :: \$10 : \$14.40$, cost of the second man's suit.

PROBLEM 457.

The surface of a pyramid is 560 sq. in: what is the surface of another similar pyramid whose volume is 9 times as great?

(*Putnam Co. Test.*)

Solution.

(1) From the above solution, $(\sqrt[3]{1})^2 : (\sqrt[3]{9})^2 :: 560 : 2420+$ sq. in.

PROBLEM 458.

The whole surface of a cylinder is 10 sq. yd.; the diameter equals the height: find the solidity.

Solution.

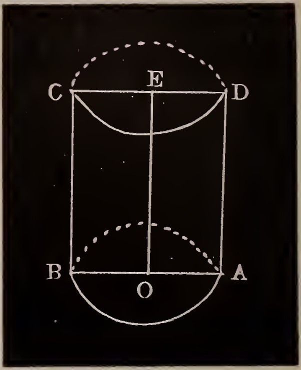

FIG. 118.

(1) Let AC be the cylinder, AB$=x$, and OE$=$CB$=x$.

(2) Area of the circle or base AOB $=\frac{1}{4}x^2\pi$, and the area of both bases$=\frac{1}{4}2x^2\pi$.

(3) $x\pi=$circumference of the base.

(4) $x^2\pi=$convex surface.

(5) $2(\frac{1}{4}x^2\pi)+x^2\pi=$whole surface.

(6) $\therefore 2(\frac{1}{4}x^2\pi)+x^2\pi=10$ sq. yd.

(7) $x^2=2.1220$ sq. yd., and $x=1.46+$ yd.$=$AB$=$OE$=$CB, the diameter and height.

(8) $(1.46+)^2\times .7854=1.665+$ sq. yd., area of base.

(9) $1.665+\times 1.46+=2.4236$ cu. yd., solidity of the cylinder.

PROBLEM 459.

I have three farms containing 192, 216 and 168 A. respectively. I wish to divide these into fields of equal size: how many fields in all, if they are the smallest possible?

Solution.

(1) The G. C. D. of 216, 192 and 168=24, the number of acres in a field.

(2) $168\div24=7$, $192\div24=8$, and $216\div24=9$ fields, respectively, making in all 24 fields.

(3) The side of each field is $\sqrt{24\times160}=56+$ rd.

PROBLEM 460.

I place a bowl into the storm,
To catch the drops of rain;
A half a globe was just its form,
Two feet across the same.
The storm was o'er, the tempest past,
I to the bowl repaired;
Six inches deep the water stood,
It being measured fair.
Suppose a cylinder, whose base
Two feet across within,
Had stood exactly in that place:
What would the depth have been?

Solution.

(1) AB=2 ft., the diameter of the globe.

(2) SKMP=segment of the sphere, or the amount of water in the half globe.

(3) PK=6 in., the depth of the water.

(4) $SK=\sqrt{\overline{SO}^2-\overline{KO}^2}$.

(5) Now as KO=6 in., $SK=\sqrt{12^2-6^2}=13.392+$.

(6) Volume of half the cylinder, $SY=\frac{1}{2}(SK^2\pi\times PK)=324\pi$.

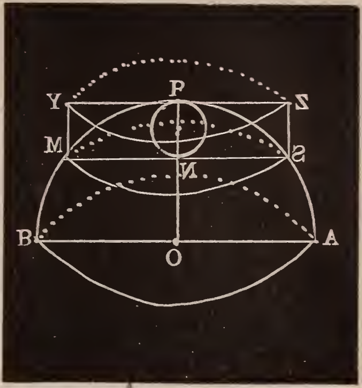

FIG. 119.

(7) Volume of the small sphere $PK=PK^3\times\frac{\pi}{6}=36\pi$.

(8) ∴ The volume of water in the vessel$=324\pi+36\pi=360\pi$.

(9) AO=the radius of the cylinder given in that data of the problem, then $\overline{SK}^2\pi=144\pi$, the area of the base of the cylinder, whose diameter is the same as the diameter of the half globe.

(10) $324\pi\div144\pi=2\frac{1}{2}$ in., the depth that the water would have been in the cylinder.

PROBLEM 461.

If a hole 12 in. in diameter be made through the center of a sphere whose diameter is 20 in., how many cu. in. of the sphere will be consumed?

Solution.

(1) Let EDMS be the sphere, and EFMS be the cylinder made by the auger.

(2) Then the volume bored out consists of a cylinder and two spherical sections, of which EDF, SMP and EFMS are vertical sections.

(3) SO=12 in., the radius of the sphere.

(4) SK=6 in., the radius

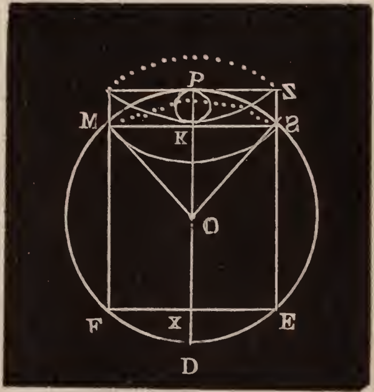

FIG. 120.

of the auger hole.

(5) PK=the diameter of the small sphere.

(6) $KO=\sqrt{(\overline{SO}^2-\overline{SK}^2)}$, or 8 in.

(7) PK=PO—KO=2 in.

(8) The volume of half the cylinder $SMYZ=\frac{1}{2}(\overline{SK}^2\pi\times KP)=36\pi$.

(9) The volume of the small sphere $PK=\overline{PK}^3\times\frac{\pi}{6}=\frac{4\pi}{3}$.

(10) $\left(36\pi+\frac{4\pi}{3}\right)\times2=\frac{224\pi}{3}=234.5728$ cu. in.=volume of both segments=EDX+SMP.

(11) Volume of the cylinder $SF=(\overline{EX}^2\times\pi\times KX)=1809.562$ cu. in.

(12) The amount consumed is 234.572+1809.562=2044.134+ cu. in.

Rule for finding a Spherical Segment.—*A spherical segment with one base is equivalent to half a cylinder having the same base and altitude, plus a sphere whose diameter is the altitude of the segment.*

PROBLEM 462.

By boring through the center, what is the diameter of an auger that will bore away one-half of a ball 6 inches in diameter?

Solution.

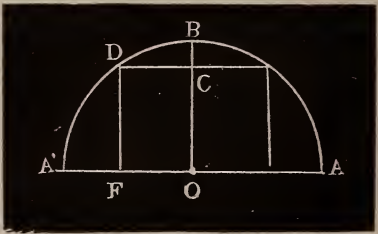

FIG. 121.

(1) Let O be the center of the ball.

(2) $OB=r=3$, $OF=CD=R$, the radius of the auger bit, and $CB=x$.

(3) Then $\overline{CD}^2=R^2=(2r-x)x$, and $CO=r-x$.

(4) The volume removed equals a cylinder, radius R, and height $2(r-x)$, together with two segments of the ball of height x, and radius R.

(5) The volume of the cylinder is $2(2r-x)\times\pi(r-x)$.

(6) The volume of the two segments is $\frac{1}{3}\pi x^2(6r-2x)$.

(7) Adding and equating to half the volume of ball $\frac{2}{3}r^2\pi$, we have $4x^3-12rx^2+12r^2x^2-4r^3=-2r^3$ . . . (1).

(8) $(1)\div4=x^3-3rx^2+3r^2x^2-r^3=-\frac{1}{2}r^3$ . . . (2).

(9) $\sqrt[3]{(2)}=x-r=r\sqrt[3]{\tfrac{1}{2}}$ . . . (3).

(10) $\therefore R^2=(2r-x)x=r^2(1-\tfrac{1}{2}\sqrt[3]{2})$ . . . (4).

(11) $2R=2r\sqrt{(1-\tfrac{1}{2}\sqrt{2})}=6\sqrt{(1-\tfrac{1}{2}\sqrt[3]{2})}$, diameter of auger bit, or 3.6496 in.

PROBLEM 463.

A blacksmith had an iron ball,
Which he did a bullet call;
He found out, by some nice trick,
It was exactly one foot thick;
And silvered o'er with plate,
And which he proved by scales and weight;
The silver's weight he did pronounce,
Was just exactly half an ounce;
He took the ball and made it hot,
And forged there out an iron rod;
The rod was just three inches thick,
Round and long, just like a stick;
Now how much silver will it take
To plate the rod that he did make,
And put it on exact as thick
As that which on the ball did stick?

Solution.

(1) The volume of the ball is $12^3\times\frac{1}{6}\pi$, or 288π cu. in.

(2) This is also the volume of the cylindrical rod.

(3) The area of the end of the rod is $(\frac{3}{2})^2\pi=2\frac{1}{4}\pi$.

(4) The length of the rod must be $288\pi\div2\frac{1}{4}\pi$, or 128 in.

(5) $3\pi\times128=384\pi$ sq. in., the convex surface of the rod.

(6) The basal surface is $2\frac{1}{4}\pi\times2=4\frac{1}{2}\pi$ sq. in.

(7) $384\pi+4\frac{1}{2}\pi=388\frac{1}{2}\pi$ sq. in., the surface of the rod.

(8) $12^2\times\pi=144\pi$ sq. in., the surface of the ball.

(9) We have by proportion, $144\pi : 388\frac{1}{2}\pi :: \frac{1}{2}$ oz. : ($1\frac{201}{576}$ oz.).

$\therefore$ It will take $1\frac{201}{576}$ oz. to plate the rod forged from the iron ball.

PROBLEM 464.

A leaden spherical shell, hollow part 6 in. in diameter and thickness of solid part 3 in., is converted into a three-inch pipe, outer measure 5 in.: find length of pipe.

Solution.

(1) Since the solid part is 3 in. thick, the shell is $6+3\times2=12$ in.

(2) Suppose this sphere to be solid; then its volume would be $12^3\times\frac{1}{6}\pi$, or 288π cu. in.

(3) The diameter of the hollow part is 6 in.; then the volume of the hollow part is $6^3 \times \frac{1}{6}\pi$, or 36π cu. in.

(4) $288\pi - 36\pi = 252\pi$, cu. in. of lead in the shell.

(5) The area of the lead pipe at one end is $(\frac{5}{2})^2\pi - (\frac{3}{2})^2\pi = \frac{16}{4}\pi$, or 4π sq. in,

(6) ∴ The length of the pipe is $252\pi \div 4\pi = 63$ in.

NOTE.—It would be well for the student to learn to use the character π instead of 3.141,592,653,989,793,238,462,643,383,279,502,884,197,169,-399,375,105 . . , for much labor can be saved, as in the above solution. For example, $\pi \div \pi = 1$. For practical purposes it is sufficiently accurate to call $\pi = 3.1416$, or 3.14159. Now, $3.1416 \div 3.1416 = 1$; $20\pi \div 2\pi = 10$.

104. The Origin of π.—At least from the time of Archimedes, π has stood for the number expressing how many times the diameter the circumference is. It is the initial letter of the Greek word περιφέρα, meaning periphery. If the diameter is taken as a unit, then π stands for the periphery, or circumference.

Prof. W. W. Beman, of the University of Michigan, says that π was first used to represent the number 3.141592 in Jones' *Synopsis Palmariorum*, London, 1706. This fact seems to have been overlooked by most writers on mathematics, as we find the statement that Euler was apparently the first to use π, Encyclopedia Britannica, ninth edition, (A. M. M.)

Archimedes, 250 B. C., the greatest mathematician of ancient times, proved that the value of π lies between $3\frac{1}{7}$ and $3\frac{10}{71}$, that is, between 3.1429 and 3.1408. Ptolemy, 150 B. C., used the value 3.1417. Metrus, of Holland, used the fraction $\frac{355}{113}$, which is correct to six places of decimals. Lambert, 1750 A. D., proved π incommensurable.

Many hare-brained visionaries still attempt to square the circle, and will continue to chase after this will-o'-the-wisp. Squaring the circle, that is, finding a square equal in area to a circle of given radius, has called out an untold amount of work, but all attempts have ended in total failure. The oldest known mathematical work, the *Rhind Papyrus* (c. 2,000 B. C.) contains the problem in the well known form, to transform a circle into a square of equal area. The writer of the papyrus, Ahmes, lays down the following rule: Cut off $\frac{1}{9}$ of a diameter and construct a square on the remainder; this has the same area as the circle. The value of π thus obtained is $(\frac{16}{9})^2 = 3.16$, not very inexact. Still farther from the correct value is that of $\pi = 3$, which is found in the Bible. I. Kings 7: 23 and II. Chron. 4: 2.

We see all this expended mental energy has been fruitless, but nevertheless has worked for advancement in the manifold realm of mathematics.

PROBLEM 465.

Four ladies own a ball of thread 3 in. in diameter: what portion of the diameter must each wind off in order to have equal shares of the thread?

Solution.

(1) Volume of the ball is $3^3\times\frac{1}{6}\pi=14.1372$ cu. in.

(2) $\frac{1}{4}$ of this is 3.5343 cu. in., the share of each lady.

(3) The last lady has a ball containing 3.5343 cu. in., whose diameter is the cube root of $(3.5343\div.5236)=1.89$ in.

(4) After the first and second have wound their shares off, the last two have a ball containing $(3.5343\times2)=7.0686$ cu. in.

(5) Its diameter is $\sqrt[3]{(7.0686\div.5236)}=2.38+$ in.

(6) Then 2.38—1.89=.49 in., third lady's share.

(7) After the first lady has wound her share off, the three remaining ladies will have a ball whose volume is $(3.5343\times3)=10.6029$ cu. in., and whose diameter is $\sqrt[3]{(10.6029\div.6236)}=2.72$ in.

(8) Then 2.72—2.38=.34 in., the second lady's share.

(9) 3—2.72=.28 in., first lady's share.

∴ The first winds off .28, the second .34, the third .49 and the fourth 1.89 in.

PROBLEM 466.

John has a ball of yarn 10 in. in diameter and a wooden ball 6 in. in diameter: how thick will be the yarn when wound on the wooden ball?

Solution.

(1) The volume of the ball of yarn is $10^3\times\frac{1}{6}\pi=\frac{1000}{6}\pi$ cu. in.

(2) The volume of the wooden ball is $6^3\times\frac{1}{6}\pi=36\pi$ cu. in.

(3) The combined volume of the balls is $\frac{1000}{6}\pi+36\pi=202\frac{2}{3}\pi$ cu. in.

(4) The diameter is $\sqrt{202\frac{2}{3}\pi\div\frac{1}{6}\pi}=10.6736$ in.

(5) The radius of the ball is $\frac{1}{2}$ of 10.6736=5.3368 in.

(6) The radius of the wooden ball is $\frac{1}{2}$ of 6=3 in.

(7) ∴ The thickness of the yarn is 5.3368—3=2.3368 in.

PROBLEM 467.

Find the volume of the largest square pyramid that can be cut from a cone 9 ft. in diameter and 20 ft. high?

Solution.

(1) The diameter of the cone is the diagonal of the largest

inscribed square, or the base of the required pyramid.

(2) Hence, the diagonal 9 squared, divided by 2, gives 40.5 sq. ft., the area of the base of the pyramid.

(3) 40.5 multiplied by $\frac{1}{3}$ of the altitude 20, gives 270 cu. ft., the volume required.

PROBLEM 468.

A cylindrical vessel 1 ft. deep and 8 in. in diameter was $\frac{13}{16}$ full of water; after a ball was dropped into the vessel it was full: find the diameter of the ball.

Solution.

(1) The area of the base is $4^2 \times \pi$, or 16π.

(2) The volume of the cylinder is $16\pi \times 12 = 192\pi$ cu. in.

(3) Now as the water occupied $\frac{13}{16}$ of the volume, the volume of the ball must have been $\frac{3}{16}$ of 192π, or 36π.

(4) The cube root of $(36\pi \div \frac{1}{6}\pi) = 6$ in., the diameter of the ball.

PROBLEM 469.

A circular table 6 ft. in diameter weighs 40 lb. and rests on four legs in its circumference, forming a square: find the least vertical pressure that must be applied to overturn it.

Solution.

(1) From Fig. 100, let ABCD be the circular table.

(2) Let A, B, C, D, be the places where the legs are attached.

(3) K is the point of application of pressure.

(4) $OL = 3\sqrt{2} \div 2 = 2.1213$ ft.

(5) $LK = 3 - 2.1213 = .8787$ ft.

(6) Let O be the center of gravity of the table; then, by a principle in Mechanics, all its weight may be considered to be at O.

(7) Then, L is the center of moments, and by the principle of moments we have, (Pressure required) $= \frac{LO \times 40}{LK} =$ 96.5+ lb.

NOTE.—From the data given, the legs are supposed to be of equal length, and also to be without weight.

PROBLEM 470.

It is desired that three men carry each $\frac{1}{3}$ of a 12 ft. log of uniform size and density. Where must the hand-stick be placed so that the one at the end of the log and the others at the ends of the stick, shall each carry equal weight? (*Hancock Co. Test.*)

Solution.

(1) When a body is supported at two points, the weight sustained at the two points respectively is inversely as their distance from the center of gravity, which in the log corresponds with its center, or 6 ft. from each end.

(2) The man at the end of the log is 6 ft. from its center.

(3) Now, that the two men at the hand-stick may carry $\frac{2}{3}$ of the log, or twice as much as the one at the end of the log, the hand-stick must be placed $\frac{1}{2}$ as far from the center of the stick, or $\frac{1}{2}$ of $6=3$ ft.

∴ The hand-stick must be placed 3 ft. from the end of the log.

NOTE.—This seems to be more of a mechanical than an arithmetical problem, for the above is solved by a well known principle of mechanics.

PROBLEM 471.

A string is wound spirally 100 times around a cone 100 ft. in diameter at the base: through what distance will a duck swim in unwinding the string, keeping it taut at all times, the cone standing on its base and at right angles to the surface of the water?

Solution.

(1) Let h=height, r=radius of base, $l=\sqrt{(h^2+r^2)}$= slant height, n= number of spirals.

(2) Let D, E be the two consecutive points in the duck's path.

(3) K,L=corresponding consecutive points of tangency of string to cone.

(4) F, G = corresponding consecutive points in base of cone.

(5) Let CD=s, FK=x.

(6) DE=ds, ML=dx.

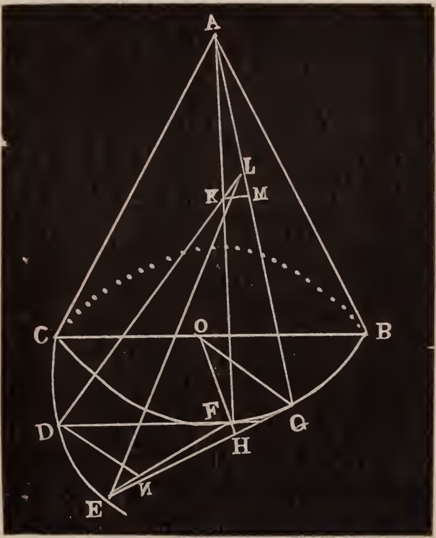

FIG. 122.

(7) $ds^2 = DN^2 + NE^2$ . . . (1).

(8) From the similar triangles GOF and DFE, r : GF=FD : DN.

(9) $\therefore$ DN=(FD.GF)$\div r$ . . . (2).

(10) From the triangles LMK and KFD, dx : MK=x : DF.

(11) FD=x.MK$\div dx$ . . . (3).

(12) MK : $(l-x)$=GF : l.

(13) MK=$[(l-x).GF]\div l$ . . . (4).

(14) But CF : $x=2\pi rn : l$.

(15) $\therefore$ GF : dx :: $2\pi rn : l$.

(16) GF=$(2\pi rn.dx)\div l$ . . . (5).

(17) (5) in (4) gives, MK=$[2\pi rn(l-x)dx]\div l^2$ . . . (6).

(18) (6) in (3) gives, FD=$[2\pi rnx(l-x)]\div l^2$ . . . (7).

(19) (7) and (5) in (2) give, PN=$[4\pi^2r^2n^2(l-x)xdx]\div rl^3$ (8).

(20) The increment of FD is, (GH+NE)=FG+NE.

(21) $\therefore$ From (7), d(FD)=$[2\pi rn(l-2x)dx]\div l^2$=(FG+NE) . . . (9).

(22) (5) in (9) gives, NE=$[-4\pi rnxdx]\div l^2$ . . . (10).

(23) (8) and (10) in (1) gives, $ds\frac{4\pi rnx}{l^2}\sqrt{1+\frac{\pi^2n^2}{l^2}(l-x)^2dx}$.

(24) $\therefore s=\frac{4\pi rn}{l^2}\int_o^l\sqrt{1+\frac{\pi^2n^2}{l^2}(l-x)^2}xdx=(4r/3\pi n)+$
$(2r/3)[\pi n-(2/\pi n)]\sqrt{1+\pi^2n^2}+2r\log(\pi n+\sqrt{1+\pi^2n^2})$.

(25) In the problem, $r=1$, $n=100$.

(26) $\therefore s=(1/75\pi)+\frac{2}{3}[100\pi-(1/50\pi)]\sqrt{1+10000\pi^2}+$
$2\log(100\pi+\sqrt{1+10000\pi^2})$.

$\therefore s=68948.771$ ft.

X. TETRAHEDRON.

105. A **Tetrahedron** is a solid having four equal faces, each of which is an equilateral triangle.

The area of one face is found by squaring its edge and multiplying by $\sqrt{3}=1.73205$. If the edge is 1, then the surface of the tetrahedron is $1^2\times\sqrt{3}\times4=6.928$.

Its volume is found by multiplying $\frac{1}{12}$ of the cube of an edge by $\sqrt{2}=1.4142$. Hence, the edge is 1; we have $1^3\times\frac{1}{12}\times\sqrt{2}=.11785$, the volume.

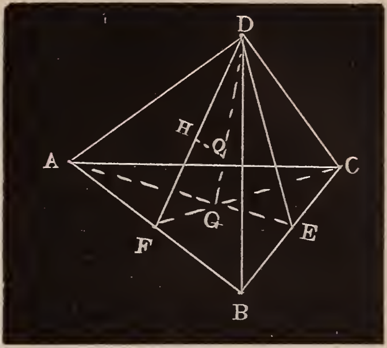

FIG. 123.

Now, let us find the altitude DG. $AB=1$, $AF=\frac{1}{2}$, $DF=\sqrt{(1^2-(\frac{1}{2})^2)}=\frac{1}{2}\sqrt{3}=$ $\frac{1}{2}$ of 1.732, or .866, which equals FC, also FG is $\frac{1}{3}$ of FC, or $\frac{1}{6}\sqrt{3}=.28866$. $DG=DF^2-FG^2$, or

$$\sqrt{[(\tfrac{1}{2}\sqrt{3})^2-(\tfrac{1}{6}\sqrt{3})^2]}=\sqrt{(\tfrac{3}{4}-\tfrac{1}{12})}=\sqrt{\tfrac{2}{3}}=\tfrac{1}{3}\sqrt{6}=.81649.$$

The center of the inscribed sphere is also the center of the tetrahedron, which is in DG at O, $\frac{1}{4}$ of DG from G is GO, or $\frac{1}{4}$ of $\frac{1}{3}\sqrt{6}=\frac{1}{12}\sqrt{6}=.20412$, the radius of the inscribed sphere.

CHAPTER XIII.

PROBLEMS.

1. If .63 gallons of wine cost \$1.47, what will $\frac{9}{20}$ of a gallon cost? *Ans.* \$1.05

2. James at college wrote to his parents that he had paid $\frac{7}{8}$ of $\frac{3}{4}$ of his debts, and still owed \$44: what were his debts? \$128.

3. If 4 pounds of cheese can be bought with 60 ct. when milk is 10 ct. a quart, how many pounds should be bought with 36 ct. when milk is 8 ct. a quart? 3 lb.

4. Bought $5\frac{1}{2}$ yd. of cloth at $\$4\frac{1}{8}$ a yard, and paid for it with wheat at $\$1\frac{4}{7}$ per bushel: how many bushels were required? 14 bu.

5. A and B can do a piece of work in 12 days, and A can do $\frac{2}{3}$ as much as B: in what time can each do it? A 30 days, B 20 days.

6. If $\frac{1}{3}$ of 6 be 3, what will $\frac{9}{10}$ of 40 be? 54.

7. A can build a wall in $18\frac{3}{4}$ days, B in $31\frac{1}{4}$ days: how long will it take both together? $11\frac{23}{32}$ days.

8. A man after doing $\frac{3}{5}$ of a piece of work in 30 days, engages an assistant and both together complete it in 6 days: in what time could the assistant do it alone? $21\frac{3}{7}$ days.

9. $\frac{3}{4}$ of $(2\frac{1}{2} \div \frac{2}{3}) + \frac{3}{5}$ of $\frac{5}{6}$ = what? $2\frac{47}{48}$.

10. When flour is \$6 a barrel, the 10-cent loaf weighs 30 oz.: what is the price of flour when it weighs 24 oz.? \$7.50 a bbl.

11. If $\frac{3}{4}$ of a yard of ribbon costs $\$\frac{5}{8}$, how many yards can be bought for $\$\frac{2}{3}$? $\frac{4}{5}$ of a yd.

12. If a street vender buy 5 bu. of chestnuts for \$18.50, and sells them at 15 ct. per liquid quart, how much does he gain? \$9.43.

13. A merchant owns $\frac{7}{11}$ of a store, and sells $\frac{3}{7}$ of his share for \$2100: what is the value of the part he still owns? \$2800.

14. A man owning $\frac{2}{5}$ of a mill, sold $\frac{2}{3}$ of his share for \$12000: what was the value of the mill? \$45000.

15. A, B and C do a piece of work for \$79.35: on the supposition that A and B do $\frac{3}{4}$ of the work, A and C $\frac{9}{10}$, and B and C $\frac{13}{20}$: what should each receive?
A \$27.60, B \$17.23 and C \$34.50.

16. Corda, being asked her age, replied: "My age! If it is multiplied by 3, and $\frac{2}{7}$ of the product tripled be, the square root of $\frac{2}{9}$ of that is 4: now, tell my age and never ask me more." 28 yr.

17. Four times $\frac{2}{3}$ of a number is 12 more than twice the number: what is the number? 18.

18. I had $\frac{2}{5}$ of my money stolen; I then earn \$60, and spend $\frac{2}{3}$ of all I now have; my uncle then gives me \$10, and after losing $\frac{1}{6}$ of what I have I find I have half as much money as I had at first: what had I at first? \$75.

19. Divide the L. C. M. of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{6}{7}$ by the G. C. D. of $83\frac{1}{3}$ and $268\frac{3}{4}$: what is the quotient? $28\frac{4}{5}$.

20. $6+4\times3-5\times2+6+2\times8\div4-9=$ what? 9.

21. $\frac{2}{3}$ of the square of $\frac{3}{4}$ of a number is 12 more than $\frac{3}{4}$ of the square of half the number: what is the number? 8.

22. Find the number, $\frac{2}{3}$ of whose cube is 10 more than the cube of its $\frac{2}{3}$. 3.

23. Squaring twice a number gives 18 more than twice its square: find it. 3.

24. Evert went $\frac{4}{7}$ of a journey on a bicycle, $\frac{5}{6}$ of the remainder by coach, and walked the last 3 miles: how long was the journey? 42 mi.

25. What is the number, to which if 15 be added and the sum multiplied by 9, and 11 taken from the product, the remainder will be 340? 24.

26. If a hen and a half could lay an egg and a half in a day and a half, how many eggs would 6 hens lay in 7 days?
28 eggs.

27. If 6 cats catch 6 mice in 6 minutes, at the same rate how many cats will it take to catch 100 mice in 100 minutes? 6 cats.

28. If an article had cost 10% less, my rate of gain would have been 15% more: what was my rate of gain? 15%.

29. What must be the asking price of cloth, costing $3.29 per yard, that I may deduct $12\frac{1}{2}$% and still gain $12\frac{1}{2}$%? $4.23.

30. Cloth marked at 30% above cost price, having depreciaıed in value, was sold at 28% off the marked price: what % did I gain or lose? I lost $6\frac{2}{5}$%.

31. John sold an article at 25% gain; had it cost him $1.25 more, he would have lost 30%: what was the cost? 1.59\frac{1}{11}$.

32. Had an article cost me 15% more, the same selling price would have given a rate of gain 12% less: find my rate of loss? 8%.

33. My merchant bought tea at 25% below wholesale cost, receiving a discount of 5% for cash payment, and sold at $12\frac{1}{2}$% above wholesale price: what per cent did he gain? $57\frac{17}{19}$%.

34. A merchant bought a piece of goods, which shrank $\frac{1}{10}$% both in length and width; he sold 7 yd. for what 6 yd. cost him: what was the loss %? $14\frac{13}{35}$%.

35. A's farm is worth 25% more than B's, and B's is worth 10% less than C's; if A should trade 75% of his farm for 80% of C's, he would lose $87.50: how much is B's farm worth? $1800.

36. If the cost of an article had been 8% less, the gain would have been 10% more: find the % gain. 15%.

37. If the cost had been $16\frac{2}{3}$% more, the gain would have been $18\frac{3}{4}$% less: what was the % of gain? $31\frac{1}{4}$%.

38. Bought 1000 bu. of corn at 16 ct. a bushel; if the loss by shrinkage and waste is 5%, for what must I sell per bushel to gain $12\frac{1}{2}$% on my investment? $.18947.

39. A mill is worth 5% less than a store, and the store 20% more than a house; the owner of the house has traded it for 75% of the mill, thereby losing $145: what was the value

of the store? Of the mill? Of the house?

Store $1200, mill $1140, house $1000.

40. I sell two horses for $430, gaining 20% on one and losing 20% on the other; I lose $20 by the transaction: find the cost of each. The first horse cost $175, the second $275.

41. I bought a certain number of hats at $2.50 each; I sold $\frac{2}{5}$ of them at 25% profit, and on the sale of the remainder I lost $15; my total gain was equal to 5%: find the number of hats. 120.

42. Jesse sold two horses for the same sum; on the one he gained 20%, and on the other he lost 20%; his whole loss was $25: what did each horse cost him?

The first cost $250, the second $375.

43. I sold an article at 20% gain; had it cost me $300 more, I would have lost 20%: find the cost. $600.

44. I gained 120% by selling rice at 8 ct. per pound: what did the rice cost per pound? $3\frac{7}{11}$ ct.

45. If $\frac{1}{2}$ an article be sold for the cost of $\frac{1}{3}$ of it, what is the rate of loss? $33\frac{1}{3}$%.

46. A merchant sold part of his goods at a profit of 20%, and the remainder at a loss of 11%; his goods cost $1000 and his gain was $100: how much was sold at a profit? 677.41\frac{29}{31}$.

47. At what price must an article which cost 30 ct. be marked, to allow a discount of $12\frac{1}{2}$% and yield a net profit of $16\frac{2}{3}$%? 40 ct.

48. Mr. Marble bought a lot of cassimeres at $3.75, and marked them "*n.no*$\frac{\tilde{r}}{o}$," his key being "John Marble": what was his gain % at the marked price? 68%.

49. Mr. Smith's key for marking goods was "Republican:" if he marked some silks "*e.bn*" and gained at that rate $11\frac{1}{9}$% on the cost, what was the cost per yard? $2.25.

50. Prof. Zerr loses 16% by selling his library for $960 less than it cost: what must he have received for it if he had gained 16%? $6960.

51. I remitted to my agent $508.95 to invest in potatoes, after deducting all expenses. He paid $46 for barrels, $38.60 for drayage, and charged $2\frac{1}{2}$% commission for buying: how

many barrels of potatoes did he buy at $2.25 per barrel?
184.

52. The Continental insured $\frac{2}{3}$ of A's machine shop at 3%; the Phœnix relieves them of $\frac{1}{2}$ the risk at 2%, and the Teutonia of $\frac{1}{4}$ of it at $1\frac{1}{2}$%. By fire the Phœnix lost $3250 more than the Teutonia: what was A's rate of loss on value of shop?
$35\frac{1}{3}$%.

53. A conical cup 6 in. deep and top 5 in. in diameter, is full of water; a ball is then dropped into it and is just immersed: how much water overflows? 19.30 cu. in.

54. A round table 10 ft. in diameter has four drop leaves of equal size, and when the leaves are down, the table is a square: find area of one leaf. 7.133 sq. ft.

55. A cylinder 60 ft. high and 4 ft. in circumference has a rope passing around it spirally, each round rising 3 ft.: how long is the rope? 100 ft.

56. Find the diagonal of a rectangular field containing $13\frac{1}{2}$ acres, whose length is to its breadth as 12 to 5.
Length 72 rd., breadth 30 rd., and diagonal 78 rd.

57. A 12-inch tile will conduct as much water as how many 3-inch tiles? 16.

58. What is the convex surface and whole surface of a right cone whose slant height is 66 ft. 8 in., radius of the base 4 ft. 2 in.?
Convex surface 870.7 sq. ft., whole surface 924.61 sq. ft.

59. In a right-angled triangle, having the side of the inscribed square 36 and the radius of the inscribed circle 20, to determine the triangle. Base 84, perpendicular 63.

60. The altitude of a triangle is 39 and base 21: required the side of the largest inscribed square. $13\frac{13}{20}$.

61. What must be the diameter of a round log 12 ft. long which, when squared, will contain 54 cu. ft.? $1\frac{1}{2}$ ft.

62. The area of a cube is 72 sq. in.: find its diagonal. 6 in.

63. The surface of a cube is 200 sq. in.: find the surface of its circumscribed sphere. 314.15926 sq. in.

64. The surface of a sphere that can just be enclosed in a cubical box is 400π sq. in.: find inner surface of the box.
2400 sq. in.

65. A ladder 20 ft. long stands 12 ft. from the base of a building, and the top rests against the wall 4 ft. below the eaves: find the height of eaves. 8 ft.

66. How many posts 7 ft. apart will it require to fence a rectangular lot containing 70756 sq. ft., the length of the lot being 4 times its breadth? 190.

67. A circular park is crossed by a straight path cutting off $\frac{1}{4}$ of the circumference; the part cut off contains 10 acres: find the diameter of the park. 150 rd., nearly.

68. If a post 4 ft. high casts a shadow 13 ft. in length, what must be the height of a post that will cast a shadow 125 ft. in length? $38\frac{1}{13}$ ft.

69. The perimeter of a right-angled triangle is 120 rd., and the radius of the inscribed circle is 10 rd.: find the sides.
30, 40 and 50 rd.

70. A lead cone is 4 in. in diameter and 1 in. high: find the diemeter of a bullet that can be made from it? 2 in.

71. The diameter of a spherical balloon is 25 ft.: how many square yards of silk were required to make it, and how many cubic feet of gas will be required to fill it? (*Putnam Co.*)
$218\frac{1}{6}$ sq. yd. of silk, and 8181.2 cu. ft. of gas.

72. If two boxes are similar, and one, with dimensions 2, 4 and 5 ft. respectively, holds eight times as much as the second, what are the dimensions of the second?
1, 2 and $2\frac{1}{2}$ ft.

73. A cylindrical cistern is 10 ft. in diameter and 20 ft. deep: give contents in barrels. 384.44 bbl.

74. Two circles have the same center. A chord, length 20 ft., is drawn across the large circle, and it will just touch the circumference of small circle. Required the area of the ring. 314.16 sq. ft.

75. Upon the diagonal of a rectangle 24 ft. by 110 ft., a triangle equivalent to the rectangle is constructed: find its altitude. $18\frac{6}{13}$ ft.

76. The ends of a cord 100 ft. long are fastened to stakes placed 80 ft. apart on level ground; a ring to which a sheep is tied plays freely on the cord: find the area of the ellipse over which the sheep can graze. 1500π sq. ft.

77. Three men, A, B and C, residing at the several vertices of a triangle, the sides of which are 65, 70 and 75 chains respectively, agree to build a school house the center of which shall be equally distant from the residences: what is the distance from each residence to the school house? 40.625 ch.

78. A tub of butter weighs 16 lb. on one end of a pair of balances, and 25 lb. on the other: find the correct weight?

20 lb.

79. An equilateral triangle has an area of one acre, and is to be enclosed by a fence: what is the length of each side?

19.2 rd.

80. How far will the head of a nail in the tire of a wheel move in driving 78.54 rd.? 100 rd.

81. What is the greatest amount of land that π rd. of rope will enclose? $\frac{1}{4}\pi$.

82. A tree 120 ft. high stands on the bank of a river 100 ft. wide: where must the tree break off so that it may remain connected at the point of breaking and its top just reach the opposite shore? $18\frac{1}{3}$ ft.

83. The length and breadth of a school room are as 8 to 5; but if it were 2 ft. longer and 2 ft. wider, the area of the floor would be 1600 sq. ft.: find dimensions. 30 and 48 ft.

84. A hawk and a dove are in the air, the hawk 150 ft. above the dove: if the dove flies straight forward in a horizontal line, and the hawk flies $1\frac{1}{4}$ times as fast directly towards the dove, how far will the hawk fly before it catches the dove?

The hawk flies $416\frac{2}{3}$ ft., the dove $333\frac{1}{3}$ ft.

85.

A wealthy man two daughters had,
And both were very fair;
To each he gave a tract of land,
One round, the other square.
At twenty shillings an acre just
Each piece its value had;
The shillings that did encompass each
For it exactly paid.
If 'cross a shilling be an inch,
Which had the greater fortune,
She that had the round or square?

86. If the sun crosses the equator at 5:20 a. m., Central Standard time, in what longitude does it cross if the sun is 10 min. slow? $7\frac{1}{2}°$ east of Greenwich.

87. If the sun crosses the equator at 7:10 a. m., Central Standard time, in what longitude does it cross if the sun is 10 min. slow? 20° west of Greenwich.

88. A mountain known to be 4 mi. high was seen from a ship 178 mi. distant: from this data calculate the diameter of the earth. 7917 mi.

89. A sells goods which cost him $160, at a certain rate of gain; B sells to C at the same rate of gain; C pays $360 for the goods: find the rate of gain for A and B. 50%.

90. In a circle 2 ft. in diameter, a five-pointed star is inscribed. The length of one side of the star is 8 in.: what area is covered by the star? 136.1 sq. in.

91. If a sphere, the diameter of which is 4 in., is depressed in a conical glass $\frac{1}{5}$ full of water, the diameter is 5 in., and altitude 6 in.: how much of the vertical axis of the sphere is immersed? .544 in., nearly.

92. The area of a field is 30 times its diagonal, and its sides are in the ratio of 3 to 4: find diagonal and sides.

Diagonal $62\frac{1}{2}$, sides $37\frac{1}{2}$ and 50.

93. Two trees 48 ft. and 60 ft. high, respectively, are standing 50 ft. apart: at what point between them must a boy stand that he may just fly his kite to the top of either tree?

12.04 ft.

94. In a store the "key" by which goods are marked was "panegyrist," the letters denoting the cost being above a line and those denoting the selling price below. Using this key as a guide, supply the blanks in the following, the gain in each being 20%: $\frac{—}{an}$, $\frac{pag}{—}$. $\frac{ps\frac{t}{y}}{an}$, $\frac{pag}{peg}$.

95. If the major axis of an ellipse is 200 ft., and the minor axis 120 ft., what is the distance between the foci? 160 ft.

96. A man sold two horses at equal prices; on one he gained 25%, and on the other he lost 25%; he lost by the transactions $20: find the cost of each horse.

$120 and $200.

97. A street passes through a semicircular arch under a railroad. If it would take $777\frac{3}{5}$ perches of masonry to fill up the entire archway, it being forty feet long, how wide is the street? 35 ft.

98. What annual rate of interest on the net sales is equivalent to a discount of 5% on the gross, if the term of credit is 60 days? $31\frac{11}{19}$%.

99. What % in advance of the cost must a merchant mark his goods, so that after allowing 4% of his sales for bad debts, an average credit of 3 months, 10% of the cost of his goods for expenses, he may make a clear gain of 14% of the first cost of the goods, money being worth 8%? $31\frac{3}{4}$%.

100. The difference between the simple interest and the true discount for 4 years is $\$11\frac{2}{3}$, and for 8 years is \$40: what is the rate % and sum of money? 5% and \$350.

101. If the remainder is 17 and the dividend 4352, what is the divisor and the quotient, if the remainder is the G. C. D. of the divisor and quotient? 51 and 85.

102. How much more water will a tile 6 in. in diameter discharge per hour than one 4 in. in diameter? $2\frac{1}{2}$ times.

103. A merchant marks his goods at 40% in advance of cost, and selling uses a pound weight $\frac{1}{8}$ oz. too light; if he throws off 10% of his marked price, find the gain per cent. $26\frac{126}{127}$%.

104. Ten men hire a coach; by getting 4 more passengers the expense to each is diminished $\$1\frac{1}{5}$: what do they pay for the coach? \$42.

105. A well is 150 ft. deep and 110 ft. in diameter: if it is one-third full of water, how much work is required to empty it? 30679598.42 ft. lb.

106. In the midst of a meadow, well covered with grass,
Just an acre was needed to tether an ass;
How long was a line that reaching all 'round,
Restricted its grazing to an acre of ground?
39.2507 yd.

107. If the area of an equilateral triangle is 300, what is the side? 26.32.

108. A coin is held 5 ft. from a wall and parallel to it; a luminous point 15 in. from the coin throws a shadow of it upon the wall: how does the size of the shadow compare with that of the coin? The shadow is 25 times the coin.

109. The hypothenuse of a right-angled triangular piece of land containing 15 acres is 100 rd.: what is the length of the sides? 60 and 80 rd.

110. A boy is running on a horizontal plane directly towards the foot of a tree 50 ft. in height: when he is 100 ft. from the foot of the tree, how much faster is he approaching it than the top? 1.118 times.

111. Find the cost of 20 planks, each 16 ft. long, 10 in. wide, and $2\frac{1}{2}$ in. thick, at $15 per M. $10.

112. I have a circular piece of ground 160 ft. in diameter, from which I take enough earth to fill a hollow cone, whose base is 16 ft. in diameter and whose depth is 30 ft.: how much is the lot lowered? $\frac{1}{10}$ ft.

113. How many quart, pint and half-pint bottles of each an equal number, can be filled from a vessel containing 8 gal. 3 qt.? 20 bottles of each

114. From the middle of the side of a square 10-acre field I run a line cutting off $3\frac{7}{8}$ acres: required the length of the line. 41 rd.

115. What shall I pay for a bond of $100, having 5 years to run, with interest at 6%, in order to make it an 8% investment? $92.86.

116. A and B shot a deer, and wishing to ascertain its weight, they procured a pole and poised it on a log—A on one end weighing 150 lb., and B on the other weighing 180 lb., after which A took the deer and exchanged places with B, when the pole again balanced: find the weight of the deer? 66 lb.

117. A carpenter has to put rafters, 13 ft. long and 6 in. thick, upon a frame 24 ft. wide. He does not want to cut the edge of the frame, and yet must have just 13 ft. from their edges to the angle of the roof. How long must be the under side of each rafter? 11 ft. 7.1 in.

118. A teacher had his wages increased each term by $33\frac{1}{3}\%$ for three succeeding terms of eight months; he now gets $64 per month: find his salary per month the first term. $27.

119. A broker sold a farm for $520, charging a certain rate of commission, and invested the proceeds, less his charges of 4% on purchase price, in city property: if his entire commission was $450, find his charges on sale of farm? 86%.

120. The aggregate face value of two notes is $108, and each has two years to run; I have both discounted at 8%, one by bank discount and the other by true discount, realizing $92: find face of each note. $50 and $58.

121. Alta and Marie bought a basket of oranges at the rate of 3 for 2 ct., and gained 50 ct. by selling them at the rate of 2 for 3 ct.: how many oranges did they buy? 60.

122. Invested $270 in stock at 25% discount, which pays 8 % annual dividends: how much must I invest in stock at 4% discount and paying 10% annual dividends, to secure an equal income? $2764.80.

123. Bought some turkeys for $26; had I bought 8 more at 10 ct. less each, all would have cost $35.60: how many did I buy? 20.

124. One agent sold a house for $3500 at 1%; another sold a farm for $100 more, and sent me $45 less than the first: what rate did the second charge? 5%.

125. What sum of money must be invested in 4% bonds, selling at 125, to give an annual income of $2500? What rate of interest is received on the investment? $78125, 3.2%.

126. A man bought a farm for $6000, and agreed to pay principal and interest in three equal annual installments: what was the annual payment, interest being 6%? $2244.658.

127. If the time past noon is $\frac{1}{2}$ of the time to midnight, what is the hour? 4 p. m.

128. If a man weighs 160 lb. avoirdupois, what will he weigh by Troy weight? $194\frac{4}{9}$ lb.

129. Duck island is 73 mi. in circumference. If two men set out from the same point in the same direction, the one

traveling at the rate of 5 mi. an hour and the other at the rate of 3 mi. an hour, in what time will they again be together?
$36\frac{1}{2}$ hr.

130. Charles Draper had a 6% bond of $800, dated Jan. 1, 1898, due Jan. 1, 1899. On July 1, 1898, he sold the bond to G. K. Joseph in such a way as to give him 8% on his investment. If Mr. Joseph borrowed the money needed to pay the note, from the bank at 10% for 90 days, what was the face of the bank note? $837.007.

131. The width of a rectangular field is $\frac{3}{4}$ of the length, and its diagonal measures 30 rd: find its area. 432 sq. rd.

132. What is the length of a cylinder 6 in. in diameter, whose solidity is the same as a 12-inch sphere? 32 in.

133. A walks 9 hr. a day and B 8 hr. a day; B's rate is 20% faster than A's: how long will it take A to descend a hill which B descends in 5 days, provided a man's rate is increased $33\frac{1}{3}$% in going down hill and decreased 25% up hill?
27 hr., or 3 da.

134. Find the volume of a pyramid cut from a cube 4 in. from a corner on each edge. $10\frac{2}{3}$ cu. in.

135. If I buy uncurrent bank notes at 10% discount, brokerage $2\frac{1}{4}$%, and sell them at par, thus gaining $348.75, what is the face value? $4500.

136. Two poles 80 ft. and 120 ft. high, respectively, are planted on the ground 100 ft. apart: if lines be drawn from the tops of the poles to the opposite bottoms, what is the distance from the ground to where the lines cross each other?
48 ft.

137. $429.30. Cleveland, O., April 13, 1873.

On demand, I promise to pay W. Jones, or order, four hundred and twenty-nine $\frac{30}{100}$ dollars. Value received, with interest at 6%. R. SMITH.

Indorsed: Oct. 2, 1873, $10; Dec. 8, 1873, $——; July 17, 1874, $200.

The exact sum due Jan. 1, 1875, was $195. Restore the lost figures of the second payment. $59.97.

138. An apple is suspended by a string from the ceiling until it comes within 4 ft. of the floor; it is swung back 6 ft.,

when it is 8 ft. from the floor: what is the height of the ceiling? $10\frac{1}{2}$ ft.

139. The area of an equilateral triangle whose base falls on the diameter and its vertex in the middle of the arc of a semicircle, is equal to 100: what is the diameter of the semicircle? 26.3214.

140. Find the volume of a rectangular piece of ice 10 in. long, 6 inches wide, and floating in water, with $\frac{1}{2}$ inch of its thickness above water, specific gravity being .9. 300 cu. in.

141. Goliath of Gath weighed 1015 lb: what was his height, if a man 5 ft. 10 in. in height and of similar proportions weighs 180 lb.? 10 ft. 4.6 in.

142. A house which cost $5400 rents for $30 per month: what is the rate per annum of interest received on the investment if the average annual expenses are $144? 4%.

CHAPTER XIV.

RULES OF MENSURATION.

RHOMBUS.

1. Area=half the product of its diagonals.
2. The sides intersect at right angles.
3. Side=$\sqrt{(\text{sum of } \frac{1}{2} \text{ the diagonals squared})}$.

CUBE.

1. Volume=$(\text{edge})^3$.
2. Diagonal=$\sqrt{(\text{edge}^2 \times 3)}$, or $\sqrt{(\text{area} \div 2)}$.
3. Area=$\text{edge}^2 \times 6$, or $\text{diagonal}^2 \times 2$.
4. All cubes are similar solids.

PYRAMID.

1. Volume=area of base$\times\frac{1}{3}$ of the altitude.
2. Lateral area=perimeter$\times\frac{1}{2}$ slant height.

FRUSTUM OF PYRAMID.

1. Volume is found by a well known rule.
2. Lateral area=half the sum of basal perimeter$\times$slant height.

CIRCLE.

1. Area=$r^2 \times \pi$, $d^2 \times \frac{1}{4}\pi$.
2. Circum.=$\sqrt{(\text{area} \times \pi)} \times 2$.
3. Radius=$\sqrt{(\text{area} \div \pi)}$.
4. All circles are similar.

CYLINDER.

1. Volume=area of base$\times$the altitude.
2. Curved surface=circumference of base$\times$altitude.

SPHERE.

1. Volume$=d^3\times\frac{1}{6}\pi$, or .5236.
2. Area$=d^2\times\pi$.
3. Diameter$=\sqrt[3]{(\text{vol.}\times\frac{1}{6}\pi)}$, or (area$\div\pi$).

CONE.

1. Volume=area of base$\times\frac{1}{3}$ of altitude.
2. Curved surface=circumference of base$\times\frac{1}{2}$ slant height.
3. Altitude$=\sqrt{(\text{slant height}^2-\text{radius}^2)}$.

MISCELLANEOUS.

1. Diameter$\times$.7071=side of inscribed square.
2. Area$\times$.6366=side of inscribed square.
3. To find the distance a nail-head in a revolving wheel moves, multiply the distance traveled by 4 and divide by π.

POINTS FOR THE STUDENT.

106. Is the quotient always an abstract number?

Show this in the following: Divide 10 apples equally among 5 boys. The quotient is not always an abstract number. It is only true when the dividend and divisor are both of the same denomination. When the divisor is abstract, the quotient is the same as the dividend. Hence, 10 apples$\div$5= 2 apples. And the world of to-day says, \$20$\div$2=\$10.

107. Take a board 8 in. square, and it will contain 64 sq. in.; now saw it so that it will make a rectangle 5×13, and it will contain 65 sq. in. Also, a board 10 in. square, saw it and place it in a rectangular shape 6×16, and it will have but 96 sq. in. Explain the paradoxes.

The fallacy in these propositions consists in the fact that when the square of 8 in. is cut as indicated, the two parts will not make a rectangle, but a figure whose sides are not exactly straight; that is, the sides are not rightly in line. To make the sides straight as inferred in the rectangle 5×13, there would be a gap in the cut sides, equal to a square inch. Similarly in the 10-inch square, the cut sides would overlap to make a true rectangle, and, therefore, embrace a surface of 4 sq. in. less.

(*School Visitor.*)

JOURNALS.

The Ohio Teacher: Edited and published by John McBurney, Cambridge, O.; published monthly; price, 75 cents per year. This is one of the best educational journals in the state. It has rendered an untold amount of good to hundreds of teachers. Every energetic teacher should have this journal on his desk.

The American Mathematical Monthly: Edited by B. F. Finkel, A. M., and J. M. Colaw, A. M., Springfield, Mo.; price, $2.00 per year. This journal is devoted entirely to mathematics. The best mathematicians in the world contribute to its pages. It ranks as one of the foremost now published.

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